G.Yu. Prokhorov

Joint Institute for Nuclear Research (JINR), BLTP, Dubna

BASED ON WORKS:

[1] Phys. Rev. Lett., 129(15):151601, (2022).

[2] Phys.Lett.B 840, 137839, (2023).

Duality of gravity and hydrodynamics: quantum anomalies

The 7th International Conference on Particle Physics and Astrophysics (ICPPA-2024), Moscow, Russia 22 to 25 October 2024

Contents

- Introduction (**Anomalies**, **Hydrodynamics**, **Polarization**...)
- Gravitational chiral anomaly and cubic gradients:
  - Derivation of the general formula (*a la Son&Surowka*):

    Kinematical Vortical Effect (KVE)
- Verification: **spin ½** and **spin 3/2**
- Development and outlook
- Conclusion

Part 1

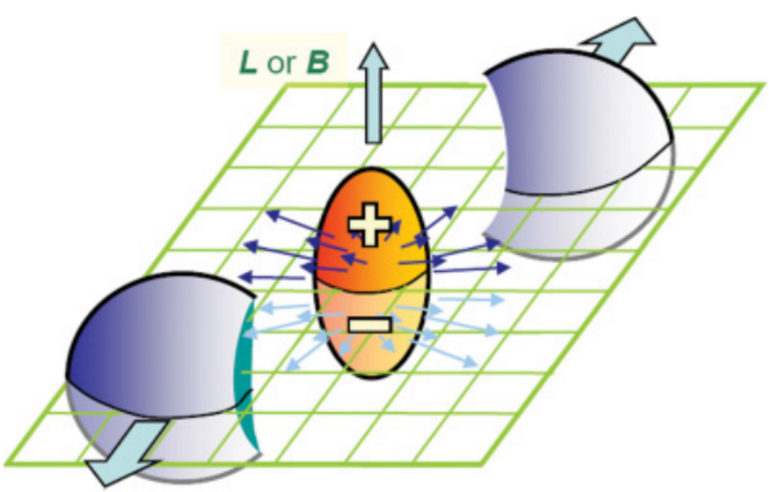
Introduction

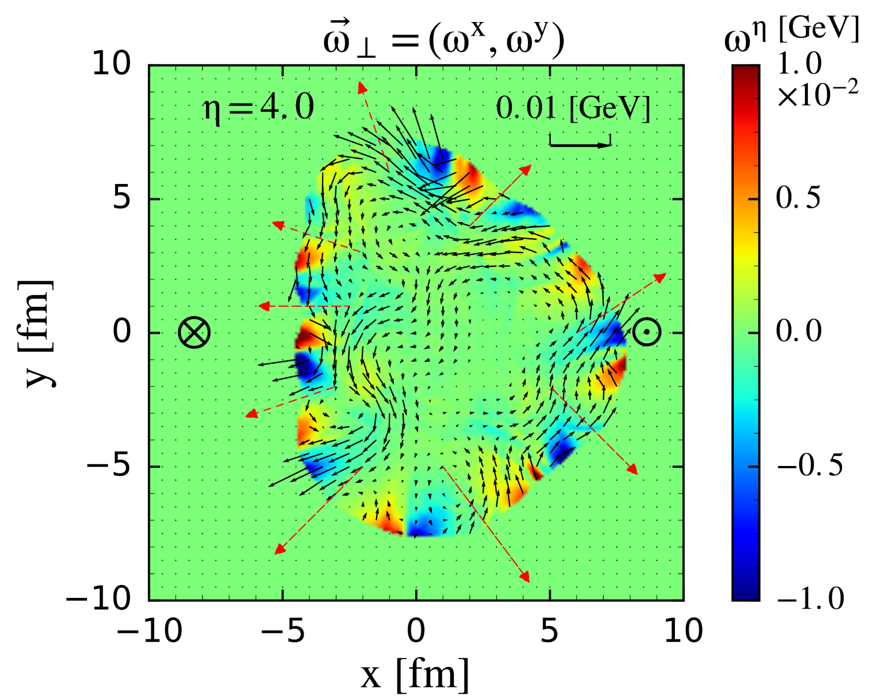
Vorticity in HIC

In non-central collisions of heavy ions, huge magnetic fields and huge angular momentum arise. Differential rotation - different at different points: **vorticity** and **vortices**.

– Rotation 25 orders of magnitude faster, than the rotation of the earth:

– The vorticity has order 10^{22} sec^{-1}

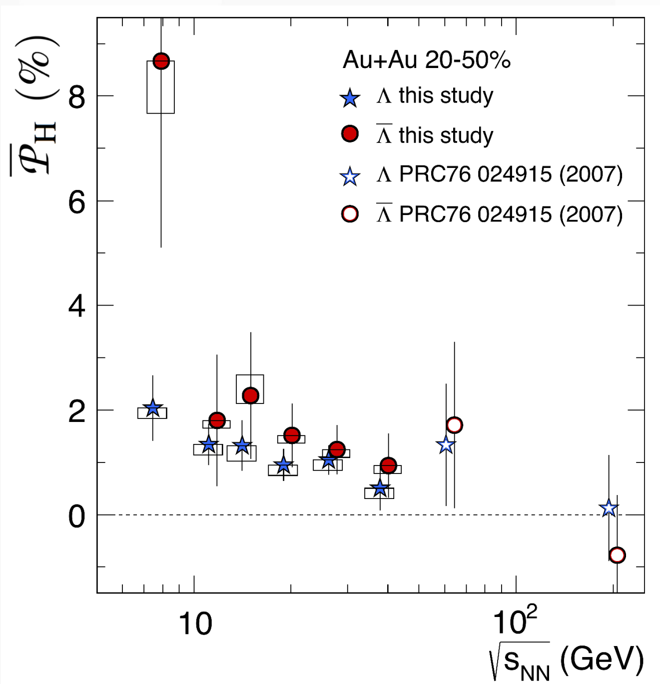




[Phys. Rev. Lett. 117, 192301 (2016)]

VORTICAL POLARISATION

The orbital angular momentum transforms into polarization - an analogue of the Barnett effect.



[Nature 548 (2017) 62-65 arXiv:1701.06657 [nucl-ex]]

STAR Collaboration

- Generation of **hyperon polarization**.
- Both **vorticity** and **acceleration** are essential for polarization.
- Pioneering theoretical prediction: **[Z. T. Liang and X. N. Wang, Phys. Rev. Lett. 94, 102301 (2005)]**
- For recent development see: **[Jian-Hua Gao, Shi-Zheng Yang, 2308.16616]**
- Also described based on **Chiral Vortical Effect** (**CVE**) – Dubna group **[Rogachevsky, Sorin, Teryaev, Phys.Rev.C82(2010) 054910],**

CVE: $$\langle j_\mu^5 \rangle = \left(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2} \right) \omega_\mu$$

– **Qualitative and quantitative correspondence!**
– Polarization from quantum anomaly ~ *spin crisis* and *gluon anomaly*: **[Efremov, Soffer, Teryaev, Nucl.Phys.B 346 (1990) 97-114]**

proton spin → hyperon polarization,
gluon field → chemical potential*4-velocity

- Also described on the basis of a thermodynamic approach: **[I. Karpenko, F. Becattini, Nucl.Phys.A 982 (2019) 519-522]**

CVE AND CME – NEW ANOMALOUS TRANSPORT

Consistency with **quantum anomaly** modifies hydrodynamic equations

[V. I. Zakharov, Lect. Notes Phys.871,295(2013)]

[D.T. Son, P. Surowka, PRL, 103 (2009) 191601]

Quantum chiral anomaly (gauge part)

$$\langle \partial_\mu \hat{j}^\mu_A \rangle = -\frac{C}{8}\varepsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}$$

Entropy current satisfies **second law** of thermodynamics $\partial_\mu s^\mu \geqslant 0$

Chiral magnetic effect (CME)

CME: $j^\mu = C\mu_5 B^\mu$

Current flows along the **magnetic field**

Chiral vortical effect (CVE)

CVE: $j^\mu_A = C\mu^2\omega^\mu$

$$\omega^\mu = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}u_\nu\partial_\alpha u_\beta$$

Current flows along the **vorticity**

Derivation **without entropy** current and generalization to the **second order** in gradients:

[Shi-Zheng Yang, Jian-Hua Gao, Zuo-Tang Liang, Symmetry 14 (2022) 5, 948]

[M. Buzzegoli, Lect. Notes Phys. 987, 53-93 (2021)]

Use **global equilibrium**

Anomalous transport from the gravitational chiral anomaly?

What about the gravitational chiral anomaly?

- The gravitational chiral anomaly grows **rapidly** with **spin**:

$$\langle \nabla_\mu \hat{j}^\mu_A \rangle_S = \frac{(S - 2S^3)}{96\pi^2 \sqrt{-g}} \varepsilon^{\mu\nu\rho\sigma} R_{\mu\nu\kappa\lambda} R_{\rho\sigma}{}^{\kappa\lambda}$$

[M. J. Duff, in First School on Supergravity (1982) arXiv:1201.0386]

Relationship with **temperature term** in CVE current:

[K. Landsteiner, E. Megias, and F. Pena-Benitez, Phys. Rev. Lett., 107:021601, (2011)]

[M. Stone and J. Kim, Phys. Rev., D98(2):025012, 2018]

leads to a **problem** when considering **higher** spins:

[G.Yu. Prokhorov, O.V. Teryaev, V.I. Zakharov,Phys.Lett.B 840, 137839, (2023)]

[G.Yu. Prokhorov, O.V. Teryaev, V.I. Zakharov, Phys.Rev.D 105 (2022) 4, L041701]

Part 2

Gravitational chiral anomaly and cubic gradients

Hydrodynamics in curved space-time

Consider an uncharged fluid of massless particles with an **arbitrary spin** in a **gravitational field**:

fluid

4-velocity of the fluid $u_\mu(x)$

Proper temperature $T(x)$

Inverse temperature vector $\beta_\mu = u_\mu / T$

Thermal vorticity tensor (analogous to the acceleration tensor) $\varpi_{\mu\nu} = -\frac{1}{2}(\nabla_\mu \beta_\nu - \nabla_\nu \beta_\mu)$

space-time

Curved space-time metric

$g_{\mu\nu}(x)$

Riemann tensor

$R_{\mu\nu\kappa\lambda}$

We consider a medium in a state of **(global) thermodinamic equilibrium**

[F. Becattini, L. Bucciantini, E. Grossi, L. Tinti, Eur. Phys. J. C 75, 191 (2015)]

[F. Becattini, Acta Phys. Polon. B 47, 1819 (2016)]

Killing equation

$$\nabla_\mu \beta_\nu + \nabla_\nu \beta_\mu = 0$$

Very close to the **Tolman-Ehrenfest's** criterion and the **Luttinger** relation

Decomposition of the tensors: hydrodynamics

- **Components of the thermal vorticity tensor** **6** components

[M. Buzzegoli, E. Grossi, F. Becattini, JHEP 10 (2017) 091]

$$\varpi_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta} w^\alpha u^\beta + \alpha_\mu u_\nu - \alpha_\nu u_\mu$$

Similar to the expansion for the electromagnetic field

- **Inverse formulas**

"thermal" acceleration

3 components

$$\alpha_\mu = \varpi_{\mu\nu} u^\nu$$

Usual “kinematic” vorticity

“thermal” vorticity pseudovector

3 components

$$w_\mu = -\frac{1}{2}\epsilon_{\mu\nu\alpha\beta} u^\nu \varpi^{\alpha\beta} \Longleftrightarrow w_\mu = \frac{\omega_\mu}{T}$$

In a state of global equilibrium

$$\alpha_\mu = \frac{a_\mu}{T}$$

Usual "kinematic" acceleration

Decomposition of the tensors: gravity

We also decompose the **Riemann tensor** into the **components:**

$$
\begin{aligned}
R_{\mu\nu\alpha\beta} \;=\; & u_\mu u_\alpha A_{\nu\beta} + u_\nu u_\beta A_{\mu\alpha} - u_\nu u_\alpha A_{\mu\beta} - u_\mu u_\beta A_{\nu\alpha} \\
& + \epsilon_{\mu\nu\lambda\rho} u^\rho (u_\alpha B^\lambda{}_\beta - u_\beta B^\lambda{}_\alpha) \\
& + \epsilon_{\alpha\beta\lambda\rho} u^\rho (u_\mu B^\lambda{}_\nu - u_\nu B^\lambda{}_\mu) \\
& + \epsilon_{\mu\nu\lambda\rho} \epsilon_{\alpha\beta\eta\sigma} u^\rho u^\sigma C^{\lambda\eta}
\end{aligned}
$$

20 components

Coincide with 3d tensors in the fluid rest frame ("**Petrov expansion**"):

[A. Z. Petrov, 1950] **[L. D. Landau and E. M. Lifschits, The Classical Theory of Fields, Vol. 2, 1975]**

Inverse formulas:

$$A_{\mu\nu} = u^\alpha u^\beta R_{\alpha\mu\beta\nu}$$

Symmetric tensor

6 components

$$B_{\mu\nu} = \frac{1}{2} \epsilon_{\alpha\mu\eta\rho} u^\alpha u^\beta R_{\beta\nu}{}^{\eta\rho}$$

Nonsymmetric traceless pseudotensor dual to the Riemann tensor

8 components

$$C_{\mu\nu} = \frac{1}{4} \epsilon_{\alpha\mu\eta\rho} \epsilon_{\beta\nu\lambda\gamma} u^\alpha u^\beta R^{\eta\rho\lambda\gamma}$$

Double dual symmetric Riemann tensor

6 components

Gradient expansion in the curved spacetime

The gravitational chiral anomaly has the **4th order** in gradients – it is to be related to the **3rd order** terms in gradient expansion of the axial current.

Hydrodynamic expansion of the axial current up to the **3rd order** in gradients:

Survive in flat spacetime

$$j_\mu^{A(3)} = \xi_1(T) w^2 w_\mu + \xi_2(T) \alpha^2 w_\mu + \xi_3(T)(\alpha w) w_\mu$$
$$+ \xi_4(T) A_{\mu\nu} w^\nu + \xi_5(T) B_{\mu\nu} \alpha^\nu$$

arbitrary coefficients

"gravitational" currents

We use only:

$$\nabla_\mu j_A^\mu = \mathcal{N} \epsilon^{\mu\nu\alpha\beta} R_{\mu\nu\lambda\rho} R_{\alpha\beta}{}^{\lambda\rho}$$

Flat space limit:
Kinematical Vortical Effect (KVE)

Let's move on to the limit of **flat space-time**. Despite the absence of a gravitational field, there **remains** a contribution to the axial current induced by the **gravitational chiral anomaly:**

Zero gravitational field

Limit $R_{\mu\nu\alpha\beta} = 0$

$$j_\mu^A = \lambda_1(\omega_\nu\omega^\nu)\omega_\mu + \lambda_2(a_\nu a^\nu)w_\mu$$

$$\frac{\lambda_1 - \lambda_2}{32} = \mathcal{N}$$

Nonzero gravitational field

$$R_{\mu\nu\alpha\beta} \neq 0$$

$$\nabla_\mu j_A^\mu = \mathcal{N}\epsilon^{\mu\nu\alpha\beta}R_{\mu\nu\lambda\rho}R_{\alpha\beta}{}^{\lambda\rho}$$

- A new type of anomalous transport – the **Kinematical Vortical Effect (KVE)**. Does not explicitly depend on temperature and density → determined only by the **kinematics** of the flow.

Discussion

- **Arbitrary** massless fields with **arbitrary** spin were considered:
- Only **conservation relation** for the current was used.

General exact result

- Although the effect is associated with gravitational anomaly, it exists in **flat space-time** (the **Cheshire cat grin**).

- In contrast to **CVE** and the **gauge** anomaly case, for **KVE** the factor from the **gravitational** anomaly is split into **two** conductivities:

$$\lambda_1 - \lambda_2 = 32\mathscr{N}$$

- Conservation laws lead to the interplay of **infrared** (e.g. vortical current) and **ultraviolet** (quantum anomaly) effects.

Part 3

Verification: spin 1/2

Transport coefficients and anomaly: spin 1/2

- In [GP, O.V. Teryaev, and V.I. Zakharov, JHEP, 02:146, 2019], [V. E. Ambrus, JHEP, 08:016, 2020], [A. Palermo, et al. JHEP 10 (2021) 077] and for ω^3 in [A. Vilenkin, Phys. Rev., D20:1807-1812, 1979] the following expression was obtained:

KVE

$$j_\mu^A = \left(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2} - \underbrace{\frac{\omega^2}{24\pi^2} - \frac{a^2}{8\pi^2}}_{\text{KVE}}\right)\omega_\mu$$

- Comparing it with the well-known anomaly [L. Alvarez-Gaume, E. Witten, Nucl. Phys., B234:269, 1984]:

$$\nabla_\mu j_A^\mu = \frac{1}{384\pi^2\sqrt{-g}}\varepsilon^{\mu\nu\alpha\beta}R_{\mu\nu\lambda\rho}R_{\alpha\beta}{}^{\lambda\rho}$$

We see that the formula is **fulfilled**:

$$\frac{\lambda_1 - \lambda_2}{32} = \mathcal{N}$$

⇨

$$\left(-\frac{1}{24\pi^2} + \frac{1}{8\pi^2}\right)/32 = \frac{1}{384\pi^2}$$

Correspondence between **gravity** and **hydrodynamics** is shown!

Verification:
spin 3/2

Rarita-Schwinger-Adler model of spin 3/2

Novel theory of spin 3/2 [Stephen L. Adler. Phys. Rev. D, 97(4):045014, 2018]:

$$S = \int d^4x \left(- \varepsilon^{\lambda\rho\mu\nu} \bar{\psi}_\lambda \gamma_5 \gamma_\mu \partial_\nu \psi_\rho + i \bar{\lambda} \gamma^\mu \partial_\mu \lambda - i m \bar{\lambda} \gamma^\mu \psi_\mu + i m \bar{\psi}_\mu \gamma^\mu \lambda \right)$$

"coupling mass"

Gravitational anomaly was found in [G. Y. Prokhorov, O. V. Teryaev, V. I. Zakharov, Phys.Rev.D 106 (2022) 2, 025022]

$$\nabla_\mu j^\mu_A = -\frac{19}{384\pi^2 \sqrt{-g}} \varepsilon^{\mu\nu\rho\sigma} R_{\mu\nu\kappa\lambda} R_{\rho\sigma}{}^{\kappa\lambda}$$

Zubarev density operator

Global Equilibrium Conditions

$$\nabla_\mu \beta_\nu + \nabla_\nu \beta_\mu = 0 \qquad \nabla_\mu \zeta = 0$$

Form of the **density operator** for a medium with **rotation** and **acceleration:**

$$\widehat{\rho} = \frac{1}{Z} \exp\left[-b_\mu \widehat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}\widehat{J}^{\mu\nu} + \zeta \widehat{Q}\right]$$

Lorentz Transform Generators

$$\widehat{J}^{\mu\nu} = \int_\Sigma \mathrm{d}\Sigma_\lambda \left(x^\mu \widehat{T}^{\lambda\nu} - x^\nu \widehat{T}^{\lambda\mu}\right)$$

$$\varpi_{\mu\nu}\hat{J}^{\mu\nu} = -2\alpha^\rho \hat{K}_\rho - 2w^\rho \hat{J}_\rho$$

$\hat{K}^\mu$ – boost (related to acceleration)

$\hat{J}^\mu$ – angular momentum (related to vorticity)

KVE IN RSA THEORY: CALCULATION

- Our *goal* is to calculate the conductivities λ_1 and λ_2 in the KVE current:

$$j^{\mu}_{A,KVE} = \lambda_1(\omega_\nu\omega^\nu)\omega^\mu + \lambda_2(a_\nu a^\nu)\omega^\mu$$

- Using the described perturbation theory, we obtain:

$$\lambda_1 = -\frac{1}{6}\int_0^{|\beta|}[d\tau]\langle T_\tau \hat{J}^3_{-i\tau_x}\hat{J}^3_{-i\tau_y}\hat{J}^3_{-i\tau_z}\hat{j}^3_A(0)\rangle_{T,c}$$

- Representing $\hat{J}_\sigma$, $\hat{K}^\mu$ through the stress-energy tensor, we obtain:

$$\lambda_1 = -\frac{1}{6T^3}\Big(C^{02|02|02|3|111} + C^{02|01|01|3|122} + C^{01|02|01|3|212} + C^{01|01|02|3|221}$$
$$-C^{01|01|01|3|222} - C^{01|02|02|3|211} - C^{02|01|02|3|121} - C^{02|02|01|3|112}\Big),$$

Typical correlator to be found: 4-point one-loop function

$$C^{\alpha_1\alpha_2|\alpha_3\alpha_4|\alpha_5\alpha_6|\lambda|ijk} = T^3\int[d\tau]d^3x\, d^3y\, d^3z\, x^i y^j z^k\langle T_\tau \hat{T}^{\alpha_1\alpha_2}(-i\tau_x,\mathbf{x})\hat{T}^{\alpha_3\alpha_4}(-i\tau_y,\mathbf{y})\hat{T}^{\alpha_5\alpha_6}(-i\tau_z,\mathbf{z})\hat{j}^\lambda_5(0)\rangle_{T,c}$$

When expanding the density operator → shift along the imaginary axis → field theory at **finite temperatures**.

KVE vs Gravitational Anomaly

The obtained formula for **cubic gradients** (KVE):

$$j^{A(3)}_\mu = \left(-\frac{53}{24\pi^2}\omega^2 - \frac{5}{8\pi^2}a^2\right)\omega_\mu$$

Gravitational chiral **anomaly**:

$$\nabla_\mu j^\mu_A = \frac{-19}{384\pi^2\sqrt{-g}}\varepsilon^{\mu\nu\alpha\beta}R_{\mu\nu\lambda\rho}R_{\alpha\beta}{}^{\lambda\rho}$$

$$\frac{\lambda_1 - \lambda_2}{32} = \mathcal{N}$$

Direct **verification**:

$$\left(-\frac{53}{24\pi^2} + \frac{5}{8\pi^2}\right)/32 = -\frac{19}{384\pi^2}$$

Coincidence of hydrodynamics and gravitational anomaly!

- For the RSA theory, the relationship between the transport coefficients in a **vortical accelerated fluid** and the **gravitational** chiral **anomaly** is shown: the factor **-19** from the anomaly is reproduced.
- Verification of the obtained formula in a very **nontrivial** case with higher spins and interaction.

Part 4

Development and Outlook

Generalization to (anti)de Sitter space

see talk of R. V. Khakimov

[R.V. Khakimov, G.Yu. Prokhorov, O.V. Teryaev, V.I. Zakharov, Phys.Rev.D 108 (2023) 12, L121701]

[R.V. Khakimov, G.Yu. Prokhorov, O.V. Teryaev, V.I. Zakharov, Phys.Rev.D 109 (2024) 10, 105001]

Previously, we considered **Ricci-flat** background $R_{\mu\nu} = 0$

Let us generalize to the case with **constant scalar curvature** (A)dS $R_{\mu\nu} = \Lambda g_{\mu\nu}$

$$\langle \hat{T}^{\mu\nu} \rangle_{s=1/2} = \left[\frac{7\pi^2}{180} T^4 + \frac{1}{72} \left(|a|^2 + \frac{R}{12} \right) T^2 - \frac{17}{2880\pi^2} \left(|a|^2 + \frac{R}{12} \right)^2 \right] \left(4u^\mu u^\nu - g^{\mu\nu} \right) + \frac{11}{960\pi^2} \left(\frac{R}{12} \right)^2 g^{\mu\nu}$$

At temperature T_{UR} , the stress-energy tensor has a vacuum form:

$$T_{UR} = \frac{a_5}{2\pi} = \sqrt{T_U^2 + T_R^2} = \frac{\sqrt{|a|^2 + R/12}}{2\pi}$$

The temperature measured by an accelerated observer in (A)dS space is determined by the **5-dimensional acceleration**! [S. Deser and O. Levin, Phys. Rev. D, 59:064004, 1999]

Duality relations are obtained that between the **accelerated** fluid to the effects of **constant curvature**.

POLARIZATION

- Signatures of **gravitational** chiral **anomaly without gravity?** Contribution to the vortical **polarization**?
  - To observe cubic terms, it is necessary that the gradients (*acceleration, vorticity, magnetic field...*) give a contribution at least of the order of temperature

$$\omega, a \sim (0.1 - 0.6)T$$

[A. Zinchenko, A. Sorin, O. Teryaev, M. Baznat, J.Phys.Conf.Ser. 1435 (2020)]

[F. Becattini, M. Buzzegoli, G. Inghirami, I. Karpenko, A. Palermo, Phys.Rev.Lett. 127 (2021) 27, 272302]

However, the cubic terms are also suppressed by the **numerical factor**

KVE:

$$j^{\mu}_{A,S=1/2} = \left(-\frac{1}{24\pi^2}\omega^2 - \frac{1}{8\pi^2}a^2\right)\omega^{\mu}$$

- The **good** news: for higher spins (e.g. 3/2) it is enhanced by **cubic growth** with spin (related to anomaly growth):

$$\lambda_1^S = \frac{S - 8S^3}{12\pi^2}$$

- **But**: should be generalized to massive particles (omega baryon is heavy).

- **Idea:** consider massless **quasiparticles** with spin 3/2 in semymetals?

[I. Boettcher, Phys. Rev. Lett. 124, 127602 (2020)]

Part 5 Conclusion

Conclusion

- The relationship between the hydrodynamic current in the third order of gradient expansion $(\omega_\nu \omega^\nu)\omega_\mu$ and $(a_\nu a^\nu)w_\mu$, the **Kinematical Vortical Effect (KVE)**, and the **gravitational chiral anomaly** has been established:
  - The axial current in a flat space-time in a vortical and accelerated fluid turns out to be associated with a quantum violation of current conservation in a curved space-time.
- The obtained formula has been **verified** directly for **spin 1/2** and **3/2**.
- This demonstrates the interplay of **infrared** and **ultraviolet** phenomena.
- Can be interpreted as a demonstration of **equivalence principle** (for **higher orders** in gradients of metrics and **quantum loop** effects)
- New contribution into vortical **polarization** (difficult to observe)?

Anomaly matching: system of equations

The divergence of the axial current transforms into the sum of **independent** terms:

$$
\begin{aligned}
\nabla_\mu j^\mu_{A(3)} = & \ (\alpha w) w^2 \left(-3T\xi_1 + T^2\xi_1' + 2T\xi_3\right) \\
& + (\alpha w)\alpha^2 \left(-3T\xi_2 + T^2\xi_2' - T\xi_3 + T^2\xi_3'\right) \\
& + A_{\mu\nu}\alpha^\mu w^\nu \left(T^2\xi_4' + 3T\xi_5 + 2T^{-1}\xi_2 + T^{-1}\xi_3\right) \\
& + B_{\mu\nu} w^\mu w^\nu \left(-2T^{-1}\xi_1 - 3T\xi_4 - T\xi_5\right) \\
& + B_{\mu\nu}\alpha^\mu \alpha^\nu \left(T^2\xi_5' - T\xi_5 - T^{-1}\xi_3\right) \\
& + A_{\mu\nu} B^{\mu\nu} \left(-T^{-1}\xi_4 + T^{-1}\xi_5\right) \\
= & \ 32\mathcal{N} A_{\mu\nu} B^{\mu\nu} \, .
\end{aligned}
$$

Principle:

In this case, the divergence is equal to the anomaly: additional terms - *macroscopic* - cannot violate the equation from the fundemental *microscopic* theory.

The coefficient in front of each pseudocalar must be equal to zero - a **system of equations** for the unknown coefficients $\xi_n(T)$.

Anomaly matching: system of equations

This **system of linear differential equations** has the form:

$$
\begin{aligned}
-3T\xi_1 + T^2\xi_1' + 2T\xi_3 &= 0 \\
-3T\xi_2 + T^2\xi_2' - T\xi_3 + T^2\xi_3' &= 0 \\
T^2\xi_4' + 3T\xi_5 + 2T^{-1}\xi_2 + T^{-1}\xi_3 &= 0 \\
-2T^{-1}\xi_1 - 3T\xi_4 - T\xi_5 &= 0 \\
T^2\xi_5' - T\xi_5 - T^{-1}\xi_3 &= 0 \\
-T^{-1}\xi_4 + T^{-1}\xi_5 - 32\mathscr{N} &= 0
\end{aligned}
$$

The factor from the **gravitational chiral anomaly**

Anomaly matching: solution

Since the theory does not include **dimensional** parameters other than **temperature**:

$$\xi_1 = T^3\lambda_1 \quad \xi_2 = T^3\lambda_2 \quad \xi_3 = T^3\lambda_3 \quad \xi_4 = T\lambda_4 \quad \xi_5 = T\lambda_5$$

The current: $$j_\mu^{A(3)} = \lambda_1\omega^2\omega_\mu + \lambda_2 a^2\omega_\mu + \lambda_3(a\omega)\omega_\mu + \lambda_4 A_{\mu\nu}\omega^\nu + \lambda_5 B_{\mu\nu}a^\nu$$

The solution looks like:

$$\frac{\lambda_1-\lambda_2}{32} = \mathcal{N} \qquad \lambda_4 = -8\mathcal{N} - \frac{\lambda_1}{2}$$

$$\lambda_3 = 0 \qquad \lambda_5 = 24\mathcal{N} - \frac{\lambda_1}{2}$$

was also shown in

[G. Y. Prokhorov, O. V. Teryaev, V. I. Zakharov, JHEP 02 (2019)]

Discussion

- Arbitrary fields with arbitrary spin were considered:

General exact result

- Only **conservation law** for the current was used.

- Although the effect is associated with an anomaly - it exists in **flat space-time** (the *Cheshire cat grin*).

- In contrast to **CVE** and the **gauge anomaly case**, the factor from the gravitational anomaly is split into **two conductivities:**

$$\lambda_1 - \lambda_2 = 32\mathcal{N}$$

- Conservation laws lead to the interplay of **infrared** (e.g. vortical current) and **ultraviolet** (quantum anomaly) effects.

Zubarev density operator

Quantum statistical mean value

$$\langle \widehat{O}(x) \rangle = \frac{1}{Z} \mathrm{tr}(\widehat{\rho}\, \widehat{O}(x))_{\mathrm{ren}}$$

statistical sum:
cancellation of disconnected correlators

Perturbation theory in the third order

$$\langle \widehat{O}(x) \rangle = \langle \widehat{O}(x) \rangle_{\beta(x)} + \frac{\varpi_{\mu\nu}}{2|\beta|} \int_0^{|\beta|} d\tau \langle T_\tau J^{\mu\nu}_{-i\tau u} \widehat{O}(0) \rangle_{\beta(x),c}$$

$$+ \frac{\varpi_{\mu\nu}\varpi_{\rho\sigma}}{8|\beta|^2} \int_0^{|\beta|} d\tau_x d\tau_y \langle T_\tau J^{\mu\nu}_{-i\tau_x u} J^{\rho\sigma}_{-i\tau_y u} \widehat{O}(0) \rangle_{\beta(x),c}$$

$$+ \frac{\varpi_{\mu\nu}\varpi_{\rho\sigma}\varpi_{\alpha\beta}}{48|\beta|^3} \int_0^{|\beta|} d\tau_x d\tau_y d\tau_z \langle T_\tau J^{\mu\nu}_{-i\tau_x u} J^{\rho\sigma}_{-i\tau_y u} J^{\alpha\beta}_{-i\tau_z u} \widehat{O}(0) \rangle_{\beta(x),c} + \dots$$

Imaginary time $\tau = i\, t$ *ordering*

Connected correlators

$$\langle \widehat{J}\widehat{O} \rangle_c = \langle \widehat{J}\widehat{O} \rangle - \langle \widehat{J} \rangle \langle \widehat{O} \rangle$$

KVE in RSA theory: calculation

- Finally, we obtain, in particular, for $C^{02|02|02|3|111}$

$$C^{02|02|02|3|111} = \frac{T}{480\pi^2}\int \frac{dp\, p\, e^{p/T}}{(1+e^{p/T})^5}\left\{126 - 291\frac{p}{T} - 472\frac{p^2}{T^2} + \left[126 + 873\frac{p}{T} + 5192\frac{p^2}{T^2}\right]e^{p/T} + \left[-126 + 873\frac{p}{T} - 5192\frac{p^2}{T^2}\right]e^{2p/T} + \left[-126 - 291\frac{p}{T} + 472\frac{p^2}{T^2}\right]e^{3p/T}\right\} = \frac{177T^3}{80\pi^2}$$

Calculating all the diagrams we obtain:

$$\lambda_1 = -\frac{1}{6}\left(2\cdot\frac{177}{80\pi^2} + 6\cdot\frac{353}{240\pi^2}\right) = -\frac{53}{24\pi^2},$$

$$\lambda_2 = -\frac{1}{6}\left(\frac{33}{40\pi^2} + \frac{53}{80\pi^2} + \frac{1}{2\pi^2} + \frac{3}{4\pi^2} + \frac{47}{80\pi^2} + \frac{17}{40\pi^2}\right) = -\frac{5}{8\pi^2}$$

Thus, the KVE in the RSA theory has the form:

$$j^{\mu}_{A,KVE} = \left(-\frac{53}{24\pi^2}\omega^2 - \frac{5}{8\pi^2}a^2\right)\omega^{\mu}$$