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Angular excitations

Conclusions 0

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Dynamical O(4)-symmetry in the light meson spectrum within the framework of the Regge approach

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Meson clustering

If one's up to analyze PDG tables, an interesting pattern is seen. Light mesons such as π , η , ρ , ω , a_J , f_J , b_J , h_J all have close masses:

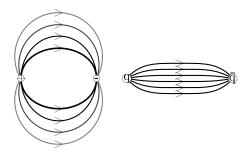
$\approx 1300 \text{ MeV}$	11 states
$\approx 1600 { m MeV}$	8 states
$\approx 2000 { m MeV}$	22 states
$\approx 2250 \text{ MeV}$	24 states

 $_{0 \bullet 0}^{\rm Introduction}$

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The hadronic string



 $\mathrm{P}\mathbf{\mu}\mathrm{c.}$ 1: electromagnetic field VS gluonic flux tube: a simple visualisation

$$M^2 = an + bJ + c$$

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O(4)-symmetry

Inside the excited unflavoured mesons, spin-orbital and spin-spin correlations are suppressed. Therefore we can describe mesons made of spinor quarks as if they were made of scalar quarks.

Total quark spin: $\vec{S} = \vec{s}_q + \vec{s}_{\bar{q}}$ Singlets and triplets: $\vec{J} = \vec{L} + \vec{S}$, with S = 0 or S = 1, respectively.

The spectrum can be fitted by a relation

$$M^2 = an_r + bL + c,$$

where n_r is the radial excitation and L is the meson angular momentum.

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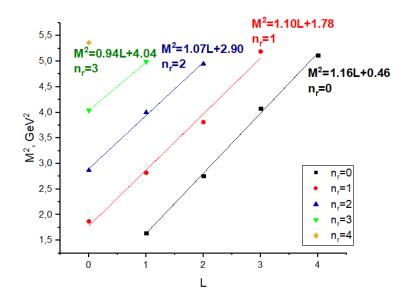
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 $n_r = constant$

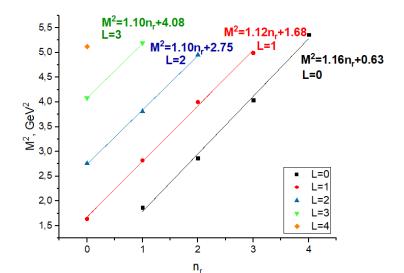


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L=constant



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Fitting this data by multiple linear regression, we acquire that in the relation $M^2 = a \cdot n_r + b \cdot L + c$ slopes for n_r and L are the same,

$$a = 1.14 \pm 0.05, b = 1.14 \pm 0.07, c \approx 0.5$$

We can conclude therefore that the whole spectrum depends on a linear combination $n_r + L$ as prescribed by O(4)-symmetry.

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The mass operator and the hamiltonian

The mass operator is identified with hamiltonian, and we suggest $\frac{m}{|p|} \ll 1$:

$$\widehat{H}\psi = E\psi, \quad H = \sqrt{p^2 + m^2} + V(r) \approx 2p + V(r)$$

Here the momentum operator contains both radial and angular parts, $p^2=p_r^2+\frac{L}{r^2}$

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About potential

We use a Cornell potential which is

$$V(\mathbf{r}) = \sigma \mathbf{r} - C_{\rm F} \frac{\alpha_{\rm s}}{\mathbf{r}} + c$$

The Casimir term C_F depends on N_c and usually is $N_c = 3$:

$$C_F \equiv \frac{N_c^2 - 1}{2N_c} = \frac{4}{3}$$

The running coupling α_s depends on a distance between quarks, but we'll not stop here.

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${\rm Large}~N_c~{\rm limit}$

An example: $\tau_{\pi^{\pm}} = 2, 6 \cdot 10^{-8}$ s, and its charge radius $\sqrt{\langle \mathbf{r}_{\pi^{\pm}}^2 \rangle} \approx 0,659$ fm, therefore $\tau_{\pi,\mathbf{r}} \approx 10^{-24}$ is the time needed to move through the whole string. So no problems? For mesons above 1 GeV which are acceptable to be considered as strings, $\tau_{\mathrm{str}} \approx 10^{-24}$ s. The decay width is

$$\tau = \frac{1}{\Gamma} = \frac{N_{\rm c}}{B \cdot M}$$

B = const and M is meson mass. If $N_c \to \infty, \tau \to \infty$ too.

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Semiclassical quantization for radial excitations

The mass operator is $M = 2p + \sigma r$.

The wave-function for fermions is antisymmetric. The Bohr-Sommerfield quantization condition should be

$$\oint p(\mathbf{r})d\mathbf{r} = \pi(\mathbf{n}_{\mathbf{r}} + \gamma) \quad \Rightarrow \quad M^2 = 2\pi\sigma(\mathbf{n}_{\mathbf{r}} + \gamma)$$

Thus in the $M^2 = an_r + bl + c$ the slope for n_r is $a = 2\pi\sigma$. If one forgets about fermionic nature of quarks, the different slope $4\pi\sigma$ appears. Radial quantum number $O \bullet$

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A collective gluon excitation

The second option: placing quarks at the 0 and ℓ , assume interaction by exchanging massive scalar particle. The mass operator is now $M = p + \sigma r$ and Bohr-Zommerfield quantization condition reads

$$\oint p(\mathbf{r})d\mathbf{r} = 2\pi(\mathbf{n}_{\mathbf{r}} + \gamma) \quad \Rightarrow \quad M^2 = 2\pi\sigma(\mathbf{n}_{\mathbf{r}} + \gamma)$$

Again, $a = 2\pi\sigma$.

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Chew-Frautchi formula

Suppose two massless quark rotating at the speed of light at radius $\ell/2$. The flux tube at the distance r from the center of mass rotates with the speed v(r) = $2r/\ell$. The mass and angular momentum of such a "solid" gluon flux tube are

$$M = 2 \int_{0}^{\ell/2} \frac{\sigma dr}{\sqrt{1 - v^2(r)}} = \frac{\pi \sigma \ell}{2}, \quad L = 2 \int_{0}^{\ell/2} \frac{\sigma rv(r) dr}{\sqrt{1 - v^2(r)}} = \frac{\pi \sigma \ell^2}{8}$$
$$\Rightarrow \quad M^2 = 2\pi \sigma L$$

This is the Chew-Frautschi formula and we've got the same slope for L which is $2\pi\sigma$.

Semiclassical look at angular excitations

Quarks stay in circular classical orbits with large r and p. There is also a centripetal acceleration acting on them. Using orbit quantization conditions and applying Newton's law, we obtain

$$\oint p(\mathbf{r})d\mathbf{r} = L\hbar, \quad p = \frac{\sigma r}{2} \quad \Rightarrow \quad L = \frac{\sigma r^2}{2}$$

After some calculations, $M^2 = 8\sigma L$ and the slope for L differs crucially from $2\pi\sigma$ for radial excitation, although $M^2(L)$ is still linear.

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Conclusions

• There is a broad mass degeneracy in light unflavoured mesons. The spectrum depends linearly on meson radial quantum number n_r and its angular momentum L,

 $M^2 = an_r + bL + c. \label{eq:matrix}$

- The analysis of experimental data shows that a \approx b \approx 1.14. The O(4) degeneracy predicts such a dependency too, discrete meson spectrum must depend on a single quantum number N = n_r + L + 1.
- Semiclassical approach is also capable to give us the needed relation between slopes, $a = b = 2\pi\sigma$, under certain conditions.