Dynamical O(4)-symmetry in the light meson spectrum within the framework of the Regge approach

Sergey Afonin, Alisa Tsymbal

Saint Petersburg State University

Meson clustering

If one's up to analyze PDG tables, an interesting pattern is seen. Light mesons such as π, η, ρ, ω, $\mathrm{a_J}$, $\mathrm{f_J}$, $\mathrm{b_J}$, $\mathrm{h_J}$ all have close masses:

$\approx$ 1300 MeV	11 states
$\approx$ 1600 MeV	8 states
$\approx$ 2000 MeV	22 states
$\approx$ 2250 MeV	24 states

The hadronic string

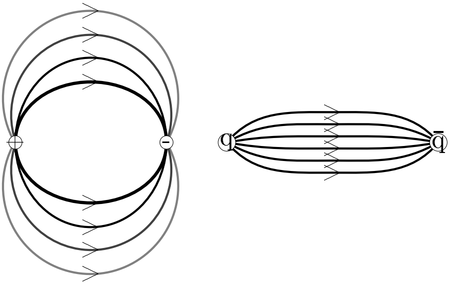


Рис. 1: electromagnetic field VS gluonic flux tube: a simple visualisation

$$M^2 = an + bJ + c$$

O(4)-symmetry

Inside the excited unflavoured mesons, spin-orbital and spin-spin correlations are suppressed. Therefore we can describe mesons made of spinor quarks as if they were made of scalar quarks.
Total quark spin: $\vec{S} = \vec{s}_q + \vec{s}_{\bar{q}}$
Singlets and triplets: $\vec{J} = \vec{L} + \vec{S}$, with $S = 0$ or $S = 1$, respectively.

The spectrum can be fitted by a relation

$$M^2 = an_r + bL + c,$$

where n_r is the radial excitation and L is the meson angular momentum.

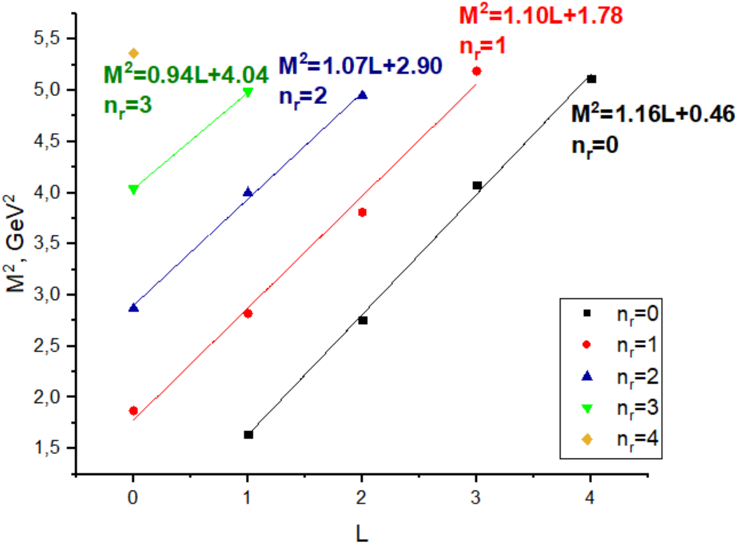
L	S	J	n = 0	n = 1	n = 2	n = 3	n = 4
0	0	0	π η	π(1300) η(1295)	π(1800) η(1760)	π(2070) η(2010)	π(2360) η(2320)
0	1	1	ρ ω	ρ(1450) ω(1420)	ρ(1700) ω(1650)	ρ(2000) ω(1960)	ρ(2270) ω(2290)
1	0	1	b1(1235) h1(1170)	b1? h1(1595)	b1(1960) h1(1965)	b1(2240) h1(2215)	
1	1	0	a0(1450) f0(1370)	a0(1710) f0(1770)	a0(2020) f0(2020)	a0? f0(2200)	
1	1	1	a1(1260) f1(1285)	a1(1640) f1?	a1(1930) f1(1970)	a1(2270) f1(2310)	
1	1	2	a2(1320) f2(1270)	a2(1700) f2(1750)	a2(2030) f2(1950)	a2(2255) f2(2300)	
2	0	2	π2(1670) η2(1645)	π2(2005) η2(2030)	π2(2285) η2(2250)		
2	1	1	ρ? ω?	ρ? ω?	ρ? ω(2220)		
2	1	2	ρ2? ω2?	ρ2(1940) ω2(1975)	ρ2(2225) ω2(2195)		
2	1	3	ρ3(1690) ω3(1670)	ρ3(1990) ω3(1945)	ρ3(2250) ω3(2255)		
3	0	3	b3(2030) h3(2025)	b3(2245) h3(2275)			
3	1	2	a2(1990) f2(2010)	a2? f2?			
3	1	3	a3(2030) f3(2050)	a3(2275) f3(2300)			
3	1	4	a4(1970) f4(2050)	a4(2255) f4?			
4	0	4	π4(2250) η4(2330)				
4	1	3	ρ3? ω3(2285)				
4	1	4	ρ4(2230) ω4(2250)				
4	1	5	ρ5(2350) ω5(2250)				

N = 0
N = 1
N = 2
N = 3
N = 4

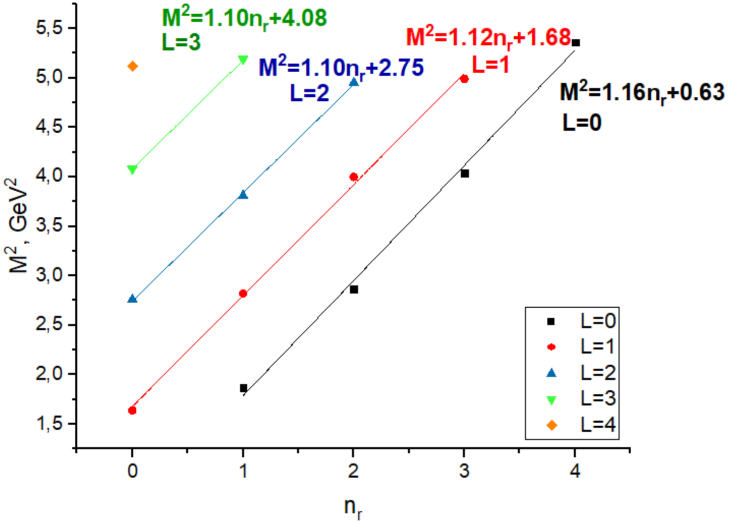
I = 1
I = 0

N = n + L

n_r=constant


M²=1.10L+1.78
n_r=1
M²=0.94L+4.04
n_r=3
M²=1.07L+2.90
n_r=2
M²=1.16L+0.46
n_r=0
M², GeV²
L
n_r=0
n_r=1
n_r=2
n_r=3
n_r=4

L=constant


M2=1.10nr+4.08
L=3
M2=1.10nr+2.75
L=2
M2=1.12nr+1.68
L=1
M2=1.16nr+0.63
L=0
M2, GeV2
nr
5,5
5,0
4,5
4,0
3,5
3,0
2,5
2,0
1,5
0
1
2
3
4
L=0
L=1
L=2
L=3
L=4

L	S	J	n = 0	n = 1	n = 2	n = 3	n = 4
0	0	0	π η	π(1300) η(1295)	π(1800) η(1760)	π(2070) η(2010)	π(2360) η(2320)
0	1	1	ρ ω	ρ(1450) ω(1420)	ρ(1700) ω(1650)	ρ(2000) ω(1960)	ρ(2270) ω(2290)
1	0	1	b1(1235) h1(1170)	b1? h1(1595)	b1(1960) h1(1965)	b1(2240) h1(2215)	
1	1	0	a0(1450) f0(1370)	a0(1710) f0(1770)	a0(2020) f0(2020)	a0? f0(2200)	
1	1	1	a1(1260) f1(1285)	a1(1640) f1?	a1(1930) f1(1970)	a1(2270) f1(2310)	
1	1	2	a2(1320) f2(1270)	a2(1700) f2(1750)	a2(2030) f2(1950)	a2(2255) f2(2300)	
2	0	2	π2(1670) η2(1645)	π2(2005) η2(2030)	π2(2285) η2(2250)		
2	1	1	ρ? ω?	ρ? ω?	ρ? ω(2220)		
2	1	2	ρ2? ω2?	ρ2(1940) ω2(1975)	ρ2(2225) ω2(2195)		
2	1	3	ρ3(1690) ω3(1670)	ρ3(1990) ω3(1945)	ρ3(2250) ω3(2255)		
3	0	3	b3(2030) h3(2025)	b3(2245) h3(2275)			
3	1	2	a2(1990) f2(2010)	a2? f2?			
3	1	3	a3(2030) f3(2050)	a3(2275) f3(2300)			
3	1	4	a4(1970) f4(2050)	a4(2255) f4?			
4	0	4	π4(2250) η4(2330)				
4	1	3	ρ3? ω3(2285)				
4	1	4	ρ4(2230) ω4(2250)				
4	1	5	ρ5(2350) ω5(2250)				

N = 0
N = 1
N = 2
N = 3
N = 4

I = 1
I = 0

N = n + L

Slopes

Fitting this data by multiple linear regression, we acquire that in the relation $\mathrm{M}^2 = \mathrm{a} \cdot \mathrm{n_r} + \mathrm{b} \cdot \mathrm{L} + \mathrm{c}$ slopes for $\mathrm{n_r}$ and L are the same,

$$\mathrm{a} = 1.14 \pm 0.05,\ \ \mathrm{b} = 1.14 \pm 0.07,\ \ \mathrm{c} \approx 0.5$$

We can conclude therefore that the whole spectrum depends on a linear combination $\mathrm{n_r} + \mathrm{L}$ as prescribed by O(4)-symmetry.

The mass operator and the hamiltonian

The mass operator is identified with hamiltonian, and we suggest $\frac{m}{|p|} \ll 1$:

$$\widehat{H}\psi = E\psi, \quad H = \sqrt{p^2 + m^2} + V(r) \approx 2p + V(r)$$

Here the momentum operator contains both radial and angular parts, $p^2 = p_r^2 + \frac{L}{r^2}$

About potential

We use a Cornell potential which is

$$V(r) = \sigma r - C_F \frac{\alpha_s}{r} + c$$

The Casimir term C_F depends on N_c and usually is $N_c = 3$:

$$C_F \equiv \frac{N_c^2 - 1}{2N_c} = \frac{4}{3}$$

The running coupling α_s depends on a distance between quarks, but we'll not stop here.

Large N_c limit

An example: $\tau_{\pi^\pm} = 2,6 \cdot 10^{-8}$ s, and its charge radius $\sqrt{< r^2_{\pi^\pm} >} \approx 0,659$ fm, therefore $\tau_{\pi,r} \approx 10^{-24}$ is the time needed to move through the whole string. So no problems? For mesons above 1 GeV which are acceptable to be considered as strings, $\tau_{str} \approx 10^{-24}$ s. The decay width is

$$\tau = \frac{1}{\Gamma} = \frac{N_c}{B \cdot M}$$

$B = \text{const}$ and M is meson mass. If $N_c \to \infty$, $\tau \to \infty$ too.

Semiclassical quantization for radial excitations

The mass operator is $M = 2p + \sigma r$.

The wave-function for fermions is antisymmertic. The Bohr-Sommerfield quantization condition should be

$$\oint p(r)dr = \pi(n_r + \gamma) \quad \Rightarrow \quad M^2 = 2\pi\sigma(n_r + \gamma)$$

Thus in the $M^2 = an_r + bl + c$ the slope for n_r is $a = 2\pi\sigma$. If one forgets about fermionic nature of quarks, the different slope $4\pi\sigma$ appears.

A collective gluon excitation

The second option: placing quarks at the 0 and ℓ, assume interaction by exchanging massive scalar particle. The mass operator is now $\mathrm{M} = \mathrm{p} + \sigma \mathrm{r}$ and Bohr-Zommerfield quantization condition reads

$$\oint \mathrm{p}(\mathrm{r})\mathrm{dr} = 2\pi(\mathrm{n_r} + \gamma) \quad \Rightarrow \quad \mathrm{M}^2 = 2\pi\sigma(\mathrm{n_r} + \gamma)$$

Again, $\mathrm{a} = 2\pi\sigma$.

Chew-Frautchi formula

Suppose two massless quark rotating at the speed of light at radius $\ell/2$. The flux tube at the distance r from the center of mass rotates with the speed $\mathrm{v(r)} = 2\mathrm{r}/\ell$. The mass and angular momentum of such a "solid" gluon flux tube are

$$\mathrm{M} = 2\int_0^{\ell/2} \frac{\sigma \mathrm{dr}}{\sqrt{1-\mathrm{v}^2(\mathrm{r})}} = \frac{\pi\sigma\ell}{2}, \quad \mathrm{L} = 2\int_0^{\ell/2} \frac{\sigma \mathrm{r v(r) dr}}{\sqrt{1-\mathrm{v}^2(\mathrm{r})}} = \frac{\pi\sigma\ell^2}{8}$$

$$\Rightarrow \quad \mathrm{M}^2 = 2\pi\sigma \mathrm{L}$$

This is the Chew-Frautschi formula and we've got the same slope for L which is $2\pi\sigma$.

Semiclassical look at angular excitations

Quarks stay in circular classical orbits with large r and p. There is also a centripetal acceleration acting on them. Using orbit quantization conditions and applying Newton's law, we obtain

$$\oint \mathrm{p(r)dr} = \mathrm{L}\hbar, \quad \mathrm{p} = \frac{\sigma \mathrm{r}}{2} \quad \Rightarrow \quad \mathrm{L} = \frac{\sigma \mathrm{r}^2}{2}$$

After some calculations, $\mathrm{M}^2 = 8\sigma\mathrm{L}$ and the slope for L differs crucially from $2\pi\sigma$ for radial excitation, although $\mathrm{M}^2(\mathrm{L})$ is still linear.

Conclusions

- There is a broad mass degeneracy in light unflavoured mesons. The spectrum depends linearly on meson radial quantum number n_r and its angular momentum L,

$$M^2 = an_r + bL + c.$$

- The analysis of experimental data shows that $a \approx b \approx 1.14$. The O(4) degeneracy predicts such a dependency too, discrete meson spectrum must depend on a single quantum number $N = n_r + L + 1$.

- Semiclassical approach is also capable to give us the needed relation between slopes, $a = b = 2\pi\sigma$, under certain conditions.