Particle production in intensive plane wave background.

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Introduction

- Particle production in oscillating field $\Phi_0 \sin(m_\phi t)$
  Enhancement by parametrical resonance
  Matter creation at preheating stage after inflation
  Kofman, Linde, Starobinskiy 94, 97
  (hep-th/9405187,hep-ph/9704452)
  Khlebnikov, Tkachev 96 (hep-ph/9608458)
  Duffaeux et al 06 (hep-ph/0602144)
- Particle production in intense plane wave $\Phi_0 \sin(\omega t - kx)$
  A.Arza. PRD 105 (2022) 3, 036004 (2009.03870)
  Intense waves can be applied to new physical directions in astrophysics.

Toy model $g\phi\chi^2$

- Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 + \frac{1}{2}(\partial_\mu\chi)^2 - \frac{1}{2}m_\phi^2\phi^2 - \frac{1}{2}m_\chi^2\chi^2 - g\phi\chi^2,$$

- Classical wave of ϕ in the initial state, $\phi(\vec{x}, t) = \Phi_0 \cos(px - \omega t)$
  Vacuum of χ in the initial state
- Pertubative production if $m_\phi > 2m_\chi$
- Non-pertubative production of χ particles even if $m_\phi < 2m_\chi$ at high amplitude Φ_0
- Direct solution of Heisenberg equation on χ amplitude

Heisenberg equations

Equations of motion,

$$(\Box + m_\phi^2)\phi = -g\chi^2, \leftarrow \text{neglect at early times}$$
$$(\Box + m_\chi^2)\chi = -2g\phi\chi.$$

Fourier for χ_k

$$\chi = \int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2\Omega_{\vec{k}}}}\left(\chi_{\vec{k}}(t)e^{i\vec{k}\cdot\vec{x}} + \chi_{\vec{k}}(t)^\dagger e^{-i\vec{k}\cdot\vec{x}}\right),$$

where $\Omega_{\vec{k}} = \sqrt{k^2 + m_\chi^2}$ and $[\chi_{\vec{k}}, \chi_{\vec{k}'}] = 0, [\chi_{\vec{k}}, \chi\dagger_{\vec{k}'}] = (2\pi)^3\delta^3(\vec{k} - \vec{k}')$.

Bogolubov transformation $A_{\vec{k}} = \chi_{\vec{k}} + \chi^\dagger_{-\vec{k}}$, equation in terms of A_k:

$$(\partial_t^2 + \Omega_{\vec{k}}^2)A_{\vec{k}} = -\omega_{\vec{p}}^2\alpha\left(\sqrt{\frac{\Omega_{\vec{k}}}{\Omega_{\vec{k}-\vec{p}}}}A_{\vec{k}-\vec{p}}e^{-i\omega_{\vec{p}}t} + \sqrt{\frac{\Omega_{\vec{k}}}{\Omega_{\vec{k}+\vec{p}}}}A_{\vec{k}+\vec{p}}e^{i\omega_{\vec{p}}t}\right),$$

where $\alpha \equiv \frac{g\Phi_0}{\omega_{\vec{p}}^2} = \frac{g\sqrt{2\rho_\phi}}{\omega_p^3}$.

Approximation A.Arza, 2022

$\chi_k \equiv a_k(t) e^{-i\Omega_k t}$ Looking for the resonance in a_k

$$e^{-i\Omega_{\vec{k}} t}(\ddot{a}_{\vec{k}} - 2i\Omega_{\vec{k}} \dot{a}_{\vec{k}}) = \sigma_{\vec{p}-\vec{k}} a^\dagger_{\vec{p}-\vec{k}} e^{i(\Omega_{\vec{p}-\vec{k}} - \omega_{\vec{p}})t},$$

RWA $\ddot{a}_k \to 0$

$$-2i\Omega_{\vec{k}} \dot{a}_{\vec{k}} = \sigma_k a^\dagger_{\vec{p}-\vec{k}} e^{i(\Omega_{\vec{k}} + \Omega_{\vec{p}-\vec{k}} - \omega_{\vec{p}})t},$$

where $\sigma_k = g\sqrt{\frac{\rho_\phi/2}{\omega^2 \Omega_k \Omega_{p-k}}}$, $\sigma_{p-k} = -\omega^2 \alpha \sqrt{\frac{\Omega_k}{\Omega_{p-k}}}$

Resonant solution $a_{\vec{k}}(t) =$

$e^{i\epsilon_{\vec{k}} t/2}\left(a_{\vec{k}}(0)(\cosh(s_{\vec{k}} t) - i\frac{\epsilon_{\vec{k}}}{2s_{\vec{k}}} \sinh(s_{\vec{k}} t)) + i\frac{\sigma_{\vec{p}-\vec{k}}}{2s_{\vec{k}}\Omega_{\vec{k}}} a^\dagger_{\vec{p}-\vec{k}}(0) \sinh(s_{\vec{k}} t)\right),$

where $s_{\vec{k}} = \frac{1}{2}\sqrt{\frac{\sigma^2_{\vec{p}-\vec{k}}}{\Omega^2_{\vec{k}}} - \epsilon^2_{\vec{k}}}$ and $\epsilon_{\vec{k}} = \epsilon_{\vec{p}-\vec{k}} = \Omega_{\vec{k}} + \Omega_{\vec{p}-\vec{k}} - \omega_{\vec{p}}$

$\ddot{a}_k \ll \Omega_k \dot{a}_k \quad \rightarrow \quad \alpha \ll 1, \mu \ll 1$

Boundary: $\alpha > \mu^2$ - instability

Any solution for arbitrary α, μ?

Without approximation

Not neglecting $\ddot{a}_k$

Ansatz: $a_{\vec{k}}(t) = e^{i\epsilon_{\vec{p}-\vec{k}}t/2}\Big[a_{\vec{k}}(0)\left(\cosh(s_{\vec{p}-\vec{k}}t) - iC_1\sinh(s_{\vec{p}-\vec{k}}t)\right) -$

$a^{\dagger}_{\vec{p}-\vec{k}}(0)\cdot iC_2\sinh(s_{\vec{p}-\vec{k}}t)\Big]$

Commutators $\quad\rightarrow\quad C_1^2 - C_2^2 = -1$

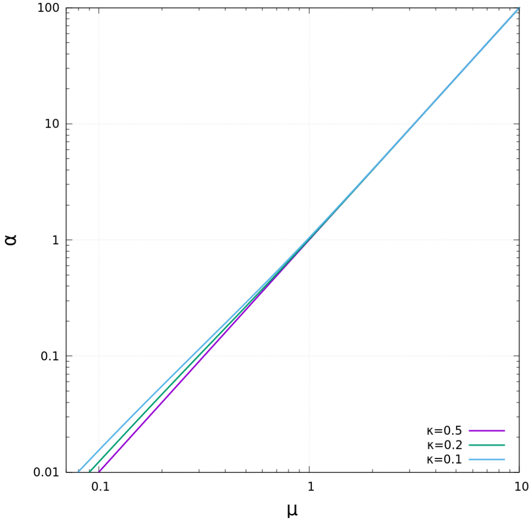
Solution

$$C_1 = \frac{\epsilon^2_{\vec{p}-\vec{k}}/4 - s^2_{\vec{p}-\vec{k}} - \Omega_{\vec{k}}\epsilon_{\vec{p}-\vec{k}}}{s_{\vec{p}-\vec{k}}(\epsilon_{\vec{p}-\vec{k}} - 2\Omega_{\vec{k}})},\ C_2 = \frac{\sigma_{\vec{p}-\vec{k}}}{s_{\vec{p}-\vec{k}}(\epsilon_{\vec{p}-\vec{k}} - 2\Omega_{\vec{k}})},$$

$$s^2_{\vec{p}-\vec{k}} = -\frac{\epsilon^2_{\vec{p}-\vec{k}}}{4} - 2\Omega^2_{\vec{k}} + \epsilon_{\vec{p}-\vec{k}}\Omega_{\vec{k}} + \sqrt{\Omega^2_{\vec{k}}\epsilon^2_{\vec{p}-\vec{k}} + 4\Omega^4_{\vec{k}} + \sigma^2_{\vec{p}-\vec{k}} - 4\epsilon_{\vec{p}-\vec{k}}\Omega^3_{\vec{k}}}.$$

$\alpha = \mu^2 -$ still boundary of instability for arbitrary α, μ

Dependence $\alpha(\mu)$



for $k = p/2$ $\quad \alpha = \mu^2;\ \mu \gg 1 \quad \rightarrow \quad \alpha = \mu^2$ for any k

Occupancy number

Approximation

$$f_{\chi,\vec{k}}(t) = \langle 0|a^\dagger_{\vec{k}}(t)a_{\vec{k}}(t)|0\rangle = \frac{\sigma^2_{\vec{p}-\vec{k}}}{4}\frac{\sinh^2(s^0_{\vec{k}}t)}{\left(s^0_{\vec{k}}\right)^2\Omega^2_{\vec{k}}},$$

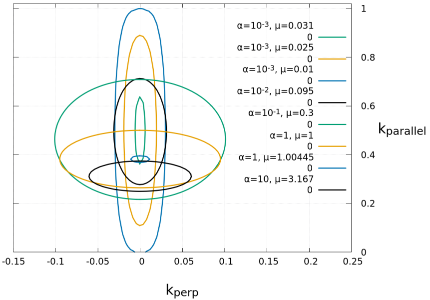
Without approximation

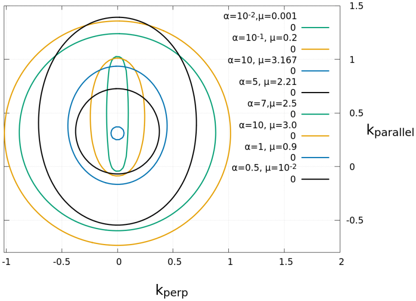
$$f_{\chi,\vec{k}}(t) = \langle 0|a^\dagger_{\vec{k}}(t)a_{\vec{k}}(t)|0\rangle = \frac{\sigma^2_{\vec{p}-\vec{k}}}{4}\frac{\sinh^2(s_{\vec{k}}t)}{s^2_{\vec{k}}(\Omega_{\vec{k}}-\epsilon_{\vec{k}}/2)^2}.$$

The total density

$$n_\chi(t) = \int \frac{d^3k}{(2\pi)^3} f_{\chi,\vec{k}}(t),$$

The contours for $s = 0$





No symmetry $k \to p - k$ at $\alpha \gg 1 \to$ fall of 2 particle production interpretation

Particle production in condensate

Equation of motion for $\phi(t) = \Phi \cos(m_\phi t)$ and $p = 0$

$$\ddot{\chi}_k + \left(k^2 + m_\chi^2 + 2g\Phi \cos(m_\phi t)\right) \chi_k = 0,$$

The Mathieu equation

$$\chi_k'' + (A_k + 2q \cos(2z)) \chi_k = 0,$$

where

$$A_k = 4\frac{k^2 + m_\chi^2}{m_\phi^2}, \qquad q = 4\frac{g\Phi}{m_\phi^2}.$$

The number of modes determines as,

$$n_k = \langle 0 | a_k(t) a_k^\dagger(t) | 0 \rangle = |\chi_k(t)|^2$$

The Heisenberg equation for condensate, $p = 0$

$$e^{-i\Omega_k t}\left[\ddot{a}_k - 2i\Omega_k \dot{a}_k + 2\alpha\omega^2\cos(m_\phi t)a_k\right] +$$
$$+e^{i\Omega_k t}\left[\ddot{a}^\dagger_{-k} + 2i\Omega_k \dot{a}^\dagger_{-k} + 2\alpha\omega^2\cos(m_\phi t)a^\dagger_{-k}\right] = 0$$

Solution

$$a_k(t) = e^{i\epsilon_k t/2}\Big(a_k(0)\left(\cosh(st) - i\frac{\epsilon^2/4 - s^2 - \Omega_k\epsilon_k}{s(2\Omega_k - \epsilon_k)}\sinh(st)\right) -$$
$$-ia^\dagger_{-k}\frac{\alpha\omega^2}{s(2\Omega_k - \epsilon_k)}\sinh(st)\Big),$$

where

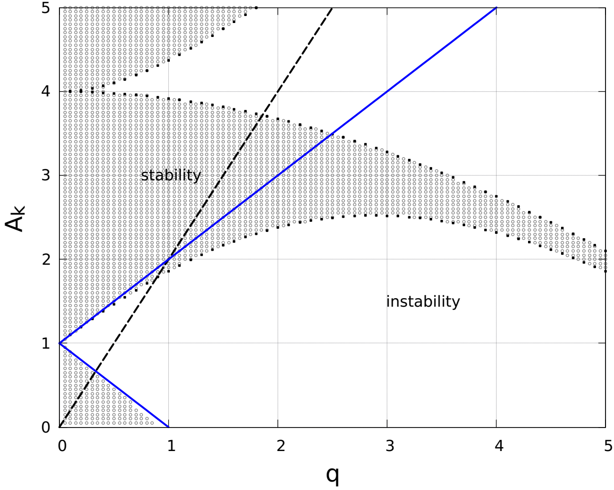
$$s = \sqrt{\Omega_k^2(2\Omega_k - \epsilon_k)^2 + \alpha^2 m_\phi^4} - \Omega_k(2\Omega_k - \epsilon_k) - \frac{\epsilon_k^2}{4}.$$

In terms of the parameters of the Mathieu equation for $s^2 = 0$:

$$q = |A_k - 1|,$$

where $A_k = 4\frac{k^2 + m_\chi^2}{m_\phi^2}$, $\qquad q = 4\frac{g\Phi}{m_\phi^2}$.

Stability diagram



$q \ll 1$ - narrow resonance, $q \gg 1$ - broad resonance

Conclusion

- For a plane wave, resonance will occur if the initial wave has a sufficiently high energy density. The threshold energy density

$$\rho_\phi \geq \frac{m_\chi^4 \omega^2}{2g^2}$$

  for any m_χ.
- It can be concluded that the interpretation through particles is not applicable to this case
- There may be other solutions for condensate, since the Mathieu equation has already been solved by mathematicians
- For a plane wave, there may be other solutions that can increase the instability region and will correspond to other peaks of the Mathieu equation

Thank you for your attention!