Dynamical O(4)-symmetry in the light meson spectrum within the framework of the Regge approach

Alisa Tsymbal

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Saint Petersburg State University

Meson clustering

If one's up to analyze PDG tables, an interesting pattern is seen. Light mesons such as π, η, ρ, ω, a_J, f_J, b_J, h_J all have close masses:

$\approx$ 1300 MeV	11 states
$\approx$ 1600 MeV	8 states
$\approx$ 2000 MeV	22 states
$\approx$ 2250 MeV	24 states

O(4)-symmetry

Inside the excited unflavoured mesons, spin-orbital and spin-spin correlations are suppressed. The discrete mesonic spectrum depends only on a quantum number $\mathrm{N} = \mathrm{n_r} + \mathrm{L}$.
$\vec{\mathrm{S}} = \vec{\mathrm{s}}_{\mathrm{q}} + \vec{\mathrm{s}}_{\bar{\mathrm{q}}}$
Singlets and triplets: $\vec{\mathrm{J}} = \vec{\mathrm{L}} + \vec{\mathrm{S}}$, with $\mathrm{S} = 0$ or $\mathrm{S} = 1$, respectively.

The spectrum can be fitted by

$$\mathrm{M}^2 = \mathrm{a}(\mathrm{n_r} + \mathrm{L}) + \mathrm{c},$$

where $\mathrm{n_r}$ is the radial excitation and L is the meson angular momentum.

L↓	S↓	J↓	n = 0	n = 1	n = 2	n = 3	n = 4
0	0	0	π η	π(1300) η(1295)	π(1800) η(1760)	π(2070) η(2010)	π(2360) η(2320)
	1	1	ρ ω	ρ(1450) ω(1420)	ρ(1700) ω(1650)	ρ(2000) ω(1960)	ρ(2270) ω(2290)
							n = 4
1	0	1	b1(1235) h1(1170)	b1? h1(1595)	b1(1960) h1(1965)	b1(2240) h1(2215)	
	1	0	a0(1450) f0(1370)	a0(1710) f0(1770)	a0(2020) f0(2020)	a0? f0(2200)	
		1	a1(1260) f1(1285)	a1(1640) f1?	a1(1930) f1(1970)	a1(2270) f1(2310)	
		2	a2(1320) f2(1270)	a2(1700) f2(1750)	a2(2030) f2(1950)	a2(2255) f2(2300)	
						n = 3	
2	0	2	π2(1670) η2(1645)	π2(2005) η2(2030)	π2(2285) η2(2250)		
	1	1	ρ? ω?	ρ? ω?	ρ? ω(2220)		
		2	ρ2? ω2?	ρ2(1940) ω2(1975)	ρ2(2225) ω2(2195)		
		3	ρ3(1690) ω3(1670)	ρ3(1990) ω3(1945)	ρ3(2250) ω3(2255)		
					n = 2		
3	0	3	b3(2030) h3(2025)	b3(2245) h3(2275)			
	1	2	a2(1990) f2(2010)	a2(2175) f2(2240)			
		3	a3(2030) f3(2050)	a3(2275) f3(2300)			
		4	a4(1970) f4(2050)	a4(2255) f4(2300)			
				n = 1			
4	0	4	π4(2250) η4(2330)				
	1	3	ρ3? ω3(2285)				
		4	ρ4(2230) ω4(2250)				
		5	ρ5(2350) ω5(2250)				
L↑	S↑	J↑	n = 0				

N = 0
N = 1, M² ≈ 1.72 GeV
N = 2, M² ≈ 2.63 GeV
N = 3, M² ≈ 3.98 GeV
N = 4, M² ≈ 5.11 GeV

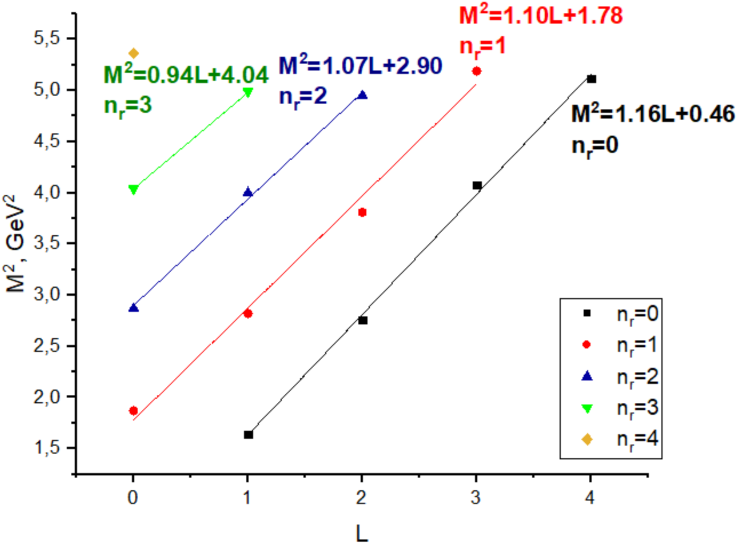
M² = 1.14 N + 0.58

I=1
I=0

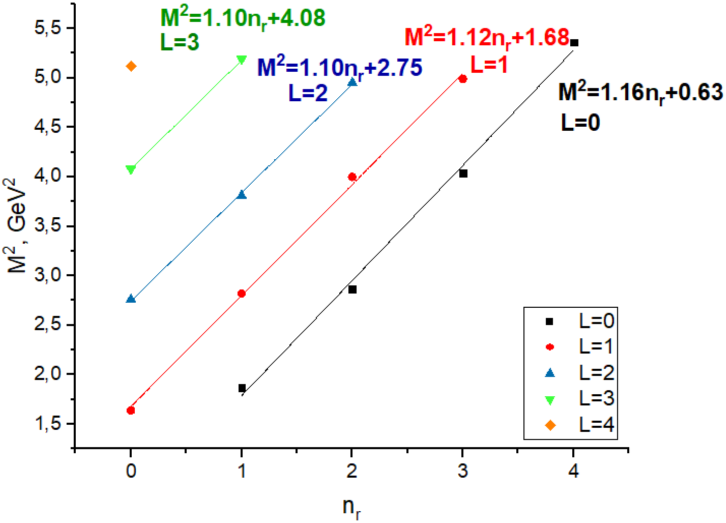
N = n + L

Резонансы с более бледным фоном не участвовали в анализе, и есть сомнения, что это связанное состояние кварк-антикварковой пары.

n_r=constant


M²=1.10L+1.78
n_r=1
M²=0.94L+4.04
n_r=3
M²=1.07L+2.90
n_r=2
M²=1.16L+0.46
n_r=0
M², GeV²
L
5,5
5,0
4,5
4,0
3,5
3,0
2,5
2,0
1,5
0
1
2
3
4
n_r=0
n_r=1
n_r=2
n_r=3
n_r=4

L=constant


M²=1.10n_r+4.08
L=3
M²=1.10n_r+2.75
L=2
M²=1.12n_r+1.68
L=1
M²=1.16n_r+0.63
L=0
M², GeV²
n_r
L=0
L=1
L=2
L=3
L=4
5,5
5,0
4,5
4,0
3,5
3,0
2,5
2,0
1,5
0
1
2
3
4

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		1	a1(1260) f1(1285)	a1(1640) f1?	a1(1930) f1(1970)	a1(2270) f1(2310)	
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	1	1	ρ? ω?	ρ? ω?	ρ? ω(2220)		
		2	ρ2? ω2?	ρ2(1940) ω2(1975)	ρ2(2225) ω2(2195)		
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		3	a3(2030) f3(2050)	a3(2275) f3(2300)			
		4	a4(1970) f4(2050)	a4(2255) f4(2300)			
				n = 1			
4	0	4	π4(2250) η4(2330)				
	1	3	ρ3? ω3(2285)				
		4	ρ4(2230) ω4(2250)				
		5	ρ5(2350) ω5(2250)				
L ↑	S ↑	J ↑	n = 0				

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Slopes

Fitting this data by multiple linear regression, we acquire that in the relation $M^2 = a \cdot n_r + b \cdot L + c$ slopes for n_r and L are the same,

$$a = 1.14 \pm 0.05, \;\; b = 1.14 \pm 0.07, \;\; c \approx 0.5$$

The mass operator

Meson: a gluon string with quark and antiquark at the ends. If L and n_r are big enough, we can use semiclassical methods. The mass operator:

$$M = 2p + \sigma r$$

Here we assume two UR quarks with $m_q = m_{\bar{q}} \ll p$ and linear confining potential σr, σ is equal to effective string tension. The maximal string length is $\ell = M/\sigma$.

Semiclassical quantization for radial excitations

The mass operator is $M = 2p + \sigma r$.

The wave-function for fermions is antisymmertic. The Bohr-Sommerfield quantization condition should be

$$\oint p(r)dr = \pi(n_r + \gamma) \quad \Rightarrow \quad M^2 = 2\pi\sigma(n_r + \gamma)$$

Thus in the $M^2 = an_r + bl + c$ the slope for n_r is $a = 2\pi\sigma$. If one forgets about fermionic nature of quarks, the different slope $4\pi\sigma$ appears.

A collective gluon excitation

The second option: placing quarks at the 0 and ℓ, assume interaction by exchanging massive scalar particle. The mass operator is now $\mathrm{M} = \mathrm{p} + \sigma \mathrm{r}$ and Bohr-Zommerfield quantization condition reads

$$\oint \mathrm{p}(\mathrm{r})\mathrm{dr} = 2\pi(\mathrm{n_r} + \gamma) \quad \Rightarrow \quad \mathrm{M}^2 = 2\pi\sigma(\mathrm{n_r} + \gamma)$$

Again, $\mathrm{a} = 2\pi\sigma$.

Chew-Frautchi formula

Suppose two massless quark rotating at the speed of light at radius $\ell/2$. The flux tube at the distance r from the center of mass rotates with the speed $\mathrm{v(r)} = 2\mathrm{r}/\ell$. The mass and angular momentum of such a "solid" gluon flux tube are

$$\mathrm{M} = 2\int_0^{\ell/2} \frac{\sigma \mathrm{dr}}{\sqrt{1-\mathrm{v}^2(\mathrm{r})}} = \frac{\pi\sigma\ell}{2}, \quad \mathrm{L} = 2\int_0^{\ell/2} \frac{\sigma \mathrm{r v(r) dr}}{\sqrt{1-\mathrm{v}^2(\mathrm{r})}} = \frac{\pi\sigma\ell^2}{8}$$

$$\Rightarrow \quad \mathrm{M}^2 = 2\pi\sigma\mathrm{L}$$

This is the Chew-Frautschi formula and we've got the same slope for L which is $2\pi\sigma$.

Semiclassical look at angular excitations

Quarks stay in circular classical orbits with large r and p. There is also a centripetal acceleration acting on them. Using orbit quantization conditions and applying Newton's law, we obtain

$$\oint \mathrm{p}(\mathrm{r})\mathrm{dr} = \mathrm{L}\hbar, \quad \mathrm{p} = \frac{\sigma \mathrm{r}}{2} \quad \Rightarrow \quad \mathrm{L} = \frac{\sigma \mathrm{r}^2}{2}$$

After some calculations, $\mathrm{M}^2 = 8\sigma\mathrm{L}$ and the slope for L differs crucially from $2\pi\sigma$ for radial excitation, although $\mathrm{M}^2(\mathrm{L})$ is still linear.

Conclusions

- There is a broad mass degeneracy in light unflavoured mesons. The spectrum depends linearly on meson radial quantum number $\mathrm{n_r}$ and its angular momentum L,

$$\mathrm{M^2 = an_r + bL + c.}$$

- The analysis of experimental data shows that $\mathrm{a \approx b \approx 1.14}$. The O(4) degeneracy predicts such a dependency too, discrete meson spectrum must depend on a single quantum number $\mathrm{N = n_r + L + 1}$.

- Semiclassical approach is also capable to give us the needed relation between slopes, $\mathrm{a = b} = 2\pi\sigma$, under certain conditions.