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Extended Vector Meson Dominance Model for Electromagnetic Nucleon Form Factors

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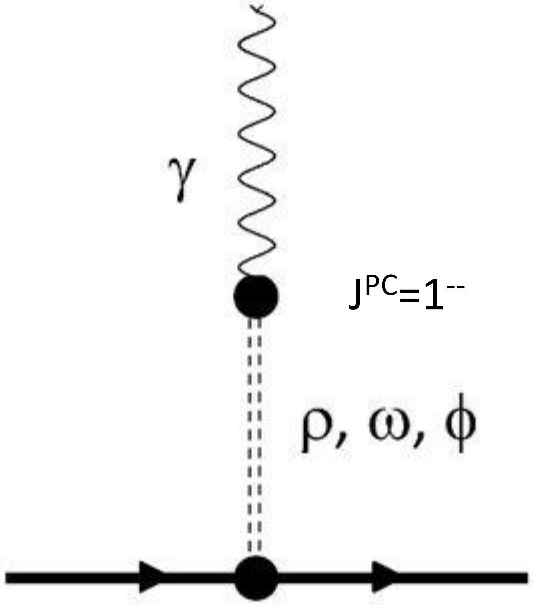
Introduction

The vector meson dominance model (VMD) was first presented in J.J. Sakurai's paper "Theory of Strong Interactions" in 1960.



$J^{PC}=1^{--}$

The main idea is that vector mesons mediate the interactions between photons and hadrons. A photon, interacting with hadrons, first transforms into vector mesons ρ^0, ω, ϕ. Such a transition is not forbidden, since the photon and vector mesons have the same quantum numbers.

J. J. Sakurai

Introduction

Decomposition of the electromagnetic nucleon current by Lorentz covariant structures determines the electro-magnetic form factors of nucleons $F_{1N},\ F_{2N}$

$$\langle p_f, s_f\, | J^{\mu}_{\text{em}}(q) |\, p_i, s_i \rangle = \overline{u}\,(p_f, s_f) \left(F_{1N}(t)\gamma^{\mu} + \frac{1}{2m_N} F_{2N}(t) i\sigma^{\mu\nu} q_{\nu} \right) u\,(p_i, s_i)$$

where $t = q^2 = -Q^2$

Sachs form factors:

$$G_{EN}(t) = F_{1N}(t) + \frac{t}{4m_N^2} F_{2N}(t)$$

$$G_{MN}(t) = F_{1N}(t) + F_{2N}(t)$$

$$G_{EN}(0) = e_N, \quad G_{MN}(0) = \mu_N$$

The isotopic components of the form factors:

$$F_{iI}(t) = \frac{1}{2} \left(F_{ip}(t) \pm F_{in}(t) \right)$$

where I = 0,1 correspond to the isoscalar (ω) and isovector (ρ) channels

Constraints imposed on the eVMD models

1. Threshold identities

$$G_{EN}\left(4m_N^2\right) = G_{MN}\left(4m_N^2\right)$$

2. Quark counting rules

$$G_{EN}(t) \sim G_{MN}(t) = O\left(1/t^2\right) \quad \text{as} \quad t \to -\infty$$

3. Scaling relations

$$G_{Ep}(t) = G_{Mp}(t)/\mu_p = G_{Mn}(t)/\mu_n \sim G_D(t) = \left(1 - \frac{t}{M_V^2}\right)^{-2}$$

(In the spacelike region for low momentum transfers)

4. Okubo-Zweig-Iizuka rule

Processes with strange mesons (ϕ) will be suppressed, since the strange quark-antiquark pair is weakly coupled to non-strange hadrons.

5. Analyticity

6. Frazer-Fulco unitarity $$ImF(t) = F^*(t) \cdot A_{\pi\pi \to N\overline{N}}$$

7. Gauge invariance

We call the model satisfying these constraints $eVMD_1$.

eVMD$_1$

In the case of zero width of vector mesons, the representation for the Sachs form factors in the eVMD$_1$ framework can be expressed as follows:

$$G_{TN}(t) = P^{TN}_{n-2}(t)\prod_V \frac{m_V^2}{m_V^2 - t} \longrightarrow G_{TN}(t) = P^{TN}_{n-2}(t)\sum_V \frac{1}{t - m_V^2} \operatorname*{res}_{u=m_V^2} \prod_{V'} \frac{m_{V'}^2}{m_{V'}^2 - u}$$

$$F_{1I}(t) = \left(\left(e_I - \mu_I \frac{t}{4m_N^2}\right) P^{Ep}_{n-3}(t) \pm \frac{1}{2} P^{En}_{n-3}(t)\right) \prod_V \frac{m_V^2}{m_V^2 - t},$$

$$F_{2I}(t) = \left(\left(-e_I + \mu_I\right) P^{Ep}_{n-3}(t) \mp \frac{1}{2} P^{En}_{n-3}(t)\right) \prod_V \frac{m_V^2}{m_V^2 - t},$$

In the basic approximation n = 3, the free parameters of the model are determined analytically without directly addressing to experimental data:

$$F_{1N}(t) = \left(e_N - \mu_N \frac{t}{4m_N^2}\right) \prod_V \frac{m_V^2}{m_V^2 - t}$$

$$F_{2N}(t) = \left(-e_N + \mu_N\right) \prod_V \frac{m_V^2}{m_V^2 - t}.$$

Only 2 parameters:

$$c_N = -\mu_N / \left(4m_N^2\right)$$

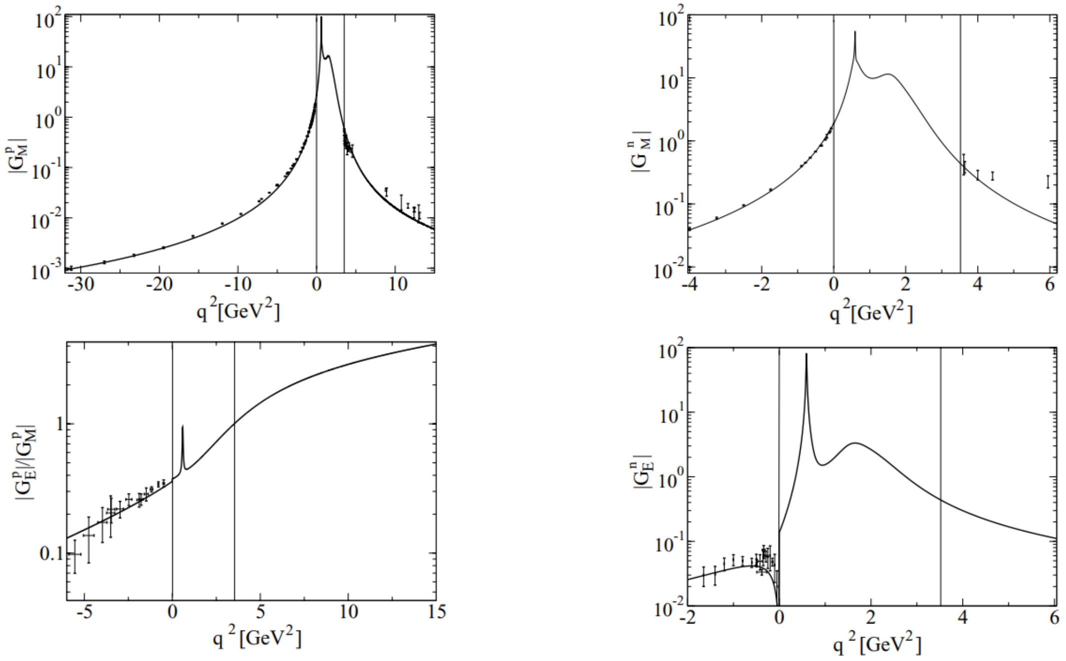
n vector mesons in each channel
Widths are zero
Mass degeneracy of I = 0,1

MFK

The MFK model [1] with three meson states in each isotopic channel also includes two free parameters c_N, which were determined from the fit of experimental data available at 2010.

$$F_1^N(Q^2) = \frac{F_1^N(0) + c^N Q^2}{\left(1+\frac{Q^2}{m_V^2}\right)\left(1+\frac{Q^2}{m_{V'}^2}\right)\left(1+\frac{Q^2}{m_{V''}^2}\right)},$$

$$F_2^N(Q^2) = \frac{F_2^N(0)}{\left(1+\frac{Q^2}{m_V^2}\right)\left(1+\frac{Q^2}{m_{V'}^2}\right)\left(1+\frac{Q^2}{m_{V''}^2}\right)}.$$



Our calculations indicate that adding the latest experimental data and simply redefining the parameters c_p and c_n is not enough to reach satisfactory agreement. A more thorough generalization of the MFK model should be provided.

3 vector mesons in each channel
Widths are not zero
Mass degeneracy of I = 0,1

[1] B. V. Martemyanov, A. Faessler, and M. I. Krivoruchenko, Electromagnetic form factors of nucleons in the extended vector meson dominance model, Phys. Rev. C 82, 038201 (2010), 0910.5589[hep-ph].

eVMD-VI

The main differences from the previous version of the model:

1. 4th vector meson with mass ~1.7 GeV and, as a consequence, an increase in the number of free parameters of the model from 2 to 6.
2. The widths become dependent from the four-momentum transfer squared.

The most general expression for the form factors takes the form:

$$F_{1I}(t) = P^{1I}_{n-2}(t) \sum_V \frac{1}{t - m_V^2 + i\sqrt{t}\Gamma_V(t)} \operatorname*{res}_{u=m_V^2} \prod_{V' \neq V} \frac{m_{V'}^2}{m_{V'}^2 - u}$$

$$F_{2I}(t) = P^{2I}_{n-3}(t) \sum_V \frac{1}{t - m_V^2 + i\sqrt{t}\Gamma_V(t)} \operatorname*{res}_{u=m_V^2} \prod_{V' \neq V} \frac{m_{V'}^2}{m_{V'}^2 - u}$$

In case n = 4:

$$P_2^{1I}(t) = \quad e_I + c_I t + d_I t^2$$

$$P_1^{2I}(t) = -e_I + \mu_I + h_I t$$

$$c_I = (c_p \pm c_n)/2$$

$$d_I = (d_p \pm d_n)/2$$

$$h_I = (h_p \pm h_n)/2$$

Here the "+" sign corresponds to the isoscalar channel, and the "-" sign to the isovector channel.

Determining the model parameters

From a large amount of form factor experiment data, 395 points were selected for this analysis, where 233 and 162 points were obtained for the space-like and time-like region, respectively.

Most data in the time-like region is represented in the form of an efficient form factor:

$$G_N^{\text{eff}}(t) = \sqrt{\frac{|G_{EN}(t)|^2 + \eta\,|G_{MN}(t)|^2}{1+\eta}}$$

where $\eta = t/\left(2m_N^2\right)$

The values of the meson masses and widths were taken in accordance with PDG:

$$m_\rho = m_\omega = 0.77\,GeV,$$

$$m_{\rho'} = m_{\omega'} = 1.25\,GeV,$$

$$m_{\rho''} = m_{\omega''} = 1.45\,GeV,$$

$$m_{\rho'''} = m_{\omega'''} = 1.70\,GeV.$$

$$\Gamma_\rho = 0.15\,GeV, \qquad \Gamma_{\rho''} = 0.4\,GeV,$$

$$\Gamma_\omega = 0.0085\,GeV, \qquad \Gamma_{\omega''} = 0.29\,GeV,$$

$$\Gamma_{\rho'} = 0.3\,GeV, \qquad \Gamma_{\rho'''} = 0.25\,GeV,$$

$$\Gamma_{\omega'} = 0.13\,GeV, \qquad \Gamma_{\omega'''} = 0.315\,GeV.$$

Determining the model parameters

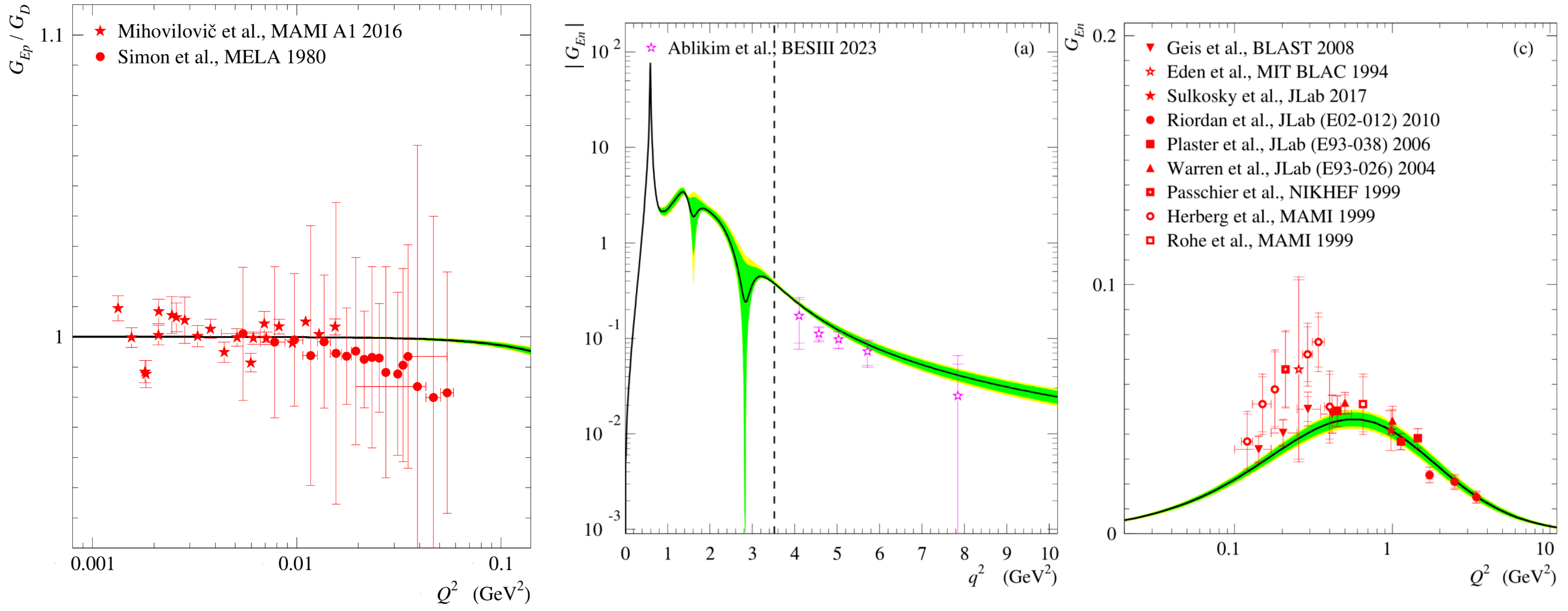
The fit of the selected data was performed using the MINUIT software package. The chi-square function was chosen as the function to be minimized.

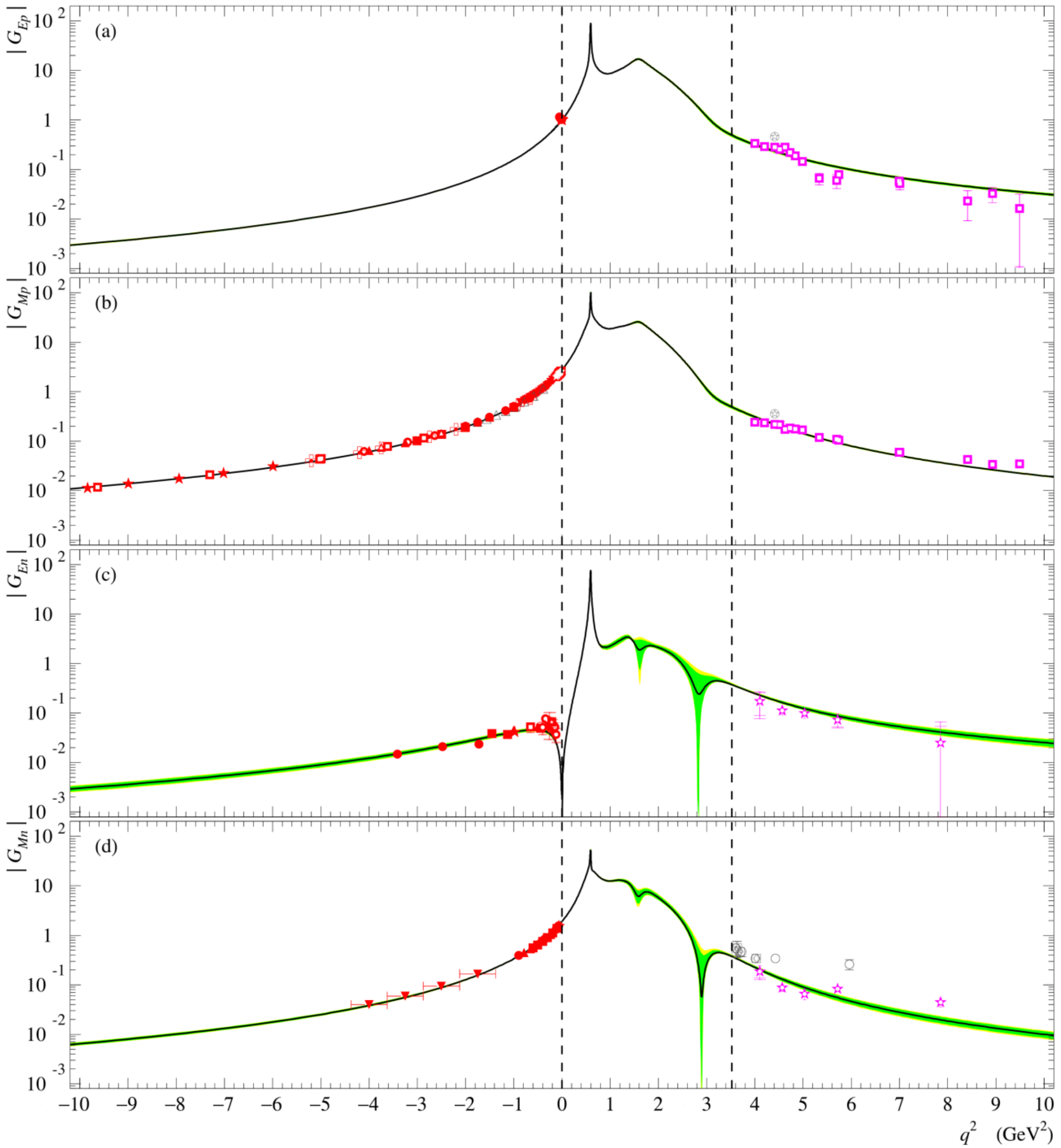
$$\chi^2(\boldsymbol{\theta}) = \sum_{t_s<0} \left(\frac{\mathcal{F}(t_s, \boldsymbol{\theta}) - \mathcal{F}_s^{\exp}}{\Delta \mathcal{F}_s^{\exp}} \right)^2 + \sum_{t_s>0} \left(\frac{|\mathcal{F}(t_s, \boldsymbol{\theta})| - |\mathcal{F}_s^{\exp}|}{\Delta |\mathcal{F}_s^{\exp}|} \right)^2$$

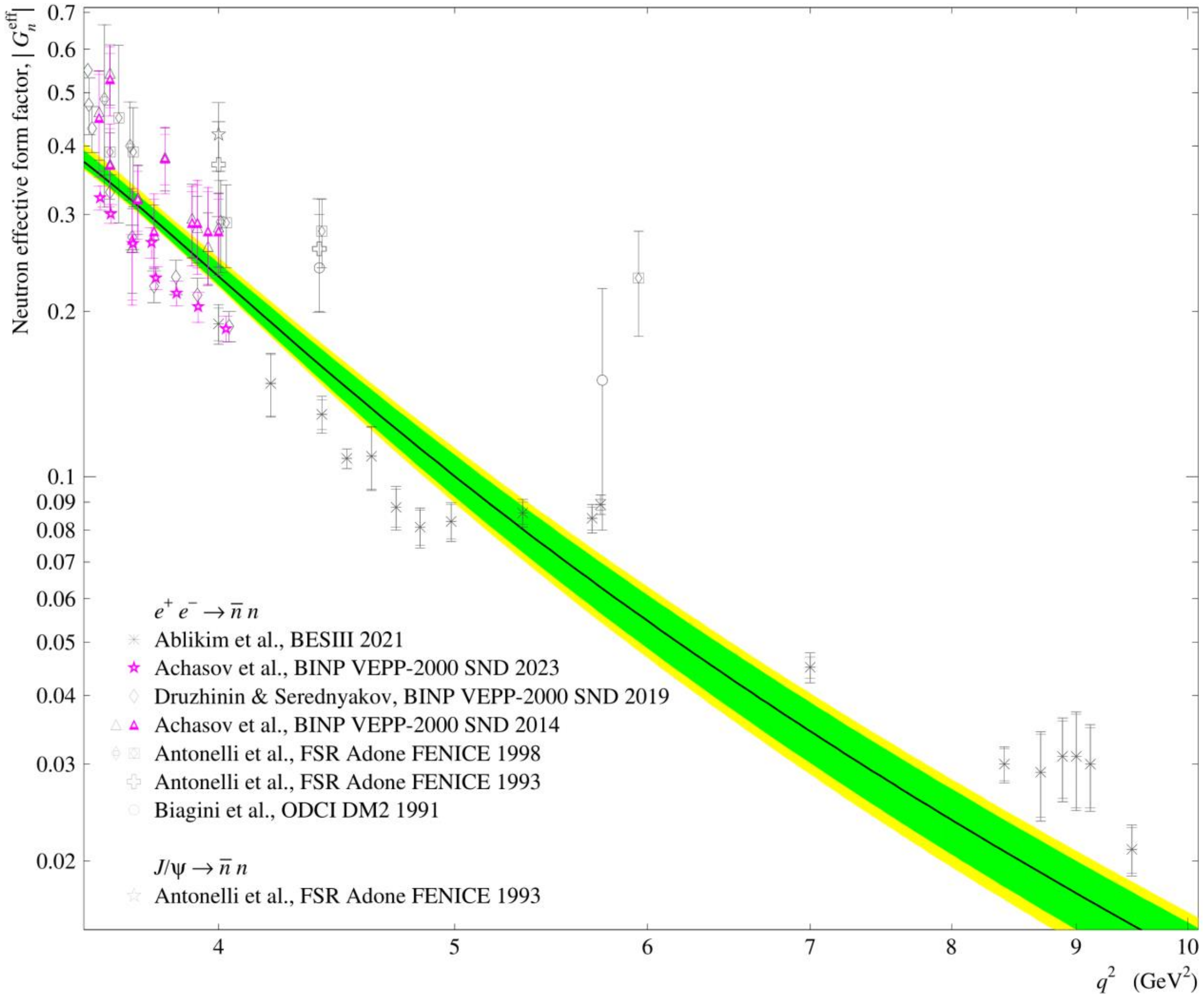
	eVMD_1	MFK	eVMD-VI
$c_p\ [\mathrm{GeV}^{-2}]$	-0.792	-0.463	$-0.812\pm0.008(0.011)$
$c_n\ [\mathrm{GeV}^{-2}]$	0.542	0.297	$0.256\pm0.016(0.022)$

The errors of the parameters given in the table correspond to one and two standard deviations (1σ and 2σ).

Results for electromagnetic form factors







The model shows good agreement with the data in both space- and time-like regions.

Nucleon radii

The electric and magnetic moments of the nucleon radii are determined by the distributions of charge density and magnetization in the nucleons:

$$\langle r^s \rangle_{TN} = \int d\mathbf{r}\, r^s \rho_{TN}(\mathbf{r}) \qquad \rho_{TN}(\mathbf{r}) = \int \frac{d\mathbf{Q}}{(2\pi)^3} e^{i\mathbf{Q}\mathbf{r}} \mathcal{G}_{TN}\left(-\mathbf{Q}^2\right)$$

where $\mathcal{G}_{EN}(t) \equiv G_{EN}(t)$, $\mathcal{G}_{MN}(t) \equiv G_{MN}(t)/G_{MN}(0)$

$$\langle r^{2s+1} \rangle_{TN} = (2s+2)! \sum_V \frac{\tilde{f}_{TN}^{VNN}}{g_{\gamma V}} \frac{1}{m_V^{2s+1}},$$

$$\langle r^{2s} \rangle_{TN} = (2s+1)! \sum_V \frac{\tilde{f}_{TN}^{VNN}}{g_{\gamma V}} \frac{1}{m_V^{2s}},$$

where $\frac{\tilde{f}_{TN}^{VNN}}{g_{\gamma V}} = -\frac{1}{m_V^2} \operatorname*{res}_{t=m_V^2} \mathcal{G}_{TN}(t).$

For second moments →

$$\frac{1}{6}\langle r^2 \rangle_{EN} = c_N + \frac{\mu_N - e_N}{4m_N^2} + e_N \sum_V \frac{1}{m_V^2},$$

$$\frac{1}{6}\langle r^2 \rangle_{MN} = \frac{c_N + h_N}{\mu_N} + \sum_V \frac{1}{m_V^2}.$$

Zemach moments

The Zemach moments of the proton charge and magnetization distributions arise in the theory of the Lamb shift and hyperfine interactions. They are determined by the convolution of the charge distribution density with the charge or magnetization distribution density of the proton:

$$\langle r^s \rangle_{ET} = \int d\mathbf{r}\, r^s \varrho_{ET}(\mathbf{r}), \qquad \varrho_{ET}(\mathbf{r}) = \int d\mathbf{r}' \rho_{Ep}(\mathbf{r}-\mathbf{r}')\, \rho_{Tp}(\mathbf{r}')\,.$$

$$\langle r^{2s+1} \rangle_{ET} = (2s+2)! \sum_{V \neq V'} \frac{\tilde{f}^{VNN}_{Ep} \tilde{f}^{V'NN}_{Tp}}{g_{\gamma V} g_{\gamma V'}} \times \frac{m_V^{2s+3} - m_{V'}^{2s+3}}{m_V^{2s+1} m_{V'}^{2s+1} (m_V^2 - m_{V'}^2)} + \frac{(2s+3)!}{2} \sum_V \frac{\tilde{f}^{VNN}_{Ep} \tilde{f}^{VNN}_{Tp}}{g_{\gamma V} g_{\gamma V} m_V^{2s+1}},$$

$$\langle r^{2s} \rangle_{ET} = \sum_{k=0}^{s} \frac{(2s+1)! \langle r^{2k} \rangle_{Ep} \langle r^{2s-2k} \rangle_{Tp}}{(2s-2k+1)!\,(2k+1)!}$$

Results for nucleon radii

	$eVMD_1$	MFK	eVMD-VI	PDG
$\langle r^2\rangle_{Ep}^{1/2}$ [fm]	0.77	0.82	0.815	0.8409 ± 0.0004
$\langle r^2\rangle_{En}$ [fm^2]	0	-0.06	-0.067	-0.1155 ± 0.0017
$\langle r^2\rangle_{Mp}^{1/2}$ [fm]	0.77	0.78	0.788	0.851 ± 0.026
$\langle r^2\rangle_{Mn}^{1/2}$ [fm]	0.77	0.79	0.791	$0.864^{+0.009}_{-0.008}$

$\langle r\rangle_{EE} = 1.034$ fm $\langle r\rangle_{EE}^{exp} = 1.085\pm0.003$ fm

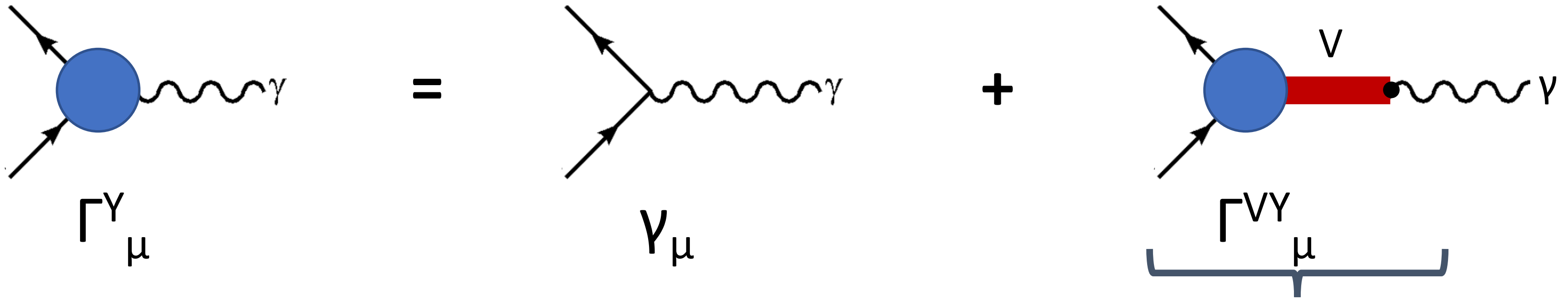
$\langle r\rangle_{EM} = 1.015$ fm $\langle r\rangle_{EM}^{exp} = 1.045\pm0.004$ fm

$\langle r^3\rangle_{EM}^{exp} = 2.71\pm0.13$ fm^3

$\langle r^3\rangle_{EM} = 2.036$ fm^3

$\langle r^3\rangle_{EM}^{exp} = 2.85\pm0.08$ fm^3

Gauge invariance of vertex functions on and off the nucleon mass shell, version VMD Kroll, T.D. Lee, Zumino



transverse contribution

$$\Lambda_\chi(p) = \frac{\chi\hat{p} + m_N}{2m_N} \qquad \chi_f, \chi_i = \pm 1$$

$$\Gamma_\mu(p_f,p_i) = \sum_{\chi_f\chi_i} \Lambda_{\chi_f}(p_f)\left(\left(g_{\mu\nu} - \frac{q_\nu q_\nu}{q^2}\right)\gamma_\nu F_{1N}^{\chi_f\chi_i} + \frac{1}{2m_N} F_{2N}^{\chi_f\chi_i} i\sigma_{\mu\nu} q_\nu + \frac{q_\mu}{q^2} F_{3N}^{\chi_f\chi_i}\right)\Lambda_{\chi_i}(p_i)$$

Ward-Green-Fradkin-Takahashi identity:

$$q^\mu \Gamma_\mu(p_f,p_i) = \hat{S}_F^{-1}(p_f) - \hat{S}_F^{-1}(p_i) = \sum_{\chi_f\chi_i} \Lambda_{\chi_f}(p_f) F_{3N}^{\chi_f\chi_i} \Lambda_{\chi_i}(p_i)$$

There are 12 form factors off-shell,
6 form factors half-off-shell
WGFT identity remove 4 form factors off-shell and 2 form factors half-off-shell

Conclusion

An updated version of the MFK model, eVMD-VI, was proposed. It includes an additional pair of experimentally observed non-strange vector mesons. As a result, the number of free parameters increases from 2 to 6. This number is still a factor of 3 lower as compared to the most popular parameterizations and models. The work also took into account the energy dependence of the decay widths of vector mesons.

The revised version of the model agrees well with experimental data of the proton and neutron form factors, their ratios, and the electric and magnetic radii of nucleons. The Zemach radii are also consistent with the experimental data and alternative theoretical estimations. The model's accuracy could be improved further by adding more parameters. However, as the number of parameters expands, their physical significance blurs.

The model can be recommended for physically justified parameterizations in numerical simulations of processes involving interactions with nucleons.