Triply heavy tetraquark spectroscopy

Elena M. Savchenko[1,2] Vladimir O. Galkin[2]

[1]Department of quantum theory and high energy physics, M.V.Lomonosov Moscow State University

[2]Federal research center "Computer science and control", Russian Academy of Sciences

The 7th international conference on particle physics and astrophysics, October 24, 2024

Introduction

- “Ordinary” hadrons:
  - baryons qqq,
  - mesons $\mathrm{q\bar{q}}$.
- Exotic hadrons:
  - tetraquarks $\mathrm{qq\overline{qq}}$,
  - pentaquarks $\mathrm{qqqq\bar{q}}$, etc.
- Searches for the $\mathrm{X_{cc\overline{cc}}}$, $\mathrm{X_{bb\overline{bb}}}$ are conducted on the Large Hadron Collider (LHC) by the LHCb, ATLAS and CMS Collaborations.

Model description I

- All parameters of the model (including the constituent masses of quarks) are fixed from previous studies of the properties of mesons and baryons.
- Quarks under the consideration:
  - $m_\mathrm{u} = m_\mathrm{d} = 0.33$ GeV,
  - $m_\mathrm{s} = 0.50$ GeV,
  - $m_\mathrm{c} = 1.55$ GeV,
  - $m_\mathrm{b} = 4.88$ GeV.

Model description II

◇ Quark content:

- $Q, Q' = c, b,\ Q \neq Q'$.
- $q = u, d, s$.
- with one open heavy flavor (without/with strangeness):
  - $QQ\overline{Q}\overline{q}$ (+ c.c.).
- with one open and another hidden heavy flavors (without/with strangeness):
  - $QQ'\overline{Q}\overline{q}$ (+ c.c.).
- with two open heavy flavors (without/with strangeness):
  - $QQ\overline{Q}'\overline{q}$ (+ c.c.).

Model description III

◇ Diquark–antidiquark bound state:

- $\{(Q_1Q_2)-(\overline{Q}_3\overline{q}_4)\}$ (+ c.c.).

◇ Diquarks under the consideration:

- nonpoint–like (the internal structure is taken into account)
- ground state $(1S)$,
- color-antitriplet $(\overline{3}_c)$,
- all masses and form factors of diquarks were calculated earlier during analyzing the properties of baryons.

Model description IV

- Ground state diquark spin:
  - $J = 0$ — scalar (S),
  - $J = 1$ — axialvector (A).
- Allowed diquark states:
  - only axialvector (A):
    - QQ.
  - both axialvector and scalar $(\mathrm{A},\ \mathrm{S})$:
    - QQ',
    - Qq.

Model description V

◇ Tetraquark's possible configurations:

- $A\overline{A}$ — any composition,
- $A\overline{S}$ — any composition,
- $S\overline{A}$ — $QQ'\overline{Q}\overline{q}$ (+ c.c.),
- $S\overline{S}$ — $QQ'\overline{Q}\overline{q}$ (+ c.c.).

Relativistic quark model I

- Relativistic Schrödinger-type quasipotential equation:

$$\left(\frac{b^2(M)}{2\mu_{\rm R}(M)} - \frac{\mathbf{p}^2}{2\mu_{\rm R}(M)}\right)\Psi_{\rm d,T}(\mathbf{p}) = \int \frac{d^3 q}{(2\pi)^3}\; V(\mathbf{p},\mathbf{q};M)\Psi_{\rm d,T}(\mathbf{q})$$

$$\mu_{\rm R} = \frac{E_1 E_2}{E_1 + E_2} = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3}$$

$$b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}$$

- Diquark–antidiquark interaction quasipotential:

$$V(\mathbf{p},\mathbf{q};M)=\frac{\langle d(\mathcal{P})|\,J_\mu\,|d(\mathcal{Q})\rangle}{2\sqrt{E_{\rm d}}\sqrt{E_{\rm d}}}\frac{4}{3}\alpha_{\rm s}D^{\mu\nu}(\mathbf{k})\frac{\langle d'(\mathcal{P}')|\,J_\nu\,|d'(\mathcal{Q}')\rangle}{2\sqrt{E_{\rm d'}}\sqrt{E_{\rm d'}}}$$

$$+\Psi^*_{\rm d}(\mathcal{P})\Psi^*_{\rm d'}(\mathcal{P}')[J_{{\rm d};\mu}J^{\mu}_{\rm d'}\,V^{\rm V}_{\rm conf.}(\mathbf{k})+V^{\rm S}_{\rm conf.}(\mathbf{k})]\Psi_{\rm d}(\mathcal{Q})\Psi_{\rm d'}(\mathcal{Q}')$$

Results I

◇ with one open heavy flavor (without/with strangeness):

Table 1: Masses $M_{QQ\bar{Q}\bar{q}}$ of the ground states and orbital excitations of triply heavy tetraquarks with one open heavy flavor and without/with strangeness ($cc\bar{c}\bar{u}$, $cc\bar{c}\bar{s}$, $bb\bar{b}\bar{u}$, $bb\bar{b}\bar{s}$ + c.c.).

dd'	nL	n_r	L	S	J	J^P	$M_{cc\bar{c}\bar{u}}$	$M_{cc\bar{c}\bar{s}}$	$M_{bb\bar{b}\bar{u}}$	$M_{bb\bar{b}\bar{s}}$
$A\bar{A}$	1S	0	0	0	0	0^+	5080	5205	14895	14998
				1	1	1^+	5104	5227	14901	15003
				2	2	2^+	5147	5267	14913	15014
	1P		1	1	0	0^-	5477	5593	15181	15281
				0	1	1^-	5480	5596	15183	15282
				1			5486	5601	15184	15284
				2			5490	5605	15185	15284
				1	2	2^-	5492	5607	15188	15287
				2			5501	5616	15188	15287
				2	3	3^-	5509	5623	15193	15292
	1D		2	2	0	0^+	5816	5926	15389	15488
				1	1	1^+	5816	5925	15390	15489
				2			5821	5930	15391	15490
				0	2	2^+	5815	5926	15392	15490
				1			5822	5931	15392	15491
				2			5830	5938	15394	15492
				1	3	3^+	5821	5931	15394	15492
				2			5831	5939	15395	15493
				2	4	4^+	5829	5938	15396	15495
$A\bar{S}$	1S		0	1	1	1^+	5060	5180	14885	14989
	1P		1		0	0^-	5424	5537	15162	15263
					1	1^-	5430	5542	15164	15265
					2	2^-	5441	5552	15168	15269
	1D		2		1	1^+	5760	5866	15370	15470
					2	2^+	5765	5871	15372	15472
					3	3^+	5773	5878	15374	15474

Analysis I

- If energetically possible, the tetraquark will fall-apart into a meson pair through the quark rearrangement.

$$\Delta = M_{\mathrm{QQ'\overline{Q}''\overline{q}}} - M^{\mathrm{lowest}}_{\mathrm{threshold}}$$

- If $\Delta < 0$, state is stable against fall-apart strong decays.
- The smaller $\Delta > 0$, the narrower is the state.

Analysis II

- Most states lie well above thresholds with $\Delta > 100$ MeV.
- Some states lie above thresholds with $50 < \Delta < 100$ MeV.
- Several states lie slightly above thresholds with $0 < \Delta < 50$ MeV.
- A number of states lie below thresholds with $\Delta < 0$.

Analysis III

- The most promising to be stable states:
  - with one open heavy flavor (without/with strangeness):

Table 2: Ground and orbitally excited states of the triply heavy tetraquarks with one open heavy flavor and without/with strangeness ($cc\bar{c}\bar{u}$, $cc\bar{c}\bar{s}$, $bb\bar{b}\bar{u}$, $bb\bar{b}\bar{s}$ + c.c.), which lie slightly above or below the meson–meson fall-apart strong decay thresholds.

$QQ'\bar{Q}''\bar{q}$	$d\bar{d}'$	nL	S	J^P	M	M_{thr}	Δ	meson pair
$cc\bar{c}\bar{u}$	$A\bar{A}$	1S	2	2^+	5147	5104	43	$D^*(2007)^0\ J/\psi(1S)$
		1P	1	2^-	5492	5421	71	$D^0\ \chi_{c2}(1P)$
			2		5501		9	
			2	3^-	**5509**	5558	**-49**	$D_2^*(2460)\ J/\psi(1S)$
		1D	2	4^+	**5829**	5850	**-21**	$D^*(2007)^0\ \psi_3(3842)$
	$A\bar{S}$	1S	1	1^+	5060	4962	98	$D^0\ J/\psi(1S)$
		1P		1^-	5430	5376	54	$D^0\ \chi_{c1}(1P)$
				2^-	5441	5421	20	$D^0\ \chi_{c2}(1P)$
		1D		3^+	5773	5708	65	$D^0\ \psi_3(3842)$
$cc\bar{c}\bar{s}$	$A\bar{A}$	1S	2	2^+	5267	5209	58	$D_s^*\ J/\psi(1S)$
		1P	1	2^-	5607	5525	82	$D_s^+\ \chi_{c2}(1P)$
			2		5616		91	
			2	3^-	**5623**	5666	**-43**	$D_{s2}^*(2573)\ J/\psi(1S)$
		1D	2	4^+	**5938**	5955	**-17**	$D_s^*\ \psi_3(3842)$
	$A\bar{S}$	1P	1	2^-	5552	5525	27	$D_s^+\ \chi_{c2}(1P)$
		1D		3^+	5878	5811	67	$D_s^+\ \psi_3(3842)$

Analysis IV

Table 2: table continues.

$QQ'\bar{Q}'\bar{q}$	$d\bar{d}'$	nL	S	J^P	M	M_{thr}	Δ	meson pair
$bb\bar{b}\bar{u}$	$A\bar{A}$	1P	1	0^-	15181	15139	42	$B^+\ \chi_{b0}(1P)$
			0	1^-	15183	15125	58	$B_1(5721)\ \eta_b(1S)$
			1		15184		59	
			2		15185		60	
			1	2^-	15188	15136	52	$B_2^*(5747)\ \eta_b(1S)$
			2					
			2	3^-	15193	15198	-5	$B_2^*(5747)\ \Upsilon(1S)$
		1D	1	3^+	15394	15488	-94	$B^*\ \Upsilon_2(1D)$
			2		15395		-93	
			2	4^+	15396	15649	-253	$B_2^*(5747)\ \chi_{b2}(1P)$
	$A\bar{S}$	1P	1	0^-	15162	15139	23	$B^+\ \chi_{b0}(1P)$
				1^-	15164	15125	39	$B_1(5721)\ \eta_b(1S)$
				2^-	15168	15136	32	$B_2^*(5747)\ \eta_b(1S)$
		1D		3^+	15374	15488	-114	$B^*\ \Upsilon_2(1D)$
$bb\bar{b}\bar{s}$	$A\bar{A}$	1P	1	0^-	15281	15226	55	$B_s^0\ \chi_{b0}(1P)$
			0	1^-	15282	15227	55	$B_{s1}(5830)^0\ \eta_b(1S)$
			1		15284		57	
			2		15284		57	
			1	2^-	15287	15239	48	$B_{s2}^*(5840)^0\ \eta_b(1S)$
			2					
			2	3^-	15292	15300	-8	$B_{s2}^*(5840)^0\ \Upsilon(1S)$
		1D	1	3^+	15492	15579	-87	$B_s^*\ \Upsilon_2(1D)$
			2		15493		-86	
			2	4^+	15495	15752	-257	$B_{s2}^*(5840)^0\ \chi_{b2}(1P)$
	$A\bar{S}$	1P	1	0^-	15263	15226	37	$B_s^0\ \chi_{b0}(1P)$
				1^-	15265	15227	38	$B_{s1}(5830)^0\ \eta_b(1S)$
				2^-	15269	15239	30	$B_{s2}^*(5840)^0\ \eta_b(1S)$
		1D		3^+	15474	15579	-105	$B_s^*\ \Upsilon_2(1D)$

Experimental data I

◇ In the fully heavy tetraquark sector there are already experimental advancements:

- While studying the double charmonium production, in 2020 the LHCb Collaboration announced the discovery of the narrow resonance X(6900).
- Several other broad structures peaking at about 6.4 and 7.2 GeV were reported.
- In 2022 ATLAS and CMS Collaborations confirmed X(6900) and hinted on a few more states, including structures at 6.4 and 7.2 GeV.

Experimental data II

- Current observation status and our predictions:

Table 3: Exotic X states observed and hinted by the LHCb, ATLAS and CMS Collaborations in di-J/ψ and $J/\psi\,\psi(2S)$ invariant mass spectra and our candidates. All masses M and total widths Γ are given in MeV.

Collaboration		Resonance	M	Γ	Our candidates: nL	S	J^{PC}	M
LHCb		X(6600)	$6400 \div 6600$					
ATLAS	m_0, model A		$6410 \pm 80^{+80}_{-30}$	$590 \pm 350^{+120}_{-200}$	1S	2	2^{++}	6367
	m_0, model B		$6650 \pm 20^{+30}_{-20}$	$440 \pm 50^{+60}_{-50}$				
	m_1, model A		$6630 \pm 50^{+80}_{-10}$	$350 \pm 110^{+110}_{-40}$				
CMS	BW_1, no interference		$6552 \pm 10 \pm 12$	$124^{+32}_{-26} \pm 33$	2S	0	0^{++}	6782
	BW_1, interference		6638^{+43+16}_{-38-31}	$440^{+230+110}_{-200-240}$				
LHCb	NRSPS, no interference	X(6900)	$6905 \pm 11 \pm 7$	$80 \pm 19 \pm 33$				
	NRSPS, interference		$6886 \pm 11 \pm 11$	$168 \pm 33 \pm 69$	2S	2	2^{++}	6868
ATLAS	m_2, model A		$6860 \pm 30^{+10}_{-20}$	$110 \pm 50^{+20}_{-10}$		0	2^{++}	6921
	m_2, model B		$6910 \pm 10 \pm 10$	$150 \pm 30 \pm 10$	1D	2	0^{++}	6899
	m_3, model β		$6960 \pm 50 \pm 30$	$510 \pm 170^{+110}_{-100}$		2	1^{++}	6904
CMS	BW_2, no interference		$6927 \pm 9 \pm 4$	$122^{+24}_{-21} \pm 18$		2	2^{++}	6915
	BW_2, interference		6847^{+44+48}_{-28-20}	191^{+66+25}_{-49-17}				
LHCb		X(7200)	$7200 \div 7400$					
ATLAS	m_3, model α		$7220 \pm 30^{+10}_{-30}$	$90 \pm 60^{+60}_{-30}$		0	0^{++}	7259
CMS	BW_3, no interference		$7287^{+20}_{-18} \pm 5$	$95^{+59}_{-40} \pm 19$	3S	2	2^{++}	7333
	BW_3, interference		7134^{+48+41}_{-25-15}	97^{+40+29}_{-29-26}				

Experimental data III

- Plenty of new experimental data are expected in the near future, including regions and mass sectors of our interest.

Conclusion I

- Masses of ground and orbitally excited states of all compositions of the triply heavy tetraquarks were calculated.
- The finite size of a diquark was taken into account.
- Diquarks and antidiquarks were considered to interact as a whole.

Conclusion II

- Triply heavy tetraquark states which are the most convenient for the experimental detection were identified.
- There are already experimental advancements in the fully heavy tetraquark sector, and our previous predictions based on the Relativistic Quark Model are consistent with them.

Publications

◇ Previous publications related to the topic:

- Masses of the $QQ\overline{Q}\overline{Q}$ tetraquarks in the relativistic diquark–antidiquark picture, Physical Review D, 2020, vol. 102, №11, p. 114030;
- Heavy Tetraquarks in the Relativistic Quark Model, Universe, 2021, vol. 7, №4, p. 94;
- Fully Heavy Tetraquark Spectroscopy in the Relativistic Quark Model, Symmetry, 2022, vol. 14, №12, p. 2504;
- Relativistic description of asymmetric fully heavy tetraquarks in the diquark–antidiquark model, The European Physical Journal A, 2024, vol. 60, №96;
- Relativistic Description of Asymmetric Fully Heavy Tetraquarks, Physics of Particles and Nuclei Letters, 2024, vol. 21, №4, p. 597–600.

Thank you for your attention!

This work was supported by the Foundation for the Advancement of Theoretical Physics and Mathematics “BASIS” grant №22-2-10-3-1.