Production of leptonic bound states in electron – positron annihilation

Fedor Martynenko (Samara University)
A. P. Martynenko, A. V. Eskin

7th International Conference on Particle Physics and Astrophysics
National Research Nuclear University " MEPhI ".

22 - 25 October 2024

Introduction. "Leptonium" systems.

Leptons of opposite electric charge ($l^\pm = e^\pm,\ \mu^\pm,\ \tau^\pm$) can form short - lived bound states under their QED interactions known as "leptonium".

- Positronium (e^+e^-)
  - M. Deutsch, Phys. Rev. 82, 455 (1951).
- Dimuonium ($\mu^+\mu^-$)
- Ditauonium ($\tau^+\tau^-$)
- Muonium ($e^\pm\mu^\mp$)
  - V. Hughes, D. McColm, K. Ziock, and R. Prepost, Phys. Rev. Lett. 5, 63 (1960).
- Tauonium ($e^\pm\tau^\mp$)
- Mu - tauonium ($\mu^\pm\tau^\mp$)

In this work we study dimuonium ($\mu^+\mu^-$) and ditauonium ($\tau^+\tau^-$) production.

- $M(\mu^+\mu^-) = 2m_\mu + E_{bind} = 211.3181576 \pm 0.0000046\ MeV$
- $E^{1S}_{bind} = -\frac{m_\mu \alpha^2}{4n^2} = -1.41\ keV$
- $\tau^{{}^1S_0,\ \mu^+\mu^-} = 0.6\ ps$, $\tau^{{}^3S_1,\ \mu^+\mu^-} = 1.8\ ps$
  - R. Gargiulo et al, J. Phys. G: Nucl. Part. Phys. 51 045004 (2024).
- $\tau^{\mu^\pm} = 2.1970\ \mu s$
- $M(\tau^+\tau^-) = 3553.88 \pm 0.18\ MeV$
- $E^{1S}_{bind} = -23.66\ keV$
- $\tau^{{}^1S_0,\ \tau^+\tau^-} = 27.60\ fs$, $\tau^{{}^3S_1,\ \tau^+\tau^-} = 20.83\ fs$
  - D. d'Enterria, R. Perez-Ramos, and H.-S. Shao, The EPJ C 82, 923 (2022).
- $\tau^{\tau^\pm} = 290.3\ fs$

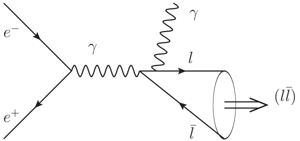
S. Navas et al. (Particle Data Group), Phys. Rev. D 110, 030001 (2024).

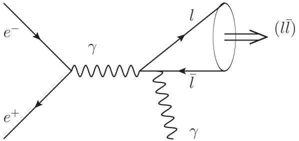
Introduction. e^+e^- accelerators.

- BES III: $\sqrt{s} \approx 3.55,\ 3.78\ GeV$, $\mathcal{L}_{int} = 20\ fb^{-1}$.
  - M. Ablikim et al 2020 Chinese Phys. C 44 040001.
- Future Super - Tau - Charm - Factory (STCF): $\sqrt{s} \approx m_\tau$, $\mathcal{L}_{int} = 1\ ab^{-1}$.
  - Zhou X. et al., 10th International Workshop on Charm Physics. P. 7 (2021).
- Belle II experiment at Super-KEKB: $\sqrt{s} \approx 10.6\ GeV$, $\mathcal{L}_{int} = 50\ ab^{-1}$.
  - Belle II Collaboration, W. Altmannshofer, et al., PTEP 2019, 123C01 (2019).
- Future circular collider (FCC-ee): $\sqrt{s} \approx 91.2\ GeV$, $\mathcal{L}_{int} = 50\ ab^{-1}$.
  - Abada, A., Abbrescia, M., AbdusSalam, S.S. et al., Eur. Phys. J. Spec. Top. 228, 261–623 (2019).
- Future linear e^+e^- (ILC, CLIC) and $\mu^+\mu^-$ colliders.
  - A. Aryshev, et al., arXiv:2203.07622[physics .acc -ph].
  - O. Brunner, et al., arXiv:2203.09186[physics .acc -ph].
  - N. Bartosik et al., 2020 JINST 15 P05001.
- Bogomyagkov, A., Druzhinin, V., Levichev, E., Milstein, A., Sinyatkin, S., EPJ Web of Conferences 181, 01032 (2018).

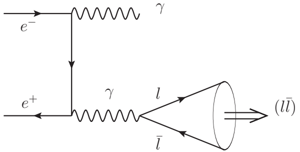
Diagrams of single leptonium production

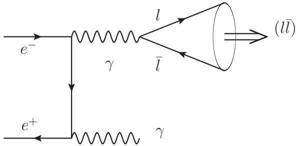
Diagrams of one - photon e^+e^- annihilation with production of $(^1S_0)$ leptonium + photon





Diagrams of two - photon e^+e^- annihilation with production of $(^3S_1)$ leptonium + photon





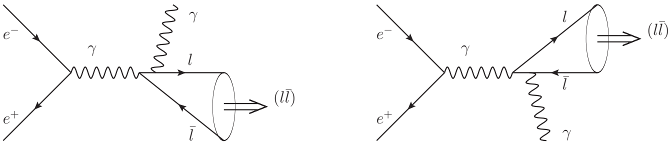
Brodsky S. J., Lebed R. F., Phys. Rev. Lett., 102, 213401 (2009)

Baier V. N., Synakh V. S., SOVIET PHYSICS JETP, 14 1122 (1962)

Baier V. N., Fadin V.S., Khoze V.A., Kuraev E.A., Phys. Rep., 78, 293 (1981)

Diagrams of single leptonium production

Diagrams of one - photon e^+e^- annihilation with production of $(^1S_0)$ leptonium + photon



The kinematics of the process in the center - of - mass reference frame

We express the momenta of produced leptons $p_{1,2}$ in terms of total momentum of bound state P and relative momentum of leptons p.

$$p_{1,2} = \frac{1}{2}P \pm p, \quad (p \cdot P) = 0, \quad P = (E_P, \boldsymbol{P}), \quad |\boldsymbol{P}| = \frac{s^2 - M^2}{2s}, \quad E_P = \frac{s^2 + M^2}{2s},$$

$$p_+ = (E_e, \boldsymbol{p_e}), \quad p_- = (E_e, -\boldsymbol{p_e}), \quad |\boldsymbol{p_e}| = \frac{1}{2}\sqrt{s^2 - 4m_e^2}, \quad E_e = \frac{s}{2},$$

where $s = p_+ + p_-$ is center - of - mass energy, $\boldsymbol{P}$ and E_P are momentum and energy of leptonium, $\boldsymbol{p_e}$ and E_e are momentum and energy of initial electron. m_e is electron mass, $M = 2m - \frac{m\alpha^2}{4}$ is leptonium mass.

Diagrams of paired leptonia production

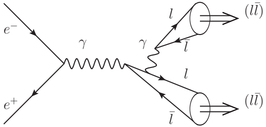
Diagrams of one - photon electron - positron annihilation with pair production of leptonia.

Diagrams (a)-(b)

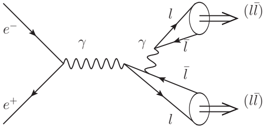
$e^+e^- \to {}^3S_1 + {}^3S_1,\ {}^3S_1 + {}^1S_0$

Diagrams (c)-(d)

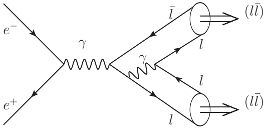
$e^+e^- \to {}^3S_1 + {}^3S_1,\ {}^1S_0 + {}^1S_0,\ {}^3S_1 + {}^1S_0$



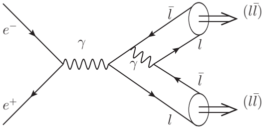
(a)



(b)



(c)



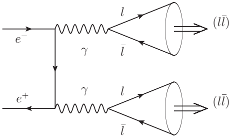
(d)

Diagrams of paired leptonia production

Diagrams of two - photon electron - positron annihilation with pair production of leptonia.

Diagram (a)

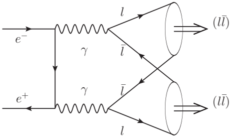
$e^+e^- \to {}^3S_1 + {}^3S_1$



(a)

Diagram (b)

$e^+e^- \to {}^3S_1 + {}^3S_1,\ {}^1S_0 + {}^1S_0,\ {}^3S_1 + {}^1S_0$



(b)

The kinematics of the process in the center - of - mass reference frame for identical final leptonia

$$p_{1,2} = \frac{1}{2}P \pm p, \quad (p\cdot P) = 0, \quad q_{1,2} = \frac{1}{2}Q \pm q, \quad (q \cdot Q) = 0,$$

$$P = (E_P, \boldsymbol{P}), \quad Q = (E_Q, -\boldsymbol{P}), \quad |\boldsymbol{P}| = \frac{1}{2}\sqrt{s^2 - 4M^2}, \quad E_P = E_Q = \frac{s}{2},$$

$$p_+ = (E_e, \boldsymbol{p_e}), \quad p_- = (E_e, -\boldsymbol{p_e}), \quad |\boldsymbol{p_e}| = \frac{1}{2}\sqrt{s^2 - 4m_e^2}, \quad E_e = \frac{s}{2}.$$

Methods

We calculate the cross sections of single and paired leptonia production in the framework of quantum electrodynamics with the account of relativistic effects.

- The production cross section:

$$\sigma_{2\to 2} = \frac{1}{64\pi^2 s^2}\frac{|\boldsymbol{P}|}{|\boldsymbol{p_e}|}\int |M|^2 d\Omega,$$

- The production amplitude can be presented as a convolution of a perturbative production amplitude of lepton, antilepton and a photon, with the wave functions of bound state (for single leptonium production):

$$M = \int \frac{d\boldsymbol{p}}{(2\pi)^3}\psi_{(l\bar{l})}(p,P)\mathcal{M}_{e^+ + e^- \to l + \bar{l} + \gamma}(P,p,p_\gamma)$$

- Relativistic wave function of leptonium after the Lorentz transformation to the reference frame of leptonium, moving with total four - momenta P:

Faustov, R.N., Ann. Phys., 78, 176–189 (1973)

S. J. Brodsky, J. R. Primack, Ann. Phys. 52, 315 (1969)

$$\psi_{(l\bar{l})}(p,P) = \Psi_{(l\bar{l})}(\boldsymbol{p})\frac{\varepsilon(\boldsymbol{p})+m}{2\varepsilon(\boldsymbol{p})}\left[\frac{\hat{v}_1 - 1}{2} - \hat{v}_1\frac{p^2}{2m(\varepsilon(\boldsymbol{p})+m)} - \frac{\hat{p}}{2m}\right]\frac{\gamma_5(\hat{v}_1+1)}{2\sqrt{2}}\times$$
$$\left[\frac{\hat{v}_1+1}{2} - \hat{v}_1\frac{p^2}{2m(\varepsilon(\boldsymbol{p})+m)} + \frac{\hat{p}}{2m}\right]$$

$$v_1 = P/M, \quad \varepsilon(\boldsymbol{p}) = \sqrt{\boldsymbol{p}^2+m^2}, \quad \Psi^{1S}_{(l\bar{l})}(p) = \frac{8\sqrt{\pi}W^{5/2}}{(p^2+W^2)^2}, \quad W = \frac{m}{2}\alpha$$

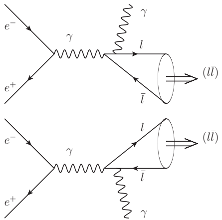
Single leptonium production

The production amplitude of single leptonium production in one - photon annihilation mechanism

$$M_{e^++e^-\to\gamma\to(l\bar{l})+\gamma} = \frac{(ie)^3[\bar{v}(p_+)\gamma^\mu u(p_-)]}{s^2}\int\frac{\Psi_{(l\bar{l})}(\boldsymbol{p})d\boldsymbol{p}}{(2\pi)^3}\frac{\varepsilon(\boldsymbol{p})+m}{2\varepsilon(\boldsymbol{p})}\times$$

$$Tr\Big\{\Big[\frac{\hat{v}_1-1}{2}-\hat{v}_1\frac{p^2}{2m(\varepsilon(\boldsymbol{p})+m)}-\frac{\hat{p}}{2m}\Big]\frac{\gamma_5(\hat{v}_1+1)}{2\sqrt{2}}\Big[\frac{\hat{v}_1+1}{2}-\hat{v}_1\frac{p^2}{2m(\varepsilon(\boldsymbol{p})+m)}+\frac{\hat{p}}{2m}\Big]$$

$$\Big(\frac{\gamma^\alpha(\hat{r}-\hat{p}_2+m)\gamma^\mu}{(r-p_2)^2-m^2}+\frac{\gamma^\mu(-\hat{r}+\hat{p}_1+m)\gamma^\alpha}{(r-p_1)^2-m^2}\Big)\Big\}\varepsilon_\gamma^\alpha,$$



Projection operators on the singlet and triplet states:

$$\Pi_{S=0}=[v(0)\bar{u}(0)]_{S=0}=\frac{\gamma_5(\hat{v}_1+1)}{2\sqrt{2}},\quad \Pi_{S=1}=\frac{\gamma^\alpha(\hat{v}_1+1)}{2\sqrt{2}}\varepsilon_\alpha$$

Polarization summation for real photons

$$\sum_{polarization}\varepsilon_\gamma^\alpha\varepsilon_\gamma^\beta=-g^{\alpha\beta}$$

For trace calculation we use FORM package

Ruijl B., Ueda T., Vermaseren J. FORM version 4.2 //arXiv preprint arXiv:1707.06453. – 2017.

Corrections

We highlight three types of corrections:

- Corrections due to taking into account binding energy $E_{bind} = -\frac{m\alpha^2}{4}$
- Relativistic corrections in the production amplitude $\sim \left(\frac{\mathbf{p}}{m}\right)^2 \approx \alpha^2$
- Relativistic corrections to the bound state wave function

Relativistic corrections in the production amplitude are determined by the powers of the relative momentum $\mathbf{p}/m$, $|\mathbf{p}| \sim W$, $W = \frac{m}{2}\alpha$. We introduce special relativistic parameters ω for them. Parameters ω are determined by momentum integrals with Coulomb wave function:

$$\frac{|\mathbf{p}|}{2m} = \sum_{n=1}^{\infty} \left(\frac{\varepsilon(p) - m}{\varepsilon(p) + m}\right)^{n+\frac{1}{2}}, \quad \psi^C_{1S}(p) = \frac{8\sqrt{\pi}W^{5/2}}{(p^2 + W^2)^2}, \quad W = \frac{m}{2}\alpha,$$

$$I^{(i)} = \int \frac{d\mathbf{p}}{(2\pi)^3} \frac{\varepsilon(p) + m}{2\varepsilon(p)} \psi^C_{1S}(\mathbf{p}) \left(\frac{\varepsilon(p) - m}{\varepsilon(p) + m}\right)^i, \quad \omega_i = \frac{I^{(i)}}{I^{(0)}}.$$

Martynenko F. A., Martynenko A. P., Eskin A. V., Phys. Rev. D., V. 110 056016 (2024)

Integrals $I^{(i)}$ are calculated analytically in complex space using residues. Leading relativistic correction of that type have the order of α^2:

$$I^{(0)} = \int \frac{d\mathbf{p}}{(2\pi)^3} \psi^C_{1S}(\mathbf{p}) \left[1 - \frac{\mathbf{p}^2}{m^2}\right] = \psi^C_{1S}(r = 0) \left[1 + \frac{3}{4}\alpha^2\right],$$

$$I^{(1)} = \int \frac{d\mathbf{p}}{(2\pi)^3} \psi^C_{1S}(\mathbf{p}) \frac{\mathbf{p}^2}{m^2} = -\frac{3}{4}\alpha^2 \psi^C_{1S}(r = 0), \quad \omega_1 = -\frac{3}{4}\alpha^2.$$

R. N. Faustov, A. P. Martynenko, J. Exp. Theor. Phys. 88, 672 (1999).

Relativistic correction to the bound state wave function

In the non-relativistic approximation the wave function of leptonic bound state was obtained from Schrodinger equation with Coulomb potential:

$$\Psi^C_{1S}(r)=\frac{W^{3/2}}{\sqrt{\pi}}e^{-W\cdot r},\quad \Psi^C_{1S}(p)=\frac{8\sqrt{\pi}W^{5/2}}{(p^2+W^2)^2}.$$

Relativistic corrections to the wave function can be calculated within the framework of perturbation theory using the following expression:

S. G. Karshenboim, V. G. Ivanov, U. D. Jentschura, and G. Soff, J. Exp. Theor. Phys. 86, 226 (1998).

W. E. Caswell and G. P. Lepage, Phys. Rev. A 20, 36 (1979).

I. B. Khriplovich and A. S. Yelkhovsky, Phys. Lett. B 246, 520 (1990).

B. A. Kniehl and A. A. Penin, Phys. Rev. Lett. 85, 1210 (2000).

$$\Delta\psi_{1S}(0)=\int \tilde{G}_{1S}(0,\mathbf{r})\Delta V(\mathbf{r})\psi^C_{1S}(\mathbf{r})d\mathbf{r},$$

where $\tilde{G}_{1S}$ is reduced Coulomb Green function, ΔV is the perturbation operator.

We consider following perturbation operators:

Breit potential:

$$\Delta V_1=\frac{\pi\alpha}{m^2}\delta(\mathbf{r})-\frac{\mathbf{p}^4}{4m^3},$$

$$\Delta V_2=-\frac{\alpha}{2m^2}\frac{1}{r}\left(\mathbf{p}^2+\frac{\mathbf{r}(\mathbf{r}\mathbf{p})\mathbf{p}}{r^2}\right),$$

$$\Delta V_3=\frac{\pi\alpha}{m^2}\left[\frac{7}{3}\mathbf{S}^2-2\right]\delta(\mathbf{r}).$$

$$\Delta\psi^{(1)}_{1S}(0)=\psi^C_{1S}(0)[-\frac{63}{128}\alpha^2].$$

$$\Delta\psi^{(2)}_{1S}(0)=\psi^C_{1S}(0)\left[\frac{1}{2}\alpha^2\ln\alpha^{-1}+\frac{5}{8}\alpha^2\right]$$

$$\Delta\psi^{(3)}_{1S}(0)=\psi^C_{1S}(0)\left[2-\frac{7}{3}S(S+1)\right]\frac{1}{4}\alpha^2\ln\alpha^{-1}$$

Relativistic correction to the bound state wave function

Vacuum polarization potential:

$$\Delta V_{vp}(r) = -\frac{\alpha^2}{3\pi}\int_1^\infty \rho(\xi)d\xi\frac{1}{r}e^{-2m_e\xi r},\ \ \rho(\xi) = \frac{\sqrt{\xi^2-1}(2\xi^2+1)}{\xi^4}.$$

$$\Delta\psi^{vp}_{1S}(0) = \psi^C_{1S}(0)\left[a_{vp}\frac{\alpha}{\pi}\right],\ a_{vp} = \int_1^\infty \frac{\rho(\xi)d\xi}{6(1+r_4\xi)}\left[2r_4^2\xi^2+7r_4\xi+2(1+r_4\xi)\ln(1+r_4\xi)+3\right]$$

Numerical value for parameter a_{vp}:

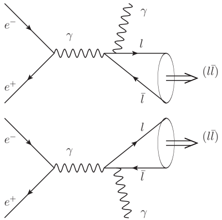
$$a_{vp}(e^+e^-) = 0.004383,\ \ a_{vp}(\mu^+\mu^-) = 0.942799,\ \ a_{vp}(\tau^+\tau^-) = 3.613251.$$

The resulting expression for the wave function of leptonic bound state with relativistic corrections:

Martynenko F. A., Martynenko A. P., Eskin A. V., Phys. Rev. D., V. 110 056016 (2024)

$$\Psi_{l\bar{l}}(0) = \psi^C_{1S}(0)\left\{1+a_{vp}\frac{\alpha}{\pi}+\left(2-\frac{7}{6}S(S+1)\right)\frac{1}{2}\alpha^2\ln\alpha^{-1}-\frac{3}{128}\alpha^2\right\}\quad .$$

Single leptonium production



After calculating the trace in FORM and extracting relativistic parameters $\sim \boldsymbol{p}^2$ we obtain for production amplitude:

$$M_{e^+ + e^- \to \gamma \to (l\bar{l}) + \gamma} = \frac{(ie)^3 \sqrt{2M} \Psi^0_{l\bar{l}}(0) [\bar{v}(p_+) \gamma^\mu u(p_-)]}{\sqrt{2} s^4 \left(1 - \frac{1}{2}\left(\frac{M^2 - 4m^2}{s^2}\right)\right)}$$

$$8 \left[1 + 8\omega_1 \left(\frac{3}{8} + \frac{m^2}{s^2} \frac{1}{\left(1 - \frac{1}{2}\left(\frac{M^2 - 4m^2}{s^2}\right)\right)}\right)\right] e^{v_1 p_\gamma \alpha \mu} \varepsilon^\alpha_\gamma,$$

Total cross section of single dileton production in singlet state:

$$\sigma_{e^+ + e^- \to \gamma \to (l\bar{l}) + \gamma} = \frac{2048 \pi^2 \alpha^3 \Psi^0_{l\bar{l}}(0)^2}{3 M s^8 \sqrt{1 - 4\frac{m_e^2}{s^2}}} \times$$

$$\frac{(M^2 - s^2)^3 (2me^2 + s^2)(-4m^2(1 + 14\omega_1) + (1 + 6\omega_1)(M^2 - 2s^2))}{(4m^2 - M^2 + 2s^2)^3}.$$

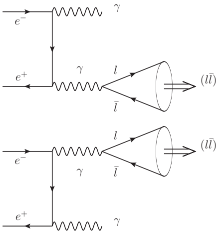
Single leptonium production

Production amplitude of single leptonium in triplet state:

$$M_{e^++e^-\to\gamma+\gamma\to(l\bar{l})+\gamma} = \frac{(ie)^3}{2\sqrt{2}M^2}\int\frac{\Psi_{(l\bar{l})}(\boldsymbol{p})d\boldsymbol{p}}{(2\pi)^3}\frac{\varepsilon(\boldsymbol{p})+m}{2\varepsilon(\boldsymbol{p})}\times$$

$$\left[\bar{v}(p_+)\left(\gamma^\mu\left(\frac{\hat{p_-}-\hat{p_\gamma}+m_e}{(p_--p_\gamma)^2-m_e^2}\right)\gamma^\alpha+\gamma^\alpha\left(\frac{\hat{p_-}-\hat{P}+m_e}{(p_--P)^2-m_e^2}\right)\gamma^\mu\right)u(p_-)\right]\varepsilon_\gamma^\alpha\varepsilon^\lambda(S_P)$$

$$Tr\left\{\left[\frac{\hat{v}_1-1}{2}-\hat{v}_1\frac{p^2}{2m(\varepsilon(\boldsymbol{p})+m)}-\frac{\hat{p}}{2m}\right]\gamma^\lambda(\hat{v}_1+1)\left[\frac{\hat{v}_1+1}{2}-\hat{v}_1\frac{p^2}{2m(\varepsilon(\boldsymbol{p})+m)}+\frac{\hat{p}}{2m}\right]\gamma^\mu\right\},$$



Total cross section:

$$\sigma_{e^++e^-\to\gamma+\gamma\to(l\bar{l})+\gamma} = \frac{256\pi^2\alpha^3\Psi^0_{l\bar{l}}(0)^2(3+26\omega_1)}{3M^3s^4(1-4\frac{m_e^2}{s^2})(s^2-M^2)}\times$$

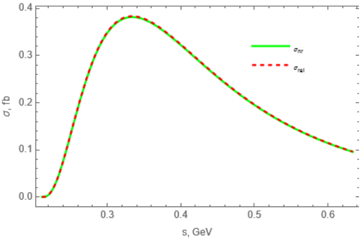
$$\left[2\coth^{-1}\left(\frac{1}{\sqrt{1-4\frac{m_e^2}{s^2}}}\right)\left(M^4-4M^2s^2\frac{m_e^2}{s^2}+s^4\left(1+4\frac{m_e^2}{s^2}-8\frac{m_e^4}{s^4}\right)\right)-\right.$$

$$\left.\sqrt{1-4\frac{m_e^2}{s^2}}\left(M^4+s^4\left(1+4\frac{m_e^2}{s^2}\right)\right)\right].$$

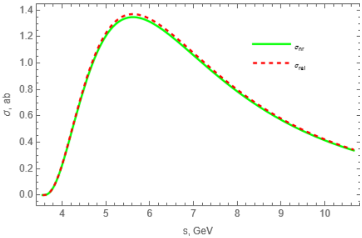
Brodsky S. J., Lebed R. F., Phys. Rev. Lett., 102, 213401 (2009)

Single leptonium production. Results

Total cross section of single production of paraleptonium ($^1S_0$) in the one-photon mechanism of e^+e^- annihilation as a function of center - of - mass energy s (GeV).



(a) $e^+e^- \to (\mu^+\mu^-) + \gamma$



(b) $e^+e^- \to (\tau^+\tau^-) + \gamma$

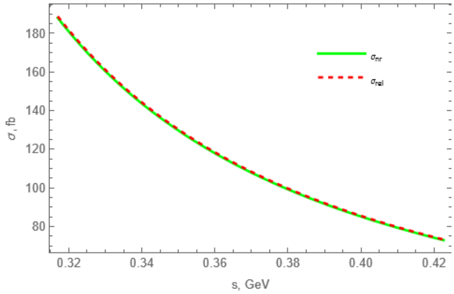
D. d'Enteria, H.-S. Shao, Phys. Lett. B 842, 137960 (2023).

The graphs show comparison of cross sections with (red dashed line) and without (green solid line) corrections. The values of cross sections in maximum are:
$\sigma_{\mu^+\mu^-}(s = 0.33\ GeV) = 0.383\ fb$, $\sigma_{\tau^+\tau^-}(s = 5.62\ GeV) = 1.371\ ab$.

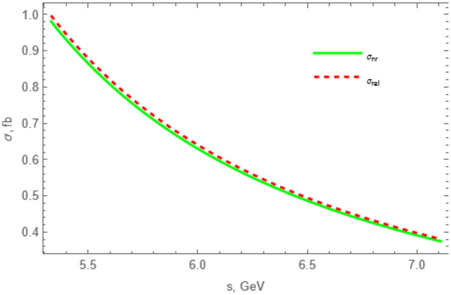
$N_{\mu^+\mu^-} \approx 8$ for BES III with integrated luminosity of 20 fb^{-1}
$N_{\tau^+\tau^-} \approx 65$ for Belle II with integrated luminosity of 50 ab^{-1} .

Single leptonium production. Results

Total cross section of single production of ortholeptonium ($^3S_1$) in the two-photon mechanism of e^+e^- annihilation as a function of center - of mass energy s (GeV) in the off-resonance region $s > 2m$.



(a) $e^+e^- \to (\mu^+\mu^-) + \gamma$



(b) $e^+e^- \to (\tau^+\tau^-) + \gamma$

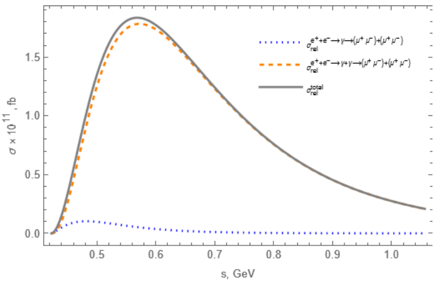
D. d'Enteria, H.-S. Shao, Phys. Lett. B 842, 137960 (2023).

The values of cross sections near threshold:
$\sigma_{\mu^+\mu^-}(s = 0.24\ GeV) = 1.04\ pb$, $\sigma_{\tau^+\tau^-}(s = 3.7\ GeV) = 21.98\ fb$.
Relativistic correction amount to 2% for ditauonium and 0.4% for dimuonium.

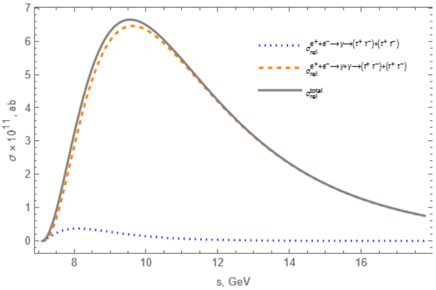
Pair leptonium production. Results

The relativistic cross section of paired production of singlet ($^1S_0$) states leptonia. Dashed and dotted lines show contributions of one - photon and two - photon annihilation mechanisms respectively. Bold line shows the total cross section.

Additional factor for pair production cross section $\alpha\psi^0_{l\bar{l}}(0)^2 \approx m^3\frac{\alpha^4}{8\pi} \approx 10^{-10}m^3$



(a)

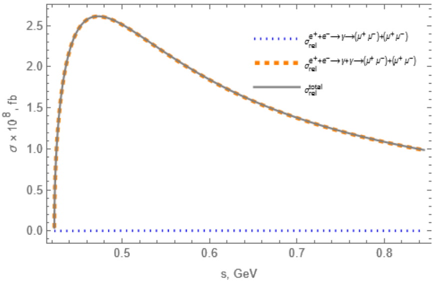


(b)

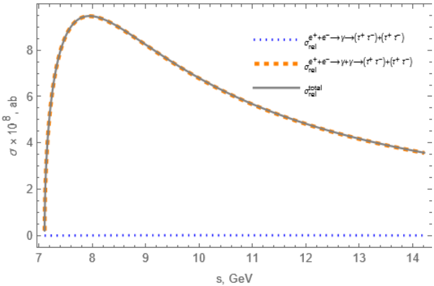
Relativistic correction amount to 4% for ditauonium pair and 1% for dimuonium pair production.

Pair leptonium production. Results

The relativistic cross section of paired production of triplet ($^3S_1$) states leptonia.



(a)



(b)

The values of $e^+e^- \to \gamma \to (l\bar{l}) + (l\bar{l})$ cross sections in the maximum are:

$\sigma_{e^++e^-\to\gamma\to(\mu\bar{\mu})+(\mu\bar{\mu})} = 0.004 \cdot 10^{-8} fb$, $\sigma_{e^++e^-\to\gamma\to(\tau\bar{\tau})+(\tau\bar{\tau})} = 0.01 \cdot 10^{-8} ab$.

Thank You!

Backup slides.

Total production amplitude in the case of pair production

$$M^{pair}_{total} = M_{e^+ + e^- \to \gamma \to (l\bar{l}) + (l\bar{l})} + M^{(a)}_{e^+ + e^- \to \gamma + \gamma \to (l\bar{l}) + (l\bar{l})} + M^{(b)}_{e^+ + e^- \to \gamma + \gamma \to (l\bar{l}) + (l\bar{l})} =$$

$$2M(4\pi\alpha)^2 \Big[\bar{v}(p_+) \Big(\frac{1}{s^6} \left(\frac{32}{s^2} T^{\mu}_{1\gamma,(c,d)} + \frac{2s^2}{M^2} T^{\mu}_{1\gamma,(a,b)} \right) \gamma^\mu +$$

$$\frac{T^{\mu\alpha}_{2\gamma,(a)}}{M^4 \cdot h_1 \cdot h_2} \Big[h_1 \gamma^\alpha (\hat{p}_- - \hat{P} + m_e) \gamma^\mu + h_2 \gamma^\mu (\hat{p}_- - \hat{Q} + m_e) \gamma^\alpha \Big] +$$

$$\frac{32 \ T^{\mu\alpha}_{2\gamma,(b)}}{s^8 \left(\frac{s^2}{4} - P.p_- - Q.p_- \right)^2} \Big[h_4 \gamma^\alpha (\hat{p}_- - \hat{p}_1 - \hat{q}_1 + m_e) \gamma^\mu +$$

$$h_3 \gamma^\mu (\hat{p}_- - \hat{p}_2 - \hat{q}_2 + m_e) \gamma^\alpha \Big] \Big) u(p_-) \Big].$$

Total production amplitude in the case of pair production

$$T^{\mu}_{1\gamma,(a,b)}=\int\frac{d\boldsymbol{p}}{(2\pi)^3}\int\frac{d\boldsymbol{q}}{(2\pi)^3}\cdot f_2\times$$
$$\Big(g_1\,Tr\big\{\psi_{(l\bar{l})}(p,P)\gamma^{\mu}(-\hat{k}+\hat{p}_1+m)\gamma^{\alpha}\psi_{(l\bar{l})}(q,Q)\gamma^{\alpha}\big\}+$$
$$g_2\,Tr\big\{\psi_{(l\bar{l})}(p,P)\gamma^{\alpha}(\hat{k}-\hat{q}_1+m)\gamma^{\mu}\psi_{(l\bar{l})}(q,Q)\gamma^{\alpha}\big\}\Big),$$

$$T^{\mu}_{1\gamma,(c,d)}=\int\frac{d\boldsymbol{p}}{(2\pi)^3}\int\frac{d\boldsymbol{q}}{(2\pi)^3}\times$$
$$\Big(g_1\,Tr\big\{\psi_{(l\bar{l})}(p,P)\gamma^{\mu}(-\hat{k}+\hat{p}_1+m)\gamma^{\alpha}\big\}\,Tr\big\{\psi_{(l\bar{l})}(q,Q)\gamma^{\alpha}\big\}+$$
$$g_2\,Tr\big\{\psi_{(l\bar{l})}(p,P)\gamma^{\alpha}(\hat{k}-\hat{q}_1+m)\gamma^{\mu}\big\}\,Tr\big\{\psi_{(l\bar{l})}(q,Q)\gamma^{\alpha}\big\}\Big).$$

$$T^{\mu\alpha}_{2\gamma,(a)}=\int\frac{d\boldsymbol{p}}{(2\pi)^3}\int\frac{d\boldsymbol{q}}{(2\pi)^3}\,Tr\big\{\psi_{(l\bar{l})}(p,P)\gamma^{\mu}\big\}\,Tr\big\{\psi_{(l\bar{l})}(q,Q)\gamma^{\alpha}\big\},$$
$$h_1=(M^2-2(Q.p_-)),\quad h_2=(M^2-2(P.p_-)).$$
$$T^{\mu\alpha}_{2\gamma,(b)}=\int\frac{d\boldsymbol{p}}{(2\pi)^3}\int\frac{d\boldsymbol{q}}{(2\pi)^3}f_1\cdot f_2\cdot Tr\Big\{\psi_{(l\bar{l})}(p,P)\gamma^{\mu}\psi_{(l\bar{l})}(q,Q)\gamma^{\alpha}\Big\},$$
$$h_3=\big(f_1-(P.p_-+Q.p_--2p.p_--2q.p_-)\big),\quad h_4=\big(f_2-(P.p_-+Q.p_-+2p.p_-+2q.p_-)\big).$$

Total production amplitude in the case of pair production

$$\frac{1}{(p_1+q_1)^2} = \frac{f_1}{\left(\frac{s^2}{4}\right)^2}, \quad f_1 = \frac{s^2}{4} - (p.p + q.q + 2p.q + P.q + Q.p),$$

$$\frac{1}{(p_2+q_2)^2} = \frac{f_2}{\left(\frac{s^2}{4}\right)^2}, \quad f_2 = \frac{s^2}{4} - (p.p + q.q + 2p.q - P.q - Q.p),$$

$$\frac{1}{(k-p_1)^2 - m^2} = \frac{2}{s^2} g_1, \quad g_1 = \left(z_1 - 2\frac{p.p - 2p.Q}{s^2}\right),$$

$$\frac{1}{(k-q_1)^2 - m^2} = \frac{2}{s^2} g_2, \quad g_2 = \left(z_1 - 2\frac{q.q - 2q.P}{s^2}\right),$$

where $z_1 = \frac{1}{1+\frac{1}{2s^2(M^2-4m^2)}}$.

Production cross section of a pair paraleptonium

$$\sigma^{pair}_{({}^1S_0+{}^1S_0)} = \frac{16384\sqrt{1-4a_2^2}\,\alpha^4\pi^3\psi^0_{ll}(0)^4}{45\sqrt{1-4a_1^2}\,a_2^4 s^8}\times$$

$$\Bigg(16\left(1-4a_1^2\right)\left(3a_2^2\left(16a_1^2\left(2a_2^4(56\omega+3)+a_2^2(44\omega+2)+12\omega+1\right)+\left(1-4a_2^2\right)^2(12\omega+1)\right)-\right.$$

$$\left.640a_1^2\left(2a_2^2+1\right)a_2^3a_3\omega+16a_3^2\omega\left(4a_1^2\left(640a_2^6+108a_2^4+76a_2^2-17\right)+3\left(1-4a_2^2\right)^2\left(10a_2^2-1\right)\right)\right)+$$

$$\frac{1}{\left(a_2^2-4a_3^2+1\right)^2}\Big(5\left(2a_1^2+1\right)\Big(128a_2^{12}a_3^2\omega+256a_2^{11}a_3^3\omega-128a_2^{10}a_3^2\left(7a_3^2-8\right)\omega-$$

$$64a_2^9a_3\left(32a_3^4-32a_3^2-3\right)\omega+4a_2^8\left(\left(256a_3^6-704a_3^4+78a_3^2-68\right)\omega-3\right)+$$

$$8a_2^7a_3\left(\left(512a_3^6-960a_3^4-66a_3^2-40\right)\omega-3\right)+a_2^6\Big(2048a_3^8\omega-4864a_3^6\omega+1656a_3^4\omega-$$

$$4a_3^2(148\omega+3)+28\omega+3\Big)+2a_2^5a_3\left(\left(-1024a_3^6+2304a_3^4-464a_3^2+66\right)\omega+3\right)+$$

$$a_2^4\left(-1024a_3^8\omega+2432a_3^6\omega-256a_3^4\omega+a_3^2(3-112\omega)+16\omega\right)+$$

$$8a_2^3a_3\left(32a_3^6-32a_3^4+a_3^2-5\right)\omega+8a_2^2a_3^2\left(16a_3^6-40a_3^4+a_3^2+9\right)\omega+$$

$$8a_2a_3^3\left(7-12a_3^2\right)\omega+2a_3^2\omega\Big)\Big)\Bigg).$$

$$a_1=\frac{m_e}{s},\quad a_2=\frac{M}{s},\quad a_3=\frac{m}{s},\quad \omega=\omega_1.$$

Production cross section of a pair of leptonia (para + ortho states)

$$\sigma^{pair}_{(^1S_0+^3S_1)} = \frac{4096(1+2a_1^2)(1-4a_2^2)^{3/2}(-1+3a_2^2-2a_2a_3+4a_3^2)\alpha^4\pi^3\psi^0_{l\bar{l}}(0)^4}{9\sqrt{1-4a_1^2}a_2^5(1+a_2^2-4a_3^2)^2s^8}\times$$

$$\Big(64a_2^6a_3\omega-384a_2^5a_3^2\omega-32a_2^4a_3\Big(3a_3^2-4\Big)\omega+a_2^3\Big(4\Big(384a_3^4+108a_3^2+41\Big)\omega+9\Big)-$$

$$2a_2^2a_3\Big(8\Big(40a_3^4+18a_3^2+5\Big)\omega+3\Big)+a_2\Big(-448a_3^4\omega+4a_3^2(80\omega+3)-44\omega-3\Big)+$$

$$64a_3^3\Big(4a_3^2-1\Big)\omega\Big).$$