Interacting color strings approach to describe puzzling long-range azimuthal correlations in p+p data

Daria Prokhorova, Evgeny Andronov

Laboratory of Ultra-High Energy Physics, St. Petersburg University

based on D. Prokhorova, E. Andronov, Physics, 6 (2024) 264

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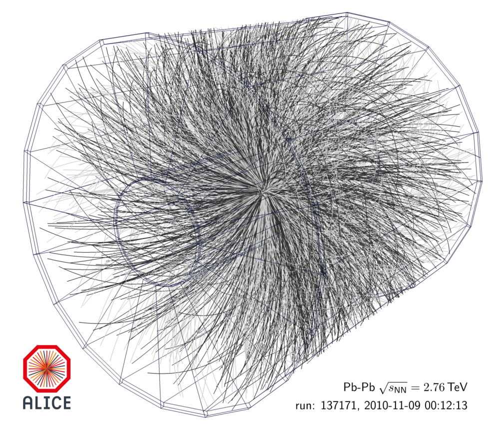
Motivation and relevance

$A + A$ collision event in high energy physics



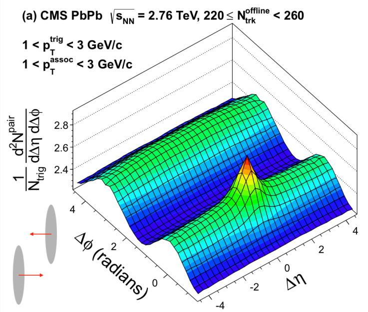
[https://cds.cern.ch/record/2032743]

- **soft** regime: produced particles with $p_T < 1$ GeV
- **perturbative** QCD calculations inapplicable
- **Q**uark-**G**luon **P**lasma formation [E. Shuryak, Sov. Phys. JETP **47** (1978) 212]

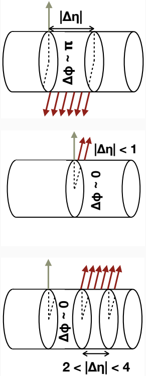
QGP **properties**: liquid phase, $T_{\rm crit} \sim 150$ MeV, $\varepsilon_{\rm crit} \sim 1$ GeV/fm^3
QGP **signals**: strangeness enhancement, azimuthal flows, jet quenching...

Two-particle angular correlation function: ridge in A+A

Near-side ($\Delta\phi \approx 0, |\Delta\eta| > 2$) ridge manifests **collectivity** in peripheral $A + A$



[CMS, Phys. Lett. B **724** (2013) 213]



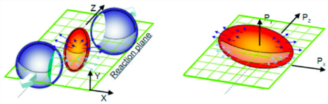
$$\eta = \frac{1}{2} \ln \left(\frac{|\vec{p}| + p_z}{|\vec{p}| - p_z} \right)$$

$$\Delta\eta = \eta_1 - \eta_2$$

$$\Delta\phi = \phi_1 - \phi_2$$

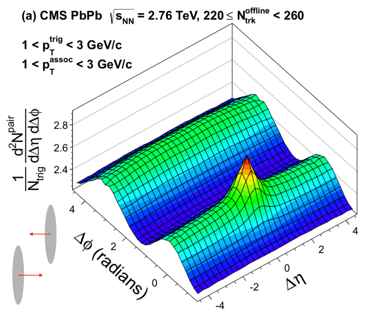
$$C(\Delta\eta, \Delta\phi) = \frac{S(\Delta\eta, \Delta\phi)}{B(\Delta\eta, \Delta\phi)}$$

- relativistic fluid converts **initial spatial anisotropies** to momentum asymmetries
- flow reflects initial conditions of QGP and **medium** transport properties



Two-particle angular correlation function: ridge in A+A and p+p

Unexpected **near-side ridge** in $p + p \sim$ peripheral $A + A$ collisions



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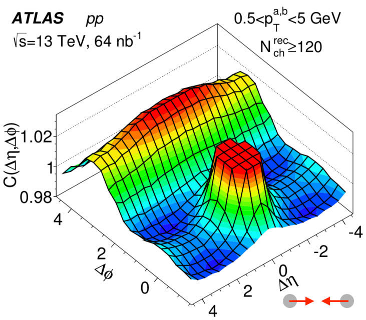
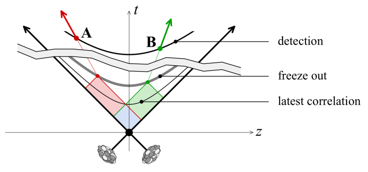


[ATLAS, Phys. Rev. C **96** (2017) 024908]

- ◇ medium produced in $p + p$ is not thermalized as in $A + A$ prior to hadronisation [V. Ambrus et al. Phys. Rev. Lett. **130** (2023) 15230]
- ◇ hydro in p+p? [R. D. Weller, P. Romatschke, **One fluid to rule them all** ... Phys. Lett. B **774** (2017) 351, Y. Zhou et al. **One fluid might not rule them all**, Nucl. Phys. A **1005** (2021) 121908]

Origin of long-range correlations from another perspective

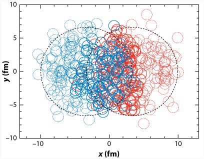
The near-side azimuthal correlations in both $A + A$ and $p + p$ collisions are **long-range in rapidity** (with $|\Delta\eta| > 1$) and



[A. Dumitru et al. Nucl. Phys. A **810** (2008) 91]

- by causality can arise only at **early times** of the collision
- are analogous to the large scale fluctuations in CMB

How to take into account **initial conditions**?

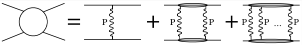


- fluctuating distributions of participant nucleons
- approaches with **longitudinally extended** colour fields:
  - Dual Parton Model [A. Capella, Phys. Rep. **236** (1994) 225]
  - String percolation model [M. Braun, C. Pajares, Nucl. Phys. B **390** (1993) 542]
  - Colour-Glass Condensate + Glasma [F. Gelis, Int. J. Mod. Phys. A **28** (2013) 1330001]

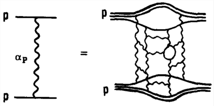
The overview of the colour string model approach

Advent of the colour string model of particle production

- pre-QCD Regge-Gribov approach: high-energy elastic scattering amplitude as multiple **Pomeron exchanges** [V. N. Gribov, JETP **53** (1967) 654]

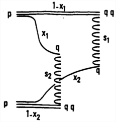


- dominant contribution of QCD topological expansion in **large N_c and N_f limit** – cylindrical diagram corresponds to the Pomeron exchange [G. Veneziano, Nucl. Phys. B **74** (1974) 365; Phys. Lett. B **52** (1974) 220; Nucl. Phys. B **117** (1976) 519]

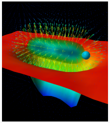


[Capella, Phys. Rep. **236** (1994) 225]

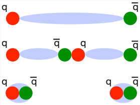
- **space-time localisation** of the cylindrical pomeron exchange diagram with unitarity cut: two-rapidity-chains fragmenting into soft particles [A. Capella et al. Phys. Lett. B **81** (1979) 68; A. B. Kaidalov, Phys. Lett. B **116** (1982) 459; X. Artru, Phys. Rep. **97** (1983) 147]



- **Cornell potential** between confined colour charges [E. Eichten et al. Phys. Rev. Lett. **34** (1975) 369]



P. Varilly, Thesis, MIT (2006)

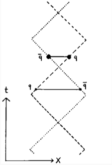


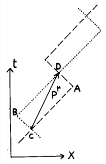
$$V(r) = -\frac{4}{3} \cdot \frac{\alpha_S}{r} + \sigma_T \cdot r,$$

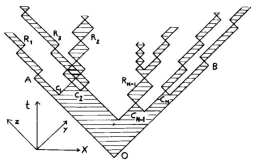
- α_S - QCD running coupling
- σ_T - string tension
- $q\bar{q}$ pair production

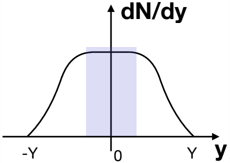
String fragmentation

- in $1+1$ space-time: massless relativistic string is a **yo-yo mode** solution of $dp/dt = \pm\sigma_T$ equation of motion [X. Artru, Phys. Rep. **97** (1983) 147]
- probabilistic string fragmentation depends on hatched area spanned by quarks' motion [B. Andersson, G. Gustafson, G. Ingelman, T. Sjostrand, Phys. Rep. **97** (1983) 31]
- colourless hadrons **uniformly** distributed over rapidity, $y = \frac{1}{2}\ln\left(\frac{p_0+p_z}{p_0-p_z}\right)$









- common approximation: **infinite** in rapidity strings
- convenient for finite experimental acceptances at **mid-rapidity** [S. Belokurova, V. Vechernin, Symmetry **12** (2020) 110]
- important to estimate the impact of finite strings' length on long-range correlations

Model of interacting colour strings finite in rapidity

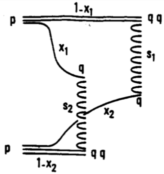
Multi-pomeron exchange in inelastic $p+p$ interaction

Step I: find a number of strings depending on collision energy

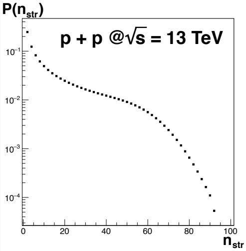
number of **cut pomerons** [A. Kaidalov et al. Phys. Lett. B **117** (1982) 247] $\rightarrow$ number of **strings** in an event: $n_{\mathrm{str}} = 2n_{\mathrm{pom}}$ [A. Capella et al. Phys. Rep. **236** (1994) 225] :

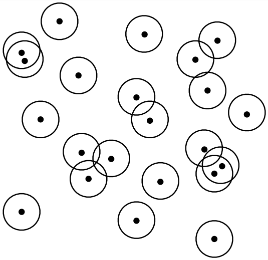
$$P(n_{\mathrm{pom}}) = C(z)\,\frac{1}{z n_{\mathrm{pom}}}\left(1-\exp(-z)\sum_{l=0}^{n_{\mathrm{pom}}-1}\frac{z^l}{l!}\right), \qquad (1)$$

$z = \frac{2w\gamma s^{\Delta}}{R^2+\alpha' \ln s}$, $w = 1.5$, $\Delta = \alpha(0) - 1 = 0.2$, $\gamma = 1.035$ GeV^{-2}, $R^2 = 3.3$ GeV^{-2}, $\alpha' = 0.05$ GeV^{-2} from [V. Vechernin, S. Belokurova, J. Phys. Conf. Ser. **1690** (2020) 012088]



[A. Capella, Phys. Rep. **236** (1994) 225]

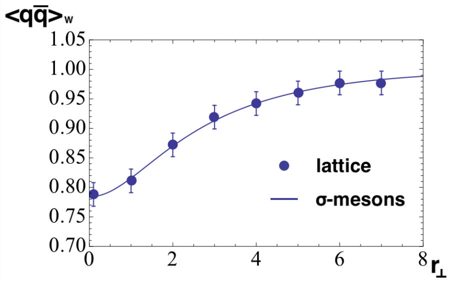




Chiral symmetry restoration in the presence of a colour string

Step II: take into account interactions of free strings

Left: lattice correlator $\langle q\bar{q}\rangle$-W [T. Iritani et al. PoS **LATTICE2013** (2014) 37] $\leftrightarrow$ QCD vacuum modifications due to the presence of a QCD string



$$\frac{\langle q\bar{q}\,(r_\perp)\,W\rangle}{\langle q\bar{q}\rangle\langle W\rangle} = 1 - CK_0\left(m_\sigma \widetilde{r_\perp}\right)$$

- K_0 - modified Bessel function
- $m_\sigma = 0.6$ GeV
- $\widetilde{r_\perp} = \sqrt{r_\perp^2 + s_{\rm str}^2}$
- $s_{\rm str} = 0.176$ fm, $C = 0.26$

Right: scalar field of σ-mesons with Yukawa potential from straight string

[T. Kalaydzhyan, E. Shuryak, Phys. Rev. D **90** (2014) 025031; Phys. Rev. C **90** (2014) 014901]

String-string transverse interaction $\leftrightarrow$ motion of 2D gas of particles

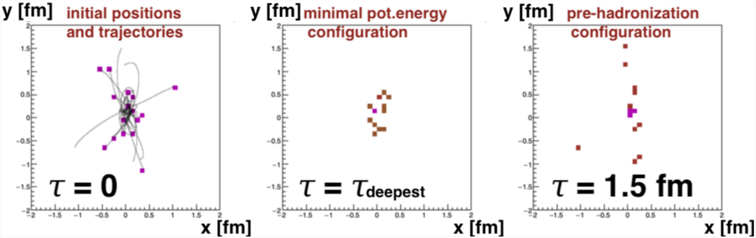
The attractive transverse evolution of the string density

Step III: find new string transverse coordinates

strings **clustering** [T. Kalaydzhyan, E. Shuryak, Phys. Rev. C **90** (2014) 01490] :

$$\ddot{\vec{r}}_i = \sum_{j \neq i} \vec{f}_{ij} = 2m_\sigma (g_N \sigma_T) \sum_{j \neq i} \frac{\vec{r}_{ij}}{\tilde{r}_{ij}} K_1(m_\sigma \tilde{r}_{ij}), \quad (2)$$

$m_\sigma = 0.6$ GeV, $g_N \sigma_T = 0.2$ - string self-interaction coupling, K_1 - first modified second-kind Bessel function



- $\tau_{\text{deepest}} \to$ largest string density
- $\tau = 1.5$ fm $\to$ conventional time before string hadronisation

Longitudinal dynamics of finite strings

Step IV: find rapidity coordinates of strings' ends

- **initial rapidity** of string end defined by current quark mass m_q and carried proton momentum fraction x_q from PDFs [A. Buckley et al. Eur. Phys. J. C **75** (2015) 132]

$$y_q^{\mathrm{init}} = \sinh^{-1}\left(\frac{x_q p_{\mathrm{beam}}}{m_q}\right) \tag{3}$$

- **rapidity loss** for massive partons at string ends due to string tension $dp_q/dt = -\sigma_T$

$$y_q^{\mathrm{loss}} = \cosh^{-1}\left(\frac{\tau^2\sigma_T^2}{2m_q^2} + 1\right), \tag{4}$$

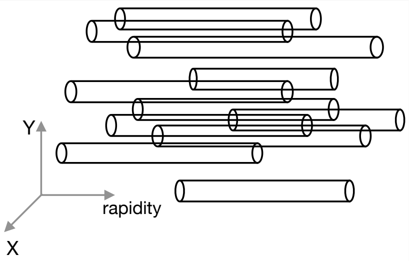
τ - same as in transverse dynamics but with periodicity

$$y_q^{\mathrm{fin}} = y_q^{\mathrm{init}} \pm y_q^{\mathrm{loss}} \tag{5}$$

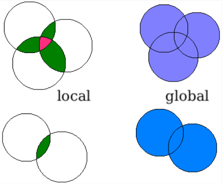
p+p event in our picture after 3D evolution of string density

Transverse evolution + longitudinal dynamics $\rightarrow$

non-uniform 3D strings density



- finite transverse strings size $\leftrightarrow$ colour confinement [M. Baker et al. Eur. Phys. J. C. **80** (2020) 514]
- how to take into account string interactions? [V. Vechernin, Phys. Atom. Nucl. **70** (2007) 1809]

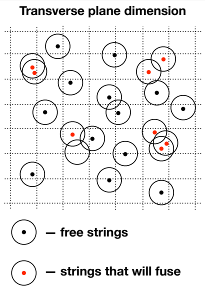


Interacting strings finite in rapidity: 3D overlaps

Step V: find overlaps of strings finite in rapidity

Cellular fusion on coarse grid — vs — Local fusion on fine lattice

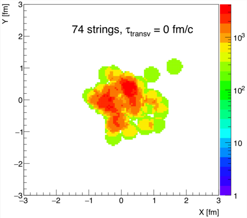
[M. Braun et al. Eur. Phys. J. C **32** (2004) 535] — [D. Prokhorova, E. Andronov, Physics, **6** (2024) 264]

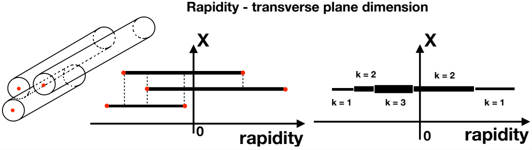


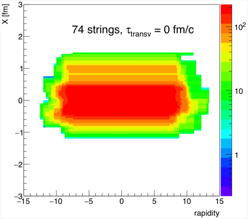
← strings' fragments with centres in the same cell and overlapping in rapidity will form a cluster

vs

strings are split to 3D fine bins with occupation numbers →



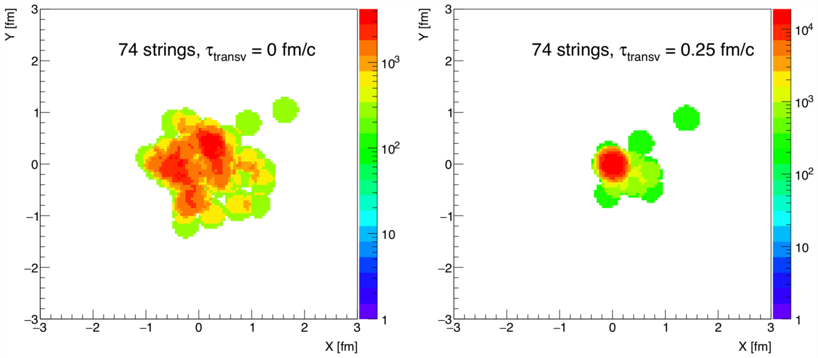




String distributions on a fine lattice

Step V: find overlaps of strings finite in rapidity

projection to **X–Y** plane before/after attractive transverse dynamics

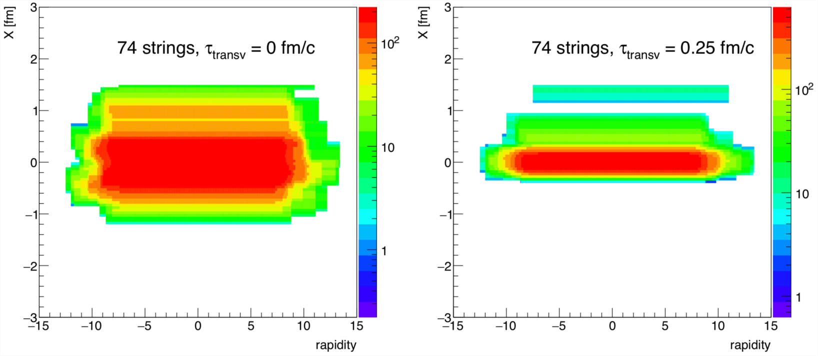


$$\ddot{\vec{r}}_i = \sum_{j \neq i} \vec{f}_{ij} = 2 m_\sigma (g_N \sigma_T) \sum_{j \neq i} \frac{\vec{r}_{ij}}{\tilde{r}_{ij}} K_1(m_\sigma \tilde{r}_{ij})$$

String distributions on a fine lattice

Step V: find overlaps of strings finite in rapidity

projection to **X–rapidity** plane before/after attractive transverse dynamics

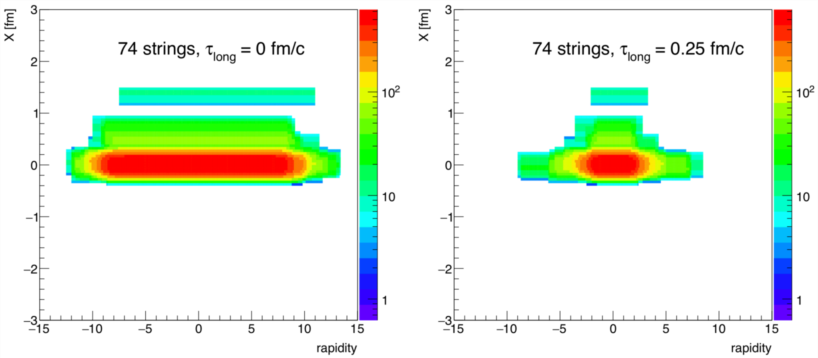


$$\ddot{\vec{r}}_i = \sum_{j\neq i} \vec{f}_{ij} = 2m_\sigma(g_N\sigma_T)\sum_{j\neq i}\frac{\vec{r}_{ij}}{\tilde{r}_{ij}}K_1(m_\sigma\tilde{r}_{ij})$$

String distributions on a fine lattice

Step V: find overlaps of strings finite in rapidity

projection to **X–rapidity** plane before/after longitudinal dynamics

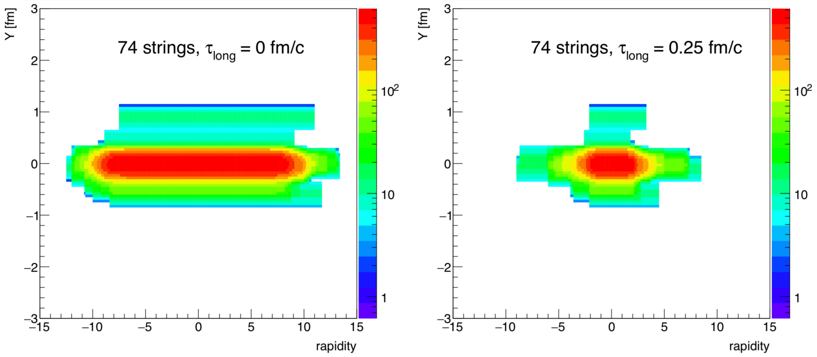


$$y_q^{\mathrm{fin}} = \sinh^{-1}\left(\frac{x_q p_{\mathbf{beam}}}{m_q}\right) \pm \cosh^{-1}\left(\frac{\tau^2\sigma_T^2}{2m_q^2}+1\right)$$

String distributions on a fine lattice

Step V: find overlaps of strings finite in rapidity

projection to **Y–rapidity** plane before/after longitudinal dynamics



$$y_q^{\mathrm{fin}} = \sinh^{-1}\left(\frac{x_q p_{\mathrm{beam}}}{m_q}\right) \pm \cosh^{-1}\left(\frac{\tau^2\sigma_T^2}{2m_q^2} + 1\right)$$

String fusion and particle production

Step VI: take into account string fusion

- mean multiplicity from a **cluster of k strings in rapidity** interval $\varepsilon_{\rm rap}$ [M. Braun et al. Int. J. Mod. Phys. A **14** (1999) 2689]

$$\boxed{\mu_{\rm bin} = \mu_0 \varepsilon_{\rm rap} \sqrt{k} \frac{S_{\rm bin}}{S_0}} \tag{6}$$

- **Poisson** multiplicity for each $\varepsilon_{\rm rap}$, $N_{\rm ch}$ is a sum of all $\varepsilon_{\rm rap}$ contributions
- particle's uniform $\phi \in [-\pi, \pi]$
- mean p_T of particles produced by a **cluster of k strings** [M. Braun et al. Phys. Rev. C **65** (2002) 024907, Kovalenko V. et al, Universe **8** (2022) 246]

$$\boxed{\langle p_T \rangle_k = p_0 k^{\beta}} \quad \beta = 1.16[1 - (\ln\sqrt{s} - 2.52)^{-0.19} \tag{7}$$

- particle p_T from **Schwinger mechanism** of pair production [J. Schwinger, Phys. Rev. **82** (1951) 664; E. Gurvich, Phys. Lett. B **87** (1979) 386; A. Casher et al. Phys. Rev. D **20** (1979) 179]

$$f(p_T) \sim \frac{\pi p_T}{2\langle p_T \rangle_k^2} \exp\left(-\frac{\pi p_T^2}{4\langle p_T \rangle_k^2}\right) \tag{8}$$

- particle species $\sim$ **exp**$(-\pi m_i^2/4\langle p_T \rangle_k^2)$

Fusion and kinetic energy of strings

Step VII: string fusion boosts string segments

- Strings' overlap modifies colour fields $\rightarrow$ strings **gain kinetic energy** ΔT pulling them towards each other [V. Abramovsky et al. JETP Lett. **47** (1988) 337]
- ΔT **parametrization** for a pair of strings in some rapidity slice with centres at 2D distance $d_{i,j}$

$$\Delta T_{i,j} = \chi d_{i,j} \exp\left(\frac{-d_{i,j}^2}{4r_0^2}\right) \tag{9}$$

χ - free model parameter, GeV/fm

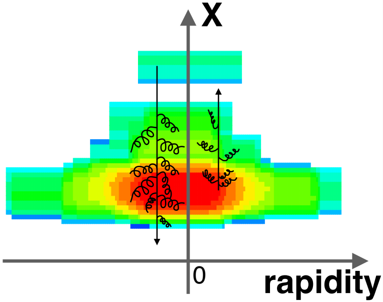
- find the vector sum of gained transverse momenta in each 2D bin in each rapidity slice covered by some number of strings
- particles produced in string's rest frame with some p_T and ϕ will get **Lorentz boost** to the laboratory frame

Introduced **correlated transverse motion** of particles produced by 3D bins that strongly depends on the degrees of strings' overlaps inside them

Fusion and particle momentum quenching

Step VIII: lose of particles' momentum in string environment

- quenching of particle's momentum due to **gluon radiation** in string medium [M. Braun, C. Pajares, Eur. Phys. J. C **71** (2011) 1558]:



$$p_{\rm fin} = (p_{\rm init}^{1/3} - \varkappa \sigma_{\rm eff}^{2/3} l)^3 \qquad (10)$$

- l - 2D particle's path
- $\varkappa$ - quenching coefficient, free model parameter
- $\sigma_{\rm eff} = 4p_0^2\sqrt{k}$

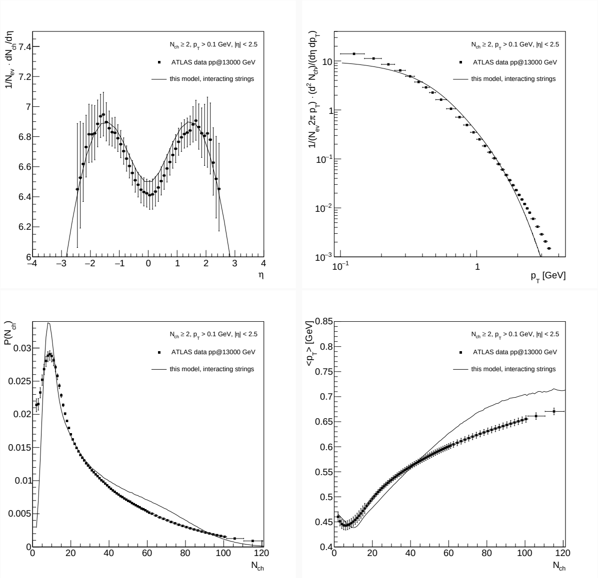
Dependence on the path of a particle in fluctuating string environment modifies ϕ and p_T

String fusion $\rightarrow$ particle boosts + momentum quenching

Results

Model parameters using ATLAS $p + p$ data at $\sqrt{s} = 13$ TeV

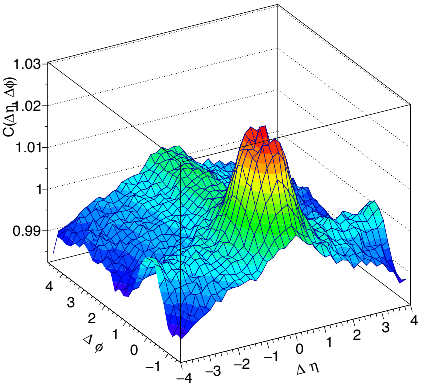
$\mu_0 = 1.14$, $\sigma_T = 0.55$ GeV/fm, $p_0 = 0.37$ GeV, $\varkappa = 0.1$, $\chi = 10^{-5}$ GeV/fm



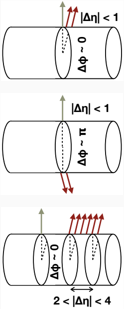
Emergent flow signal in two-particle correlation function

Model result for p+p with longitudinal + transverse dynamics + string fusion + particles boosts + particles' momentum quenching

- ✓ $\Delta\eta \approx 0$, $\Delta\phi \approx 0$ peak and $\Delta\eta \approx 0$ structure - ρ-resonance decay
- ✓ near-side ridge at $\Delta\phi \approx 0$ for wide $\Delta\eta$ similar to ATLAS $p+p$!!!



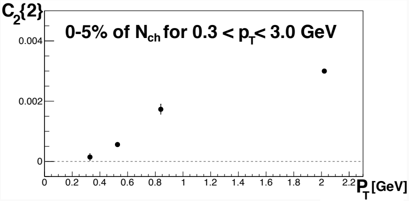
$$C(\Delta\eta, \Delta\phi) = \frac{S(\Delta\eta, \Delta\phi)}{B(\Delta\eta, \Delta\phi)}$$



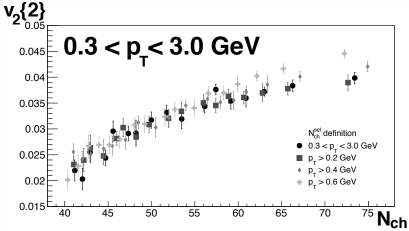
$0-10\%$ event class (N_{ch}), particles: $|\eta| < 2.5$, $0.3 < p_T < 3.0$ GeV

Elliptic flow harmonic $v_2\{2\}$ with event multiplicity and p_T

Model result for p+p with longitudinal + transverse dynamics + string fusion + particles boosts + particles' momentum quenching



particles $|\eta| < 2.5$



0.5% width event classes, particles $|\eta| < 2.5$

Two-particle cumulants in ϕ [S. Wang et al. Phys. Rev. C **44** (1991) 1091]

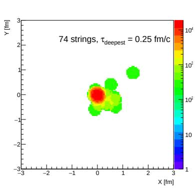
$$c_2\{2\} = \langle\langle e^{2i(\phi_1-\phi_2)}\rangle\rangle$$
$$v_2\{2\} = \sqrt{c_2\{2\}}$$

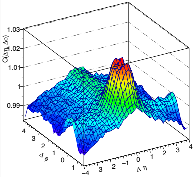
$\langle\langle ...\rangle\rangle$ - average of event pairs averaged over all events

- **flow grows** with event multiplicity
- **splitting** of flow signal for central events with different fractions of soft particles
- particles' momentum quenching in string medium → **larger anisotropy for** particles with **higher** p_T

Model result:

- ✓ qualitative description of near-side ridge and $c_2\{2\}$
- ✓ no away-side ridge
- ✓ obtained **core-corona** event structure similar to [Y. Kanakubo et al. EPJ Web Conf. **845** (2023) 0101]
  - ◇ **corona:** only momentum quenching
  - ◇ **low occupancy core regions:** multi-directional particle boosts and $\Delta\phi \approx \pi$
  - ◇ **hot core region:** single dominant direction of strong particle boosts with $\Delta\phi \approx 0$
- ✓ τ_{deepest} as a core-corona separation parameter **may be better tuned** [K. Werner et al. Phys. Rev. Lett. **106** (2011) 122004]





Conclusions

Collective behaviour in interacting string model: conclusions

Models based on **colour strings** as particle emitting sources can describe **collective behaviour** if

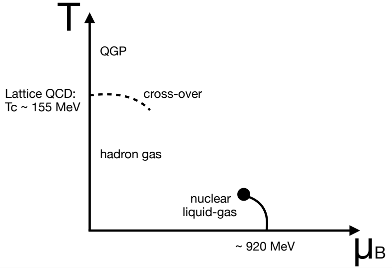
- **inhomogeneity** in the string density distribution is considered
  - 3D initial conditions are found **dynamically** from
    - ✓ transverse attractive interaction of strings
    - ✓ strings' longitudinal finitness and length oscillations
  - string fusion non-uniformly modifies string tension in the areas of multiple strings' overlaps
- **interactions** prior to hadronisation is taken into account:
  - string-string: attraction of overlapped strings due to fusion, which results in particle boosts
  - particle-string: momentum quenching in string environment

Advantage: developed model is applicable to **both** p+p and A+A

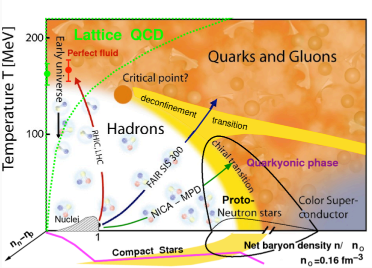
Thank you for your attention!

BACKUP

Phase diagram of strongly interacting matter



A. Bzdak: "The rest is everybody's guess"



Current view

Questions to answer:

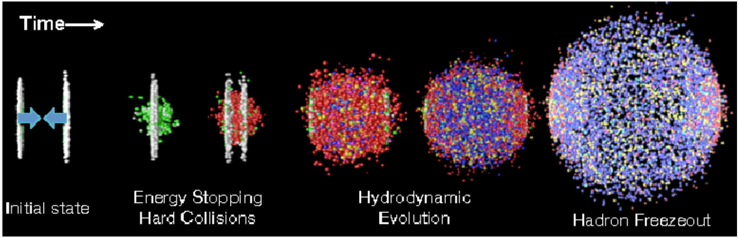
- whether cross-over turns into a first-order phase transition at $\mu_B > 0$?
- whether QCD critical point exists?
- what is the role of colour confinement in hadron production?

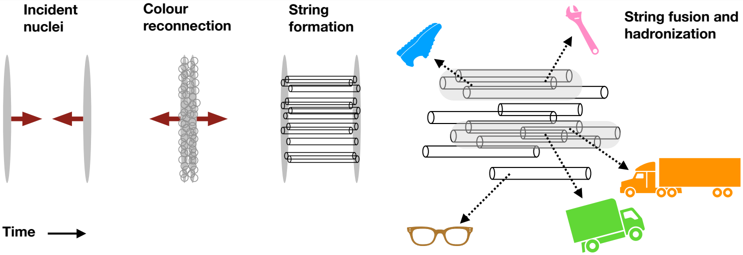
Tools available:

- lattice QCD calculations
- controlled experiments on high energy hadron and ion collisions

QGP vs colour strings scenarios

fluctuating positions of nucleons + hydro QGP

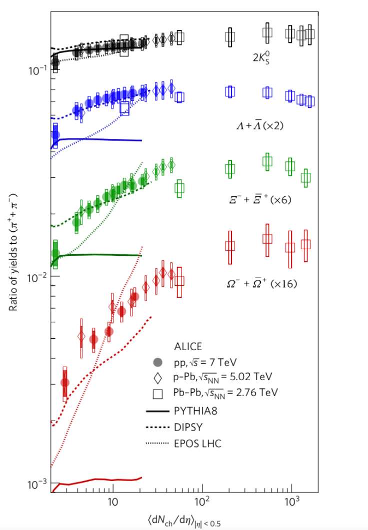




colour strings + their interaction

possible hybrid: colour strings + QGP [C. Shen, B. Schenke, Phys. Rev. C **97** (2018) 024907]

QGP-like behaviour in small systems: strangeness enhancement



[ALICE, Nature Phys. **13** (2017) 535]

- **prediction** of strangeness enhancement in QGP [J. Rafelski et al. Phys. Rept. **142** (1986) 167]
- **experimental** observations:
  - relative yields of strange hadrons grow with N_{ch} in $p+p$
  - at large N_{ch} they reach the level of $p+A$ and $A+A$

Is there a **common underlying mechanism** connected to the QGP formation in both $p+p$ and $A+A$?

Strangeness in Multi-Pomeron exchange model

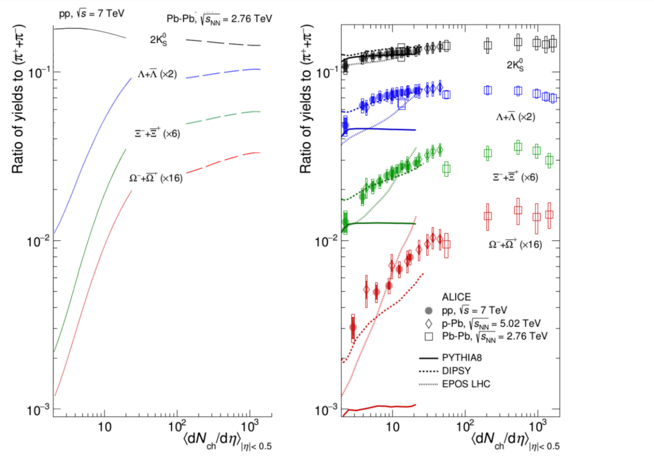


Figure 17. Multiplicity dependence of the strange and multi-strange particle yields ($K^0_s, \Lambda, \Xi, \Omega$) divided by charged pion multiplicity. Left plot—our model calculation for *pp* collisions at $\sqrt{s}$ = 7 TeV (solid lines) and for *Pb-Pb* collisions (dashed lines) at $\sqrt{s_{NN}}$ = 2.76 TeV (this work). Right plot [11]—experimental data (dots) and prediction of other models.

[V. Kovalenko et al. Universe **8** (2022) 246]

Measures of Forward-Backward rapidity correlations

Correlation coefficient [S. Uhlig et al. Nucl. Phys. B **132** (1978) 15]

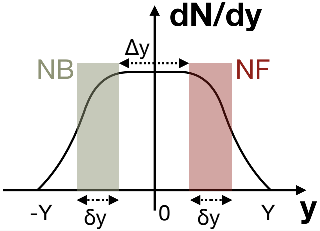
$$b_{B-F} = \left. \frac{d\langle N_B(N_F)\rangle}{dN_F} \right|_{N_F=\langle N_F\rangle} \qquad (11)$$

For **linear** $\langle N_B(N_F)\rangle$ [A.Capella, J. Tran Thanh Van, Phys. Rev. D 1984, **29**, 2512–2516]

$$b_{\rm corr}[N_F, N_B] = \frac{\langle N_F N_B\rangle - \langle N_F\rangle\langle N_B\rangle}{\langle N_B^2\rangle - \langle N_B\rangle^2} \qquad (12)$$

Strongly intensive $\Sigma[N_F, N_B]$ [E. Andronov, Theor. Math. Phys. **185** (2015) 1383] independent of volume and its event-by-event fluctuations for independent particle production [M. Gorenstein, M. Gazdzicki, Phys. Rev. C **84** (2011) 014904]

$$\Sigma[N_F, N_B] = \frac{\langle N_F\rangle\omega[N_B] + \langle N_B\rangle\omega[N_F] - 2(\langle N_F N_B\rangle - \langle N_F\rangle\langle N_B\rangle)}{\langle N_F\rangle + \langle N_B\rangle} \qquad (13)$$



- N_F or N_B - multiplicities in Forward or Backward rapidity intervals
- $\langle..\rangle$ - averaging over events
- $\omega[A] = (\langle A^2\rangle - \langle A\rangle^2)/\langle A\rangle$ - scaled variance of extensive event variable A

Model formalism for independent particle sources

3 types of strings with respect to Forward и Backward windows in rapidity:

- n_{for} - short: producing particles only in **Forward** window
- n_{back} - short: producing particles only in **Backward** window
- n_{long} - long: producing particles both in **Forward** and **Backward** windows

The probability to have some **string configuration** C:
$\sum_C q\left(C\right) \equiv \sum_{n_{\text{long}}, n_{\text{for}}, n_{\text{back}}} q\left(n_{\text{long}}, n_{\text{for}}, n_{\text{back}}\right) = 1$

Multiplicities in rapidity windows: $N_F = \sum_{k=1}^{n_{\text{long}}} N_F^{(k)} + \sum_{s=1}^{n_{\text{for}}} N_F^{(s)}$,
$N_B = \sum_{k=1}^{n_{\text{long}}} N_B^{(k)} + \sum_{t=1}^{n_{\text{back}}} N_B^{(t)}$.

Joint distribution: $P(N_F, N_B) = \sum_C q\left(C\right) P_C\left(N_F, N_B\right)$.

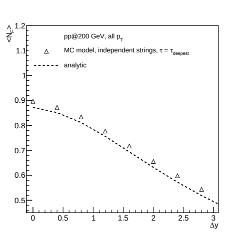
Comparison of analytical and numerical calculations

$$\langle N_F \rangle = \overline{\mu} \cdot \left(\overline{n_{\rm long}} + \overline{n_{\rm for}} \right), \tag{14}$$

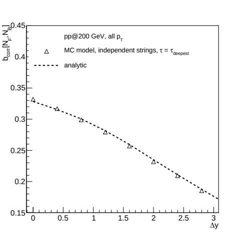
$$b_{\rm corr}[N_F, N_B] = \frac{\overline{\mu} \cdot \left(D_{n_{\rm long}} + 2 \cdot cov\left(n_{\rm long}, n_{\rm for}\right) + cov\left(n_{\rm for}, n_{\rm back}\right)\right)}{\overline{\mu} \cdot \left(D_{n_{\rm long}} + 2 \cdot cov\left(n_{\rm long}, n_{\rm for}\right) + D_{n_{\rm for}} \right) + \overline{n_{\rm long}} + \overline{n_{\rm for}}}, \tag{15}$$

$$\Sigma[N_F, N_B] = 1 + \overline{\mu} \cdot \frac{D_{n_{\rm back}} - cov\left(n_{\rm for}, n_{\rm back}\right)}{\overline{n_{\rm long}} + \overline{n_{\rm for}}}. \tag{16}$$

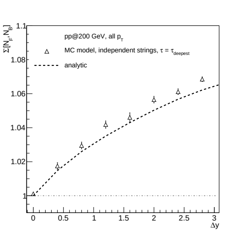
Independent sources, evolving till $\tau_{\rm deepest}$:



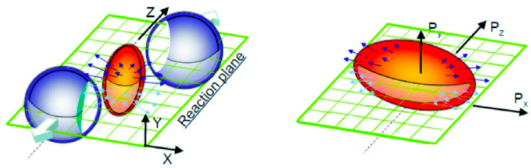
$\langle N_F \rangle$



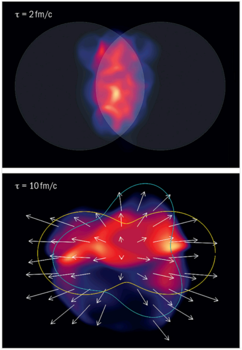
$b_{\rm corr}[N_F, N_B]$



$\Sigma[N_F, N_B]$



[M. Aggarwal et al. Adv. Nucl. Phys. **257** (2021) 161]

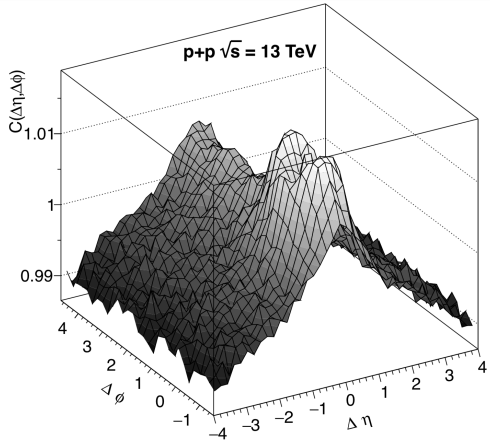


[https://cerncourier.com/a/going-with-the-flow/; Credit: MUSIC arXiv:1209.6330]

- Paradigm: initial spatial anisotropies **are converted** to momentum asymmetries
- **Fourier expansion** of the single-particle distribution in the azimuthal angle, ϕ, [S. Voloshin, Y. Zhang, Z. Phys. C **70** (1996) 665]

$$E\frac{d^3N_{\mathrm{ch}}}{d^3p} = \frac{1}{2\pi}\frac{d^2N_{\mathrm{ch}}}{p_T dp_T dy}\left(1 + 2\sum_{n=1}^{\infty} v_n \cos(n(\phi - \Psi_{RP}))\right) \qquad (17)$$

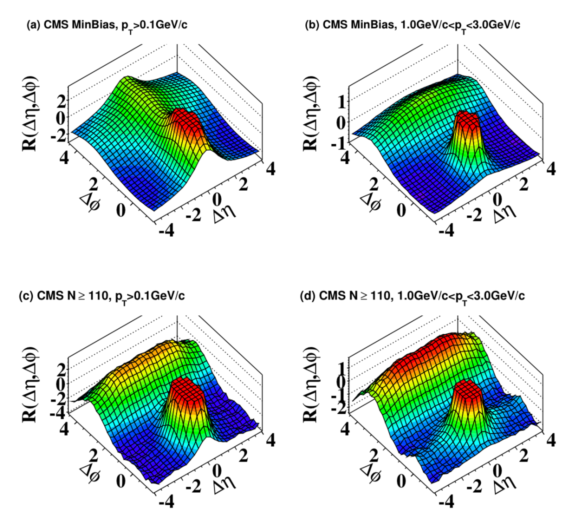
Two-particle angular correlation function, p+p@ 13 TeV



$C(\Delta\eta, \Delta\phi)$ calculated for particles with $|\eta| < 2.5$ and $0.3 < p_T < 3.0$ GeV. Presented for event class with particle selection $p_T^{\mathrm{cent}} > 0.2$ GeV and $\langle N_{\mathrm{ch}} \rangle \approx 53$.

- only particle momentum quenching, no boosts
- no near-side ridge is visible

CMS two-particle angular correlation functions, p+p@ 7 TeV



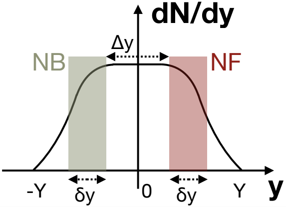
(a) minimum bias events with pT > 0.1 GeV/c, (b) minimum bias events with 1 < pT < 3 GeV/c, (c) high multiplicity (≥ 110) events with pT > 0.1 GeV/c and (d) high multiplicity (≥ 110) events with 1 < pT < 3 GeV/c [CMS, JHEP **09** (2010) 091]

Found relationships between the studied quantities

It has been shown that:

1. $\Sigma[N_F, N_B] \approx \sigma^2(C)$ - variance of event-by-event **asymmetry coefficient** distribution [PHOBOS, Phys. Rev. C **74** (2006) 011901(R)]:

$$C = \frac{N_F - N_B}{\sqrt{N_F + N_B}} \tag{18}$$



2. With notations from [M. Kitazawa, X. Luo, Phys. Rev. C **96** (2017) 024910], one finds for **cumulants** $\langle .. \rangle_c$ and **factorial cumulants** $\langle .. \rangle_{\rm fc}$ of $N_F - N_B$ distribution:

$$\Sigma[N_F, N_B] = \frac{\langle q_{(b)}^2 \rangle_{\rm c}}{\langle q_{(a)} \rangle_{\rm c}} = 1 + \frac{\langle q_{(b)}^2 \rangle_{\rm fc}}{\langle q_{(a)} \rangle_{\rm fc}}, \tag{19}$$

- $q_{(a)} = N_F + N_B$, $q_{(b)} = N_F - N_B$
- connection of $\Sigma[N_F, N_B]$ to ratios of (factorial) cumulants is another way to see its strong intensity

Cumulants and factorial cumulants of $N_F - N_B$ distribution

First-order and second-order cumulants for the joint probability distribution, $P(N_F, N_B)$, in terms of the moments of the same distribution:

$$\langle q_{(a)} \rangle_{\rm c} = \langle N_F \rangle + \langle N_B \rangle, \tag{20}$$

$$\langle q_{(b)} \rangle_{\rm c} = \langle N_F \rangle - \langle N_B \rangle, \tag{21}$$

$$\langle q^2_{(a)} \rangle_{\rm c} = \langle N_F^2 \rangle - \langle N_F \rangle^2 + \langle N_B^2 \rangle - \langle N_B \rangle^2 + 2 \cdot \left(\langle N_F N_B \rangle - \langle N_F \rangle \langle N_B \rangle \right), \tag{22}$$

$$\langle q^2_{(b)} \rangle_{\rm c} = \langle N_F^2 \rangle - \langle N_F \rangle^2 + \langle N_B^2 \rangle - \langle N_B \rangle^2 - 2 \cdot \left(\langle N_F N_B \rangle - \langle N_F \rangle \langle N_B \rangle \right), \tag{23}$$

$$\langle q_{(a)} \cdot q_{(b)} \rangle_{\rm c} = \langle N_F^2 \rangle - \langle N_F \rangle^2 - \langle N_B^2 \rangle + \langle N_B \rangle^2. \tag{24}$$

Factorial cumulants in terms of the cumulants:

$$\langle q_{(a)} \rangle_{\rm fc} = \langle q_{(a)} \rangle_{\rm c}\,, \tag{25}$$

$$\langle q_{(b)} \rangle_{\rm fc} = \langle q_{(b)} \rangle_{\rm c}\,, \tag{26}$$

$$\langle q^2_{(a)} \rangle_{\rm fc} = \langle q^2_{(a)} \rangle_{\rm c} - \langle q_{(a^2)} \rangle_{\rm c} = \langle q^2_{(a)} \rangle_{\rm c} - \langle q_{(a)} \rangle_{\rm c}\,, \tag{27}$$

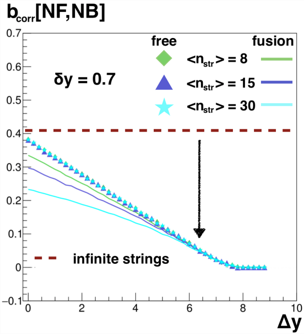
$$\langle q^2_{(b)} \rangle_{\rm fc} = \langle q^2_{(b)} \rangle_{\rm c} - \langle q_{(b^2)} \rangle_{\rm c} = \langle q^2_{(b)} \rangle_{\rm c} - \langle q_{(a)} \rangle_{\rm c}\,, \tag{28}$$

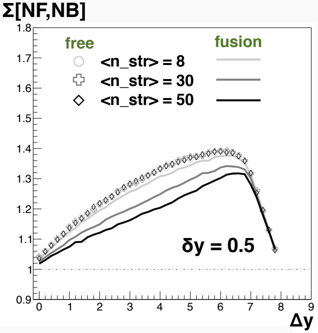
$$\langle q_{(a)} \cdot q_{(b)} \rangle_{\rm fc} = \langle q_{(a)} \cdot q_{(b)} \rangle_{\rm c} - \langle q_{(ab)} \rangle_{\rm c} = \langle q_{(a)} \cdot q_{(b)} \rangle_{\rm c} - \langle q_{(b)} \rangle_{\rm c}\,. \tag{29}$$

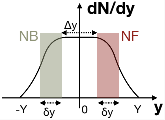
Rapidity correlations: $b_{\mathrm{corr}}[N_F, N_B]$ and $\Sigma[N_F, N_B]$ results

Toy model (short strings + fusion) results for:

$$b_{\mathrm{corr}}[N_F, N_B] = \frac{\langle N_F N_B\rangle - \langle N_F\rangle\langle N_B\rangle}{\langle N_B^2\rangle - \langle N_B\rangle^2} \qquad \Sigma[N_F, N_B] = \frac{\langle N_F\rangle\omega[N_B] + \langle N_B\rangle\omega[N_F] - 2(\langle N_F N_B\rangle - \langle N_F\rangle\langle N_B\rangle)}{\langle N_F\rangle + \langle N_B\rangle}$$



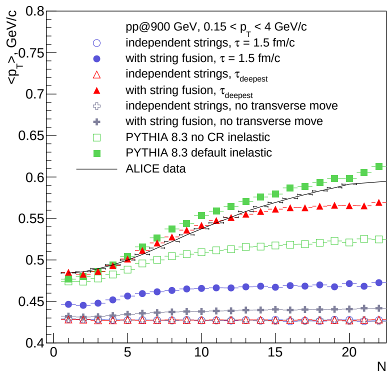




- ✓ finite strings make long-range correlations dependent on Δy even without explicit short-range correlations
- ✓ string fusion splits the values of $\Sigma[N_F, N_B]$: the larger the string density, the smaller the $\Sigma[N_F, N_B]$ value, while it is strongly intensive for free strings

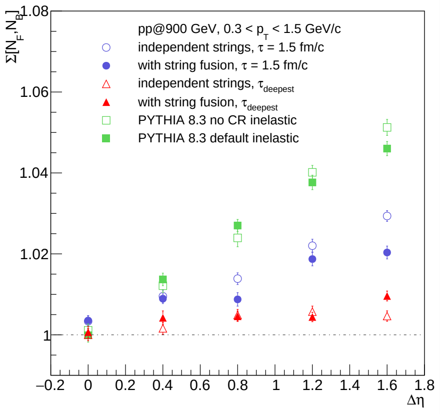
Results for $\langle p_T \rangle - N$ at $\sqrt{s} = 900$ GeV

Elaborated model (longitudinal + transverse dynamics + fusion) result **vs** ALICE data, dependence on the string density evolution time τ



- ✓ good slope of $\langle p_T \rangle - N$ correlation for τ_{deepest}, absent for free strings
- ✓ PYTHIA result w/wo Colour Reconnection resembles model behaviour

F-B multiplicity fluctuations at $\sqrt{s} = 900$ GeV: $\Sigma[N_F, N_B]$



[D. Prokhorova, E. Andronov, G. Feofilov, Physics **5** (2023) 636]

Conclusion: $\Sigma[N_F, N_B]$ depends on the formation of string clusters and grows with $\Delta\eta$ due to appearance of short strings and not short-range correlations.