Cross channel properties of the photon generalized parton distributions

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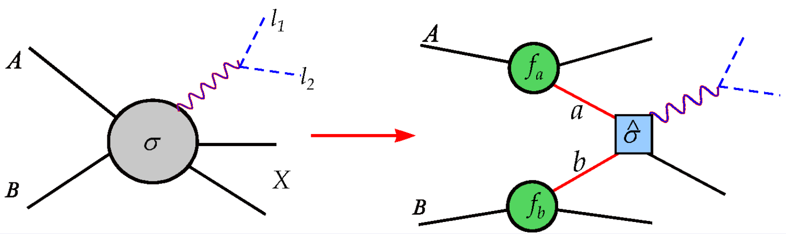
Hard processes in QCD

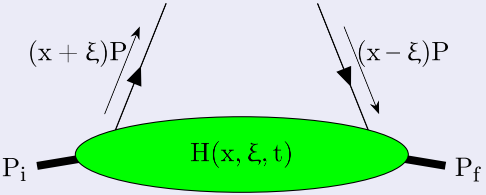
Factorization theorems



$$\frac{d\sigma}{dY} = \int f(x_1)f(x_2)\frac{d\sigma_{Hard}(x_1,x_2,Y)}{dY}dx_1dx_2$$

Generalized Parton Distributions

Nonperturbative part of the exclusive process



GPD - Generalized Parton Distribution

$$H(x, \xi, t)$$

x - parton collinear momentum fraction

$\xi = \frac{P_i^+ - P_f^+}{P_i^+ + P_f'^+}$ - skewness", transferred collinear momentum fraction

$t = (P_i - P_f)^2$ - squared transferred momentum

Experiments

1. CERN:
   - COMPASS
   - AMBER/COMPASS-II
2. JLAB: CLAS
3. KEK: J-PARC
4. DESY: HERA
   - HERMES
   - ZEUS
   - H1

Future experiments:

1. EIC @ BNL
   (possible extraction of GPDs from DDVCS)
2. SPD @ JINR
   (extraction of convoluted GPDs from DVCS)

TMD as one of the main objectives of the both facilities is also connected to GPD[a]

[a] Tanmay Maji, Chandan Mondal, D. Chakrabarti, and O. V. Teryaev. "Relating transverse structure of various parton distributions". In: *JHEP* 01 (2016), p. 165. DOI: 10.1007/JHEP01(2016)165. arXiv: 1506.04560 [hep-ph].

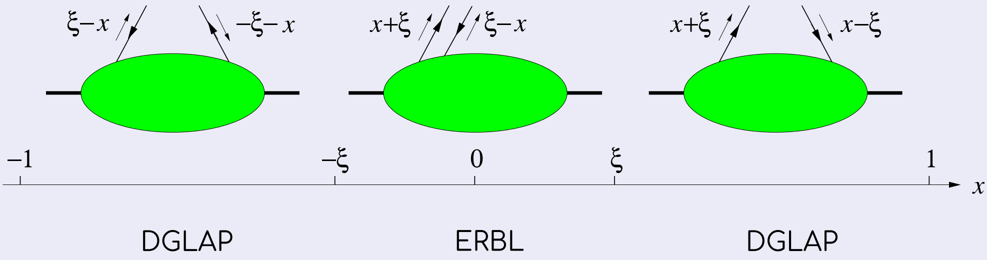
Generalized parton distributions

Definition by lightcone correlators of a hadron with spin $\frac{1}{2}$

$$
\begin{aligned}
F^q &= \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p'|\, \bar{q}(-z)\, \gamma^+ q(z)\, |p\rangle \Big|_{z^+=0,\, \vec{z}=0} \\
&= \frac{1}{2P^+}\left[H^q(x,\xi,t)\, \bar{u}(p')\gamma^+ u(p) + E^q(x,\xi,t)\, \bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m} u(p) \right], \\
\tilde{F}^q &= \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p'|\, \bar{q}(-z)\, \gamma^+\gamma_5\, q(z)\, |p\rangle \Big|_{z^+=0,\, \vec{z}=0} \\
&= \frac{1}{2P^+}\left[\tilde{H}^q(x,\xi,t)\, \bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}^q(x,\xi,t)\, \bar{u}(p')\frac{\gamma_5\Delta^+}{2m} u(p) \right]
\end{aligned}
\tag{1}
$$

GPD properties

Kinematic regions. $\xi < 1$



GPD properties

Helicity components

	Hadron helicity	
	retained	changed
for sum over parton helicities	$H(x, \xi, t)$	$\tilde{H}(x, \xi, t)$
for difference over parton helicities	$E(x, \xi, t)$	$\tilde{E}(x, \xi, t)$

Forward limit

$$\begin{aligned} H^q(x,0,0) &= q(x), & \tilde{H}^q(x,0,0) &= \Delta q(x) & \text{for } x > 0, \\ H^q(x,0,0) &= -\bar{q}(-x), & \tilde{H}^q(x,0,0) &= \Delta \bar{q}(-x) & \text{for } x < 0 \end{aligned} \quad (2)$$

T-invariance

$$H^q(x, \xi, t) = H^q(x, -\xi, t) \quad (3)$$

GPD properties

Formfactor sum rules

$$\int_{-1}^{1} \mathrm{d}x\, H^q(x,\xi,t) = F_1^q(t), \qquad \int_{-1}^{1} \mathrm{d}x\, E^q(x,\xi,t) = F_2^q(t),$$
$$\int_{-1}^{1} \mathrm{d}x\, \tilde{H}^q(x,\xi,t) = g_A^q(t), \qquad \int_{-1}^{1} \mathrm{d}x\, \tilde{E}^q(x,\xi,t) = g_P^q(t), \tag{4}$$

Ji sum rule for the angular momentum of partons

$$\frac{1}{2}\int_{-1}^{1} \mathrm{d}x\, x[H(x,\xi,0)+E(x,\xi,0)] = J, \tag{5}$$

where J - parton contribution to the spin of the hadron[1]

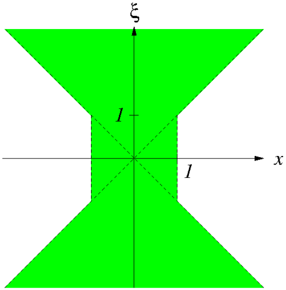
[1] Xiang-Dong Ji. "Gauge-Invariant Decomposition of Nucleon Spin". In: *Phys. Rev. Lett.* 78 (1997), pp. 610–613. DOI: 10.1103/PhysRevLett.78.610. arXiv: hep-ph/9603249.

Generalized distribution amplitudes

GDA is defined as t-channel version of GPD

$$\Phi^q(z,\zeta,s) = \int \frac{dx^-}{2\pi} e^{i(2z-1)(p+p')^+x^-/2} \times {}_{out}\langle \pi^+(p)\pi^-(p')|\, \bar{q}(-\frac{1}{2}x)\,\gamma^+ q(\frac{1}{2}x)\ |0\rangle\Big|_{x^+=0,\, x=0}\,, \quad (6)$$

Analytic continuation



$$H(x, \xi) = \mathrm{sgn}(\xi)\Phi\left(\frac{x}{\xi}, \frac{1}{\xi}\right) \quad (7)$$

GPD can be analytically continued to the unphysical kinematical region via GDA[2]

[2] O.V. Teryaev. "Crossing and radon tomography for generalized parton distributions". In: *Phys.Lett.* B510 (2001), pp. 125–132. DOI: 10.1016/S0370-2693(01)00564-0. arXiv: hep-ph/0102303 [hep-ph].

Double distributions

Lorentz covariance implies polynomiality property

$$\int_{-1}^{1} dx\, x^n H(x,\xi,t) = \sum_{\substack{i=0\\ \text{even}}}^{n} (2\xi)^i A_{n+1,i}(t) + \mathrm{mod}(n,2)\,(2\xi)^{n+1} C_{n+1}(t)$$

n-th moment of GPD must be ξ polynomial of the order $n+1$.

Double distribution parametrization

$$H(x,\xi,t) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha\, \delta(x-\beta-\xi\alpha)\, f(\beta,\alpha,t) \qquad (8)$$

Double distributions

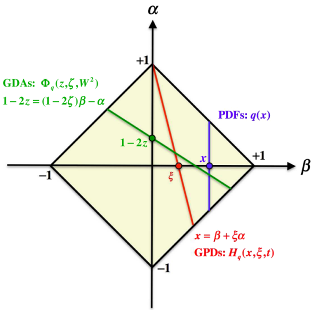
$$H(x, \xi, t) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \, \delta(x - \beta - \xi\alpha) \, f(\beta, \alpha, t) \quad (9)$$

where f support region is $|\beta| + |\alpha| < 1$

Double Distributions

GPD↔DD connection is a Radon transform[3,4]

$$H(x, \xi, t) = \int d\beta\, d\alpha\, \delta(x - \beta - \xi\alpha)\, f(\beta, \alpha, t) \qquad (10)$$

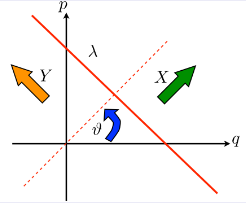


[3] O.V. Teryaev. "Crossing and radon tomography for generalized parton distributions". In: *Phys.Lett.* B510 (2001), pp. 125–132. DOI: 10.1016/S0370-2693(01)00564-0. arXiv: hep-ph/0102303 [hep-ph].

[4] Andrei V. Belitsky, Dieter Mueller, A. Kirchner, and A. Schafer. "Twist three analysis of photon electroproduction off pion". In: *Phys. Rev. D* 64 (2001), p. 116002. DOI: 10.1103/PhysRevD.64.116002. arXiv: hep-ph/0011314.

Radon Transform

2-Dimensional RT



$$f^{\sharp}(p, \phi) = \int_{-\infty}^{\infty} f(p\cos\phi + t\sin\phi, -p\sin\phi + t\cos\phi)dt,$$

Original paper[5], modern reviews with applications[6,7]

[5] J. Radon. “Über die Bestimmung von Functionen durch ihre Integralwerte längs gewisser Mannigfaltigkeiten”. In: *Ber. Verh. Sächs. Akad. Wiss. Leipzig, Math-Nat. Kl.* 69 (1917), pp. 262–277.

[6] И.М. Гельфанд, С.Г. Гиндикин, and М.И. Граев. *Избранные задачи интегральной геометрии*. Изд-во, Добросвет, 2000. ISBN: 5-7913-0034-4.

[7] P. Facchi and M. Ligabo. “Classical and quantum aspects of tomography”. In: *AIP Conference Proceedings* 1260 (2010), pp. 3–34. DOI: 10.1063/1.3479322. arXiv: 1001.5169v1 [math-ph].

RT - is a basis of the computer tomography

In 1979 Allan M. Cormack and Godfrey N. Hounsfield received the Nobel Prize in Physiology or Medicine "for the development of computer assisted tomography"

Inverse RT

2-Dimensional

$$f(x,y) = -\frac{1}{2\pi^2}\int_0^\infty\int_0^{2\pi}\frac{\partial_p f^\sharp(\phi, s + x\cos\phi + y\sin\phi)}{s}d\phi ds \quad (11)$$

[8] И.М. Гельфанд, С.Г. Гиндикин, and М.И. Граев. *Избранные задачи интегральной геометрии*. Изд-во, Добросвет, 2000. ISBN: 5-7913-0034-4.

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$$f(x,y) = -\frac{1}{2\pi^2}\int\limits_0^\infty\int\limits_0^{2\pi}\frac{\partial_p f^\sharp(\phi, s + x\cos\phi + y\sin\phi)}{s}d\phi ds \quad (11)$$

3-Dimensional

$$f(x) = -\frac{1}{8\pi^2}\int\limits_{\mathbb{S}^2}\partial_p^2 f^\sharp(\omega, \langle\omega, x\rangle)d\omega \quad (12)$$

[8] И.М. Гельфанд, С.Г. Гиндикин, and М.И. Граев. *Избранные задачи интегральной геометрии*. Изд-во, Добросвет, 2000. ISBN: 5-7913-0034-4.

Inverse RT

2-Dimensional

$$f(x,y) = -\frac{1}{2\pi^2}\int_0^{\infty}\int_0^{2\pi}\frac{\partial_p f^{\sharp}(\phi, s + x\cos\phi + y\sin\phi)}{s}d\phi ds \quad (11)$$

3-Dimensional

$$f(x) = -\frac{1}{8\pi^2}\int_{\mathbb{S}^2}\partial_p^2 f^{\sharp}(\omega, \langle\omega, x\rangle)d\omega \quad (12)$$

Odd dimensional inverse RT is local!

It can be shown by the solution of wave equation (which is connected to the inverse RT) what **Huygens principle is strictly applicable only in the odd dimensional space**[8].

[8] И.М. Гельфанд, С.Г. Гиндикин, and М.И. Граев. *Избранные задачи интегральной геометрии*. Изд-во, Добросвет, 2000. ISBN: 5-7913-0034-4.

Inverse RT for GPD

Double distribution reconstruction procedure has been derived[9] as inverse RT

DD from GPD by inverse RT

$$F(\beta, \alpha) = -\frac{1}{2\pi^2} \int_{-\infty}^{\infty} \frac{dx}{x} \int_{-\infty}^{\infty} d\xi \partial_x H(x + \beta + \alpha\xi, \xi), \quad (13)$$

[9] O.V. Teryaev. “Crossing and radon tomography for generalized parton distributions”. In: *Phys.Lett.* B510 (2001), pp. 125–132. DOI: 10.1016/S0370-2693(01)00564-0. arXiv: hep-ph/0102303 [hep-ph].

D - term

Motivated by polynomiality property (Lorentz covariance)

$$H^q(x,\xi,t) = \int d\beta\, d\alpha\, \delta(x-\beta-\xi\alpha)\, f^q(\beta,\alpha,t) + \mathrm{sgn}(\xi) D^q\left(\frac{x}{\xi}, t\right) \quad (14)$$

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EMT decomposition for a spin-0 hadron

$$\langle P'|\hat{T}_{\mu\nu}(0)|P\rangle = \bar{N}(p')\Big[M_2(t)\frac{\bar{P}_\mu \bar{P}_\nu}{m_N} + J(t)\frac{i\bar{P}_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho}{m_N} + d(t)\frac{1}{5m_N}\left(\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2\right) \pm \bar{c}(t) g_{\mu\nu}\Big] N(p)\,. \qquad (15)$$

Formfactor $d(t)$ is connected with the hadron D-term $D(\alpha, t)$. Namely it is a 1st Gegenbauer polynomial coefficient of the D-term.

Pressure distribution inside the proton[10]

The pressure distribution inside the proton

V. D. Burkert[1]*, L. Elouadrhiri[1] & F. X. Girod[1]

The proton, one of the components of atomic nuclei, is composed of fundamental particles called quarks and gluons. Gluons are the carriers of the force that binds quarks together, and free quarks are never found in isolation—that is, they are confined within the composite particles in which they reside. The origin of quark confinement is one of the most important questions in modern particle and nuclear physics because confinement is at the core of what makes the proton a stable particle and thus provides stability to the Universe. The internal quark structure of the proton is revealed by deeply virtual Compton scattering[1,2], a process in which electrons are scattered off quarks inside the protons, which subsequently emit high-energy photons, which are detected in coincidence with the scattered electrons and recoil protons. Here we report a measurement of the pressure distribution experienced by the quarks in the proton. We find a strong repulsive pressure near the centre of the proton (up to 0.6 femtometres) and a binding pressure at greater distances. The average peak pressure near the centre is about 10^{35} pascals, which exceeds the pressure estimated for the most densely packed known objects in the Universe, neutron stars[3]. This work opens up a new area of research on the fundamental gravitational properties of protons, neutrons and nuclei, which can provide access to their physical radii, the internal shear forces acting on the quarks and their pressure distributions.

The basic mechanical properties of the proton are encoded in the gravitational form factors (GFFs) of the energy–momentum tensor[1,4,5]. Graviton–proton scattering is the only known process that can be used to directly measure these form factors[4,6], whereas generalized parton distributions[2,7,8] enable indirect access to the basic mechanical properties of the proton[2].

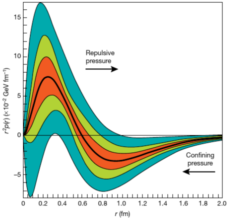
A direct determination of the quark pressure distribution in the proton (Fig. 1) requires measurements of the proton matrix element of the energy–momentum tensor[5]. This matrix element contains three scalar GFFs that depend on the four-momentum transfer t to the proton. One of these GFFs, $d_1(t)$, encodes the shear forces and pressure distribution on the quarks in the proton, and the other two, $M_2(t)$ and $J(t)$, encode the mass and angular momentum distributions. Experimental information on these form factors is essential to gain insight into the dynamics of the fundamental constituents of the proton. The framework of generalized parton distributions (GPDs)[2,7,8] has provided a way to obtain information on $d_1(t)$ from experiments. The most effective way to access GPDs experimentally is deeply virtual Compton scattering (DVCS)[1,2], where high-energy electrons (e) are scattered from the protons (p) in liquid hydrogen as $e\,p \to e'\,p'\,\gamma$, and the scattered electron (e'), proton (p') and photon (γ) are detected in coincidence. In this process, the quark structure is probed with high-energy virtual

(2) We then define the complex CFF, $\mathcal{H}$, which is directly related to the experimental observables describing the DVCS process, that is, the differential cross-section and the beam-spin asymmetry.
(3) The real and imaginary parts of $\mathcal{H}$ can be related through a dispersion relation[14–16] at fixed t, where the term $D(t)$, or D-term, appears as a subtraction term[17].
(4) We derive $d_1(t)$ from the expansion of $D(t)$ in the Gegenbauer polynomials of ξ, the momentum transfer to the struck quark.
(5) We apply fits to the data and extract $D(t)$ and $d_1(t)$.
(6) Then, we determine the pressure distribution from the relation between $d_1(t)$ and the pressure $p(r)$, where r is the radial distance from the proton's centre, through the Bessel integral.

The sum rules that relate the second Mellin moments of the chiral-even GPDs to the GFFs are[1]:

$$\int x\,[H(x,\xi,t)+E(x,\xi,t)]\,\mathrm{d}x = 2J(t)$$

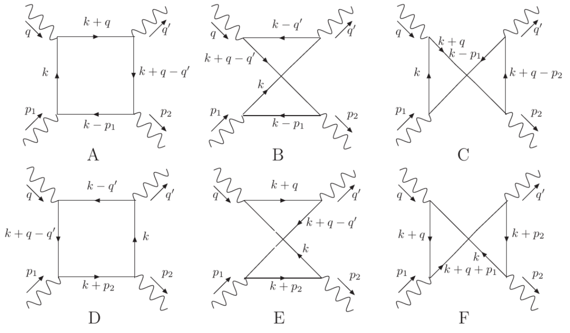
$$\int xH(x,\xi,t)\mathrm{d}x = M_2(t) + \frac{4}{5}\xi^2 d_1(t)$$



[10] V. D. Burkert, L. Elouadrhiri, and F. X. Girod. "The pressure distribution inside the proton". In: *Nature* 557.7705 (2018), pp. 396–399. DOI: 10.1038/s41586-018-0060-z. URL: https://doi.org/10.1038/s41586-018-0060-z.

Photon as a toy model for QCD[11,12]

Photon GPD u GDA were derived from the leading twist $\gamma^* + \gamma$ DVCS amplitude (For simplicity $\Delta_\perp = 0$ and $m_q \to 0$ were taken).



[11] S. Friot, B. Pire, and L. Szymanowski. "Deeply virtual compton scattering on a photon and generalized parton distributions in the photon". In: *Phys.Lett.* B645 (2007), pp. 153–160. DOI: 10.1016/j.physletb.2006.12.038. arXiv: hep-ph/0611176 [hep-ph].

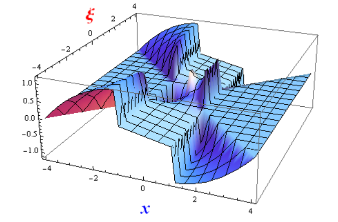
[12] M. El Beiyad, B. Pire, L. Szymanowski, and S. Wallon. "Diphoton Generalized Distribution Amplitudes". In: *Phys.Rev.* D78 (2008), p. 034009. DOI: 10.1103/PhysRevD.78.034009. arXiv: 0806.1098 [hep-ph].

Photon GPD and GDA from the DVCS amplitude

DVCS amplitude tensor is decomposed by:

$$T^{\mu\nu\alpha\beta}(\Delta_T = 0) = \frac{1}{4} g_T^{\mu\nu} g_T^{\alpha\beta} W_1 + \frac{1}{8}\left(g_T^{\mu\alpha} g_T^{\nu\beta} + g_T^{\nu\alpha} g_T^{\mu\beta} - g_T^{\mu\nu} g_T^{\alpha\beta}\right) \not{W_2} + \frac{1}{4}\left(g_T^{\mu\alpha} g_T^{\nu\beta} - g_T^{\mu\beta} g_T^{\alpha\nu}\right) W_3$$

Using $H(x,\xi) = \mathrm{sgn}(\xi)\Phi(\frac{x}{\xi}, \frac{1}{\xi})$ photon GPD was continued[13] to the unphysical region via GDA.



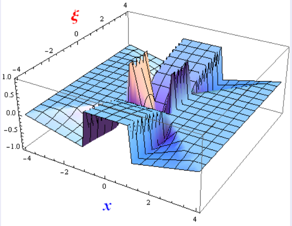


Figure: Left: $H_1(x,\xi)$ (unpolarized). Right: $H_3(x,\xi)$ (polarized)

[13] I. R. Gabdrakhmanov and O. V. Teryaev. “Analyticity and sum rules for photon GPDs”. In: *Phys. Lett. B* 716 (2012), pp. 417–424.

Photon DD

Photon DDs has been derived[14] via inverse RT. Consider here and after the coefficient $\frac{N_C e_q^2}{4\pi^2} \ln \frac{Q^2}{m^2}$.

$$F_1(\beta, \alpha) = [2(1 - |\beta| - |\alpha|) - 1 + \delta(\alpha)]\mathrm{sgn}(\beta),$$
$$F_3(\beta, \alpha) = \delta(\alpha) - 1.$$

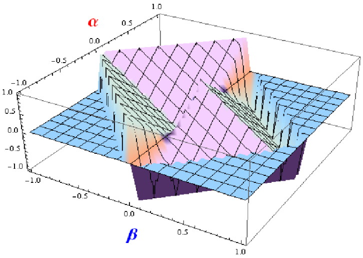


Figure: $F_1(\beta, \alpha)$ regular part

[14] I. R. Gabdrakhmanov and O. V. Teryaev. "Analyticity and sum rules for photon GPDs". In: *Phys. Lett. B* 716 (2012), pp. 417–424.

Photon virtual quark cloud mechanical stability

$$H^q(x,\xi,t) = \int d\beta\, d\alpha\, \delta(x-\beta-\xi\alpha)\, f^q(\beta,\alpha,t) + \mathrm{sgn}(\xi) D^q\left(\frac{x}{\xi}, t\right) \quad (16)$$

[15] I. R. Gabdrakhmanov and O. V. Teryaev. "Analyticity and sum rules for photon GPDs". In: *Phys. Lett. B* 716 (2012), pp. 417–424.

[16] M.V. Polyakov. "Generalized parton distributions and strong forces inside nucleons and nuclei". In: *Phys.Lett.* B555 (2003), pp. 57–62. DOI: 10.1016/S0370-2693(03)00036-4. arXiv: hep-ph/0210165 [hep-ph].

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Photon D-term calculated in[15]

$$\begin{aligned} D_1(\alpha) &= (|\alpha|-1)(2|\alpha|+1)\mathrm{sgn}(\alpha) \\ D_3(\alpha) &= 0 \end{aligned}$$

Negative sign (for positive α) of the D-term (as well as $d_1(0) = -\frac{5}{6}$) is in accordance with stability criteria for nucleons in vacuum[16].

[15] I. R. Gabdrakhmanov and O. V. Teryaev. "Analyticity and sum rules for photon GPDs". In: *Phys. Lett. B* 716 (2012), pp. 417–424.

[16] M.V. Polyakov. "Generalized parton distributions and strong forces inside nucleons and nuclei". In: *Phys.Lett.* B555 (2003), pp. 57–62. DOI: 10.1016/S0370-2693(03)00036-4. arXiv: hep-ph/0210165 [hep-ph].

Quark GPD decomposition

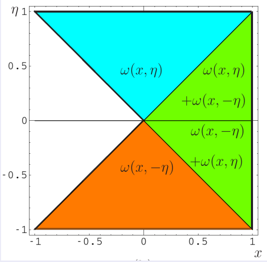
Let's assume only quark GPD ($0 < \xi < 1, -\xi < x < 1$) and DD ($0 < \beta < 1, |\alpha| < 1 - \beta$). So GPD can be decomposed[17]:

$$H^q(x,\xi,t) = \theta(x+\xi)\omega(x,\xi,t) + \theta(x-\xi)\omega(x,-\xi,t)\,,$$

where $\omega(x,\xi,t) = \frac{1}{\xi}\int_0^{\frac{x+\xi}{1+\xi}} d\beta\, F(\beta,(x-\beta)/\xi,t)\,.$

While the full GPD:

$$H_i(x,\xi,t) = H^q(x,\xi,t) \pm H^q(-x,\xi,t)\,,$$



And the same for GDA: $\Phi(z,\zeta,t) = \theta(\zeta - z)\varpi(z,\zeta,t) - \theta(z-\zeta)\varpi(-z,-\zeta,t)\,,$ where $\omega(x,\xi,t) = \varpi(\frac{x}{\xi},\frac{1}{\xi},t)\,.$

[17] Dieter Mueller and A. Schafer. "Complex conformal spin partial wave expansion of generalized parton distributions and distribution amplitudes". In: *Nucl. Phys. B* 739 (2006), pp. 1–59. DOI: 10.1016/j.nuclphysb.2006.01.019. arXiv: hep-ph/0509204.

Analytic continuation from DGLAP to ERBL region

The DGLAP → ERBL continuation (up to the D-term) has been derived by D.Müller[18] for a specific class of GPD (satisfying the light-front wave function overlap (LFWF) representation) by inverse Laplace transform:

$$H^{ERBL}(x,\xi,t) = -\frac{1}{2\pi i}\frac{x+\xi}{1-x}\int_{-i\infty}^{i\infty}\frac{dr}{r-\frac{x+\xi}{1-x}}\frac{H^{DGLAP}\left(\frac{rx}{x+\xi+rx},\frac{r\xi}{x+\xi+rx},t\right)}{x+\xi+rx}. \quad (17)$$

The procedure was later generalized[19] for any GPD.

So it is possible to analytically reconstruct GPD (up to the D-term) in the whole support area (GDA as well) by using only GPD in the DGLAP region.

[18] Dieter Müller. "Double distributions and generalized parton distributions from the parton number conserved light front wave function overlap representation". In: (Nov. 2017). arXiv: 1711.09932 [hep-ph].

[19] I. R. Gabdrakhmanov, D. Müller, and O. V. Teryaev. "Inverse Radon transform at work". In: *Phys. Part. Nucl. Lett.* 16.6 (2019), pp. 625–637.

Analytic continuation from DGLAP to ERBL region

Let's apply the (17) to the known photon GPD in the outer DGLAP region

Closing the contour by the second and third quadrants and assuming for ex. $0 < x < \xi$ we get:

$$H_1(x,\xi,t) = \frac{x\xi - (2\xi+1)x^2}{\xi^2(\xi+1)} + 1 \tag{18}$$

for the unpolarized ERBL GPD and:

$$H_3(x,\xi,t) = -\frac{x-\xi^2}{\xi(\xi+1)} \tag{19}$$

for the polarized one.

Which after (anti)symmetrization give us already known photon ERBL GPDs (up to the D-term).

- Photon proved to be a useful toy model to exclusive QCD processes
  - Purely QED amplitudes gave specialists easy tool to analytically construct QCD phenomenological distributions
  - But at the same time experimental test of them is technically
- Integral geometry approach is quite useful in the reconstruction of double distributions from GPDs also leading to the whole new set of methods
- Inversion procedure by inverse Laplace transform gives possibility to reconstruct (up to the D-term) GPD in the full kinematic area by only DGLAP region
- These procedures can be applied:
  - to develop analytical and numerical computational tools for GPD/DD reconstruction
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Thanks for your attention!