

# The Schwinger effect: Double assistance by photon and intermediate plane wave

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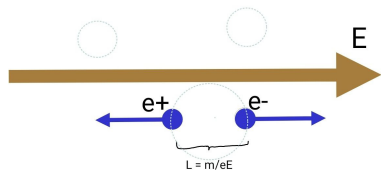


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# The Schwinger effect



Nonperturbative over small parameter  $e$ ,

$$E_{LAB} \sim 10^{16} \text{ V/m},$$

Sauter 1931, Euler Heisenberg 1936,  
Schwinger 1951

$$\Gamma = VT \cdot \frac{(eE)^2}{(2\pi)^3} e^{-\frac{\pi m^2}{eE}}$$

$$E_S = m^2/e = 10^{18} \text{ V/m}$$

$$e^{-\pi E_S/E_{LAB}} \sim e^{-90}$$

No chance of detection? Need to enhance the process!

Sauter pulse  $E = E_0/ch^2(\omega t)$

$$\Gamma \propto \exp\left(-\frac{2\pi m^2}{eE(\sqrt{1+\gamma^2}+1)}\right), \quad \gamma = \frac{\omega m}{eE} - \text{Keldysh parameter}$$

$\gamma \gg 1 \rightarrow$  less suppressed

Nikishov; Nikishov, Ritus 1970

# Dynamical assistance//Catalysis for the Schwinger effect

2 components of EM field  $\rightarrow$  Schwinger p.p. less suppressed

## Dynamically assisted Schwinger mechanism

Schutzhold Gies Dunne PRL 2008 arXiv:0807.0754

$$\vec{E}(t) = \frac{E_0}{\cosh(\Omega t)} \vec{e}_z + \frac{\varepsilon}{\cosh(\omega t)} \vec{e}_z$$

where  $E_S \gg E_0 \gg \varepsilon > 0$ ,  $m \gg \omega \gg \Omega > 0$ ,  $\gamma = \frac{m\omega}{eE_0}$

$$\Gamma \propto \exp \left[ -\frac{m^2}{eE_0} \left( 2 \arcsin \frac{\pi}{2\gamma} + \frac{\pi^2}{2\gamma^2} \sqrt{4\gamma^2 - \pi^2} \right) \right], \quad \Omega \rightarrow 0$$

## Catalysis of Schwinger Vacuum Pair Production

= Photon decay

Dunne Gies Schutzhold PRD 2009 arXiv:0908.0948

Worldline Inst:

Monin, Voloshin PRD 2010 arXiv:1001.3354

$$\Gamma \propto \exp \left( -\frac{m^2}{eE} \left[ \left( 2 + \frac{\omega^2}{2m^2} \right) \arctan \left( \frac{2m}{\omega} \right) - \frac{\omega}{m} \right] \right) \rightarrow \exp \left( -\frac{8m^2}{3eE} \frac{m}{\omega} \right), \quad \omega \gg 1$$

Less suppressed at  $\omega > m$

# Assistance + Catalysis = Double Assistance

## Time-dependent field vs plane wave

Torgrimsson Oertel Schützhold PRD 2016 arXiv:1607.02448  
Torgrimsson PRD 2019 arXiv:1812.04607

Double assistance:

Const electric field  $E$  + photon  $\Omega \sim m$  + intermediate field  $\varepsilon E(t)$ ,  
 $E(t) = E/ch^2(\omega t)$  or  $E(t) = E \sin(\omega t)$ .

$\varepsilon \sim 10^{-3} - 10^{-2}$ ,  $\omega \sim (10^{-3} - 10^{-2})m$  – freq. of X-ray laser

Torgrimsson Schneider Schützhold PRD 2018 arXiv:1712.08613

Dynamical assistance (not double!) with plane wave  $\varepsilon E \cos \omega(t - x)$

Double assistance with photon and plane wave?

# Worldline formalism

Scalar QED:  $S_E = \int d^4x \left( -\frac{1}{4} F_{\mu\nu}^2 + |D_\mu \phi|^2 + m^2 |\phi|^2 \right)$ .  
 $A_\mu$  — classical EM field.  $Z[A_\mu] = \int D\phi^* D\phi e^{-S_E[A_\mu, \phi^*, \phi]} = e^{-W[A_\mu]}$ .  
Effective action:

$$W[A_\mu] = \int d^4x \left( -\frac{1}{4} F_{\mu\nu}^2 - \log \det (-D_\mu^2 + m^2) \right).$$

Schwinger *proper time* representation:

Schwinger, 1951

$$W[A_\mu] = \int d^4x \left( -\frac{1}{4} F_{\mu\nu}^2 + \int_0^\infty \frac{ds}{s} e^{-m^2 s} \text{Tr} \left( e^{D_\mu^2 s} \right) \right).$$

Operator  $(-D_\mu^2)$  can be interpreted as QM Hamiltonian.

$$\text{Tr} \left( e^{s D_\mu^2} \right) = \int d^4x \langle x_\mu | e^{-s(-D_\mu^2)} | x_\mu \rangle = \int_{p.b.c.} D x_\mu e^{-\int_0^s d\tau \left( \frac{\dot{x}_\mu^2}{4} + ie A_\mu \dot{x}_\mu \right)}.$$

# The Schwinger effect with Worldline Instantons

Affleck, Alvarez, Manton, 1982

The Schwinger effect (vacuum pair production):  $\Gamma = 2 \operatorname{Im} W[A_\mu]$

$$\Gamma = 2 \operatorname{Im} \int_0^\infty \frac{ds}{s} e^{-m^2 s} \int_{p.b.c.} D x_\mu e^{-\int_0^1 d\tau \left( \frac{\dot{x}_\mu^2}{4s} + i e A_\mu \dot{x}_\mu \right)}.$$

Take integrals over  $x_\mu$  and  $s$  in the saddle point approximation

EOMs:  $\frac{\ddot{x}_\mu}{2s} - e F_{\mu\nu} \dot{x}_\nu = 0$  and  $-m^2 + \frac{\int_0^1 \dot{x}_\mu^2 d\tau}{4s^2} = 0$ .

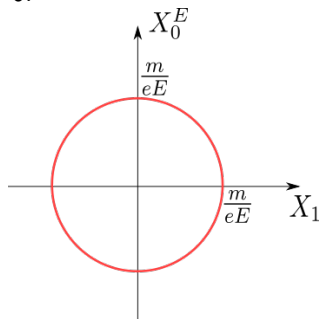
Uniform constant electric field  $E$ .

The leading order closed solution is a circle:

$$\begin{aligned} x_0^{cl} &= \frac{m}{eE} \sin(2\pi\tau), & x_1^{cl} &= \frac{m}{eE} \cos(2\pi\tau), \\ x_2^{cl} &= x_3^{cl} = 0, & s &= \frac{2\pi}{eE}. \end{aligned}$$

The action on the solution  $x_\mu^{cl}$  is  $S[x_\mu^{cl}] = \frac{\pi m^2}{eE}$ .

Condition:  $S[x_\mu^{cl}] \gg 1$ ,  $\Gamma \propto e^{-S[x_\mu^{cl}]}$ .



$S[x_\mu^{cl}] \propto \text{area}$

# The Schwinger effect with Worldline Instantons

- Pre-exponential factor: integrate over quadratic fluctuations  $\delta x_\mu$  near classical solution  
 $x_\mu = x_\mu^{cl} + \delta x_\mu$
- Zero mode: 4-volume  $VT$
- Negative mode:  $\delta x_\mu \propto x_\mu^{cl}$   
 $i$  in pre-exp factor  $\rightarrow \Gamma = 2 \text{Im}W[A_\mu] \neq 0$ .

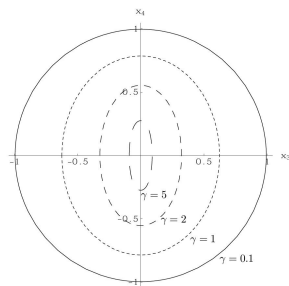
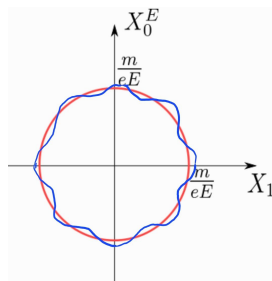
$$\Gamma = VT \cdot \frac{(eE)^2}{(2\pi)^3} e^{-\frac{\pi m^2}{eE}}$$

**Sauter pulse**  $E = E_0/ch^2(\omega t)$

WInst - [Dunne Schubert PRD 2005 hep-th/0507174](#)

$$\Gamma \propto \exp\left(-\frac{2\pi m^2}{eE(\sqrt{1+\gamma^2}+1)}\right)$$

$$\gamma = \frac{\omega m}{eE} - \text{Keldysh parameter}$$



# How to deal with external photons in Worldline formalism

Each photon vertex — functional derivative over  $A_\mu$ . In scalar QED

$$V[k_\mu, \varepsilon_\mu(k)] = \int_0^1 d\tau \varepsilon_\mu(k) \dot{x}_\mu(\tau) e^{ik^\mu x^\mu(\tau)}.$$

N-point photon amplitude

Schubert, 2001, hep-th/0101036

$$M[k_1, \varepsilon(k_1); \dots; k_N, \varepsilon(k_N)] = (-ie)^N \int_0^\infty \frac{ds}{s} e^{-m^2 s} (4\pi s)^{-2} \langle V[k_1, \varepsilon(k_1)] \dots V[k_N, \varepsilon(k_N)] \rangle$$

Here  $\langle V_1 \dots V_N \rangle \equiv \int_{p.b.c} D x_\mu [V_1 \dots V_N] \exp\left(-\int_0^s d\tau \left(\frac{\dot{x}_\mu^2}{4} + ie A_\mu \dot{x}_\mu\right)\right)$ .

New extra terms  $ik_j^\mu x^\mu(\tau_j)$  in the exponent from  $V_j$ ,  $j = 1..N$ .

## The Optical theorem

Cross-sections of pair production are proportional to the imaginary part of the corresponding amplitude (depending on the number of initial photons):

- The photon decay:  $\Gamma_{\gamma \rightarrow e^+ e^-} = \frac{1}{2\omega} \text{Im} \mathcal{M}[k, \varepsilon(k); k, \varepsilon^*(k)]$ .
- $2\gamma$  scattering:  $\sigma_{\gamma\gamma \rightarrow e^+ e^-} \propto \text{Im} \mathcal{M}[k_1, \varepsilon(k_1); k_2, \varepsilon(k_2); k_1, \varepsilon^*(k_1); k_2, \varepsilon^*(k_2)]$ .
- ...



# Photon decay $\gamma \rightarrow e^+e^-$ in EM background

Decay width: 
$$\Gamma_{\gamma \rightarrow e^+e^-} = \frac{1}{2\omega} \varepsilon_\mu(k) \varepsilon_\nu^*(k) \text{Im} \Pi_{\mu\nu}(k).$$

$$\text{Im} \Pi_{\mu\nu}(k) = \text{Im} \int_0^\infty \frac{ds}{s} \int_{p.b.c.} D x_\mu \oint d\tau_1 \oint d\tau_2 \dot{x}_\mu(\tau_1) \dot{x}_\nu(\tau_2) e^{-S_m[x_\mu; \tau_1, \tau_2]},$$

where

$$S_m[x_\mu; \tau_1, \tau_2] = m^2 s + \int_0^1 d\tau \left( \frac{\dot{x}_\mu^2}{4s} + ieA_\mu \dot{x}_\mu \right) - ik_\mu (x_\mu(\tau_1) - x_\mu(\tau_2)).$$

The saddle point over  $s$ : 
$$m^2 - \frac{\int_0^1 d\tau \dot{x}_\mu^2}{4s^2} = 0.$$

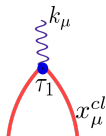
EOMs: 
$$\frac{\ddot{x}_\mu}{2s} - ieF_{\mu\nu} \dot{x}_\nu = -ik_\mu (\delta(\tau - \tau_1) - \delta(\tau - \tau_2)).$$

Break for the derivatives: 
$$\frac{\dot{x}_\mu(\tau_1+0) - \dot{x}_\mu(\tau_1-0)}{2s} = -ik_\mu$$

Saddle points  $\tau_{1(2)}$  for the classical trajectory  $x_\mu^{cl}(\tau)$

The strength of the break proportional to  $k_\mu$

Closed trajectories with saddle points both in E and H



# Worldline instantons for the photon decay $\gamma \rightarrow e^+ e^-$ in electric and magnetic backgrounds

Ext. photon  $k_\mu = (\omega, 0, \omega, 0)$

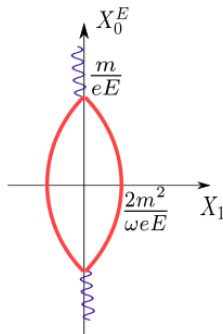
Electric field  $(E, 0, 0)$  or  
Magnetic field  $(0, 0, H)$ .

Classical solution —  
two arcs of circle (electric)  
two hyperbolas (magnetic)  
connected at  $\tau_1 = 0, \tau_2 = 0.5$

$$\Gamma \propto e^{-S}.$$

In the limit  $\omega \gg 2m$

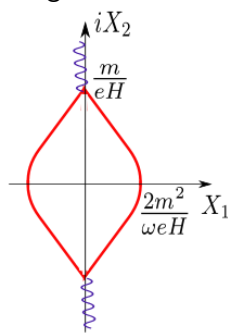
Electric field



$$S = \frac{8m^3}{3weE}$$

Monin, Voloshin, 2010

Magnetic field



(projection on  $(ix_2, x_1)$ )

$$S = \frac{8m^3}{3weH}$$

PS 2013

# Breit-Wheeler process $\gamma\gamma \rightarrow e^+e^-$ in external electric field

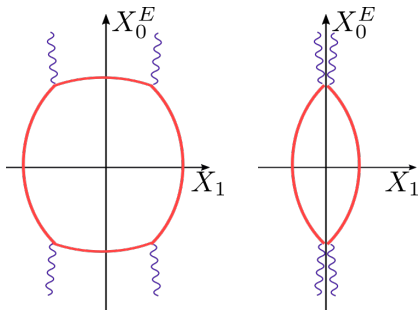
Breit-Wheeler cross-section is proportional to the imaginary part of 4-point photon amplitude:

$$\sigma_{\gamma\gamma \rightarrow e^+e^-}(k_1, k_2) \propto \text{Im} \int_0^\infty \frac{ds}{s} e^{-m^2 s} \int_{p.b.c} Dx_\mu \prod_{j=1}^4 \left( \oint d\tau_j \dot{x}_\mu \varepsilon_\mu(k_j) \right) \cdot \exp \left( - \int_0^s d\tau \left( \frac{\dot{x}_\mu^2}{4} + ieA_\mu \dot{x}_\mu \right) - ik_1^\mu x_\mu(\tau_1) - ik_2^\mu x_\mu(\tau_2) + ik_1^\mu x_\mu(\tau_3) + ik_2^\mu x_\mu(\tau_4) \right).$$

General solution of E.O.M.s – 4 arcs of a circle, connected at 4 points  $\tau_1 \dots \tau_4$ , see Fig, left panel.

On E.O.M.s up & down arcs shrink into points, see Fig, right panel.

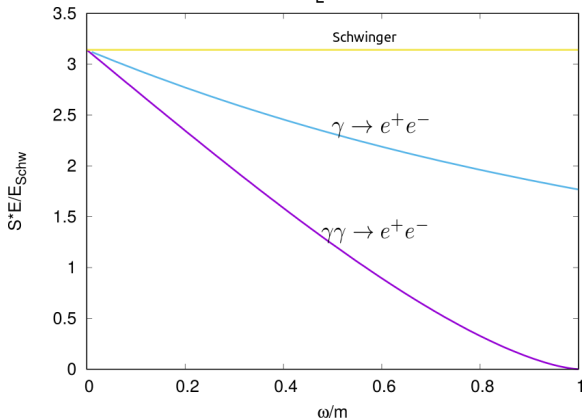
(the same for  $N$  photons in the initial state)



# Breit-Wheeler pair production. Head-on collision.

Configuration:  $k_1^\mu = (\omega, 0, \omega, 0)$ ,  $k_2^\mu = (\omega, 0, -\omega, 0)$ ,  $\vec{E} = (E, 0, 0)$ .

Suppression exponent: 
$$S[x_\mu^{cl}] = \frac{m^2}{eE} \left[ \pi - 2 \arcsin \frac{\omega}{m} - 2 \frac{\omega}{m} \sqrt{1 - \left(\frac{\omega}{m}\right)^2} \right]$$



$2\gamma$  – less suppressed than  $1\gamma$

$\omega/m = 1$  – threshold for Breit-Wheeler without external field

# Pair production in N-photon interaction

Configuration:  $K^\mu = (\omega_\Sigma = \sum_{i=1}^N \omega_i, \vec{k}_\Sigma = \sum_{i=1}^N \vec{k}_i), \quad \vec{E} = (E, 0, 0).$

$$P_{N\gamma \rightarrow e^+e^-}^{non-pert} \propto \text{Im} M_{2N} \sim e^{-S_N}$$

$$S_N = \frac{m^2}{eE} \left( \left( 2 + \frac{k_\Sigma^2}{2m^2} \right) \arctan \frac{\sqrt{4m^2 - \omega_\Sigma^2 + k_\Sigma^2}}{\omega_\Sigma} - \frac{\omega \sqrt{4m^2 - \omega_\Sigma^2 + k_\Sigma^2}}{2m^2} \right)$$

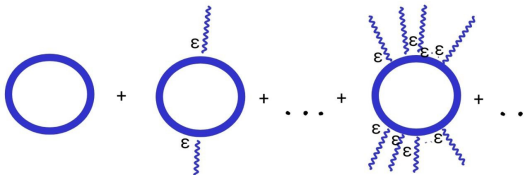
in agreement with [Torgrimsson Schneider Oertel Schuetzhold JHEP 2017](#) (WKB method)

# Dynamical assistance by a plane wave

Torgimsson Schneider Schuetzhold PRD 2018

$$\vec{E} = E_0 \vec{e}_z + \varepsilon E_0 \cos \omega(t - x) \vec{e}_z$$

perturbation theory over  $\varepsilon$



$$P = \sum_{N=1}^{\infty} \varepsilon^{2N} P_N,$$

$$P_N : \omega_{\Sigma} = N\omega, \quad k_{\Sigma} = N\Omega$$

$$\varepsilon^{2N} P_N = \exp \left( 2N \log \varepsilon - \frac{m^2}{eE} \left[ \left( 2 + \frac{N^2 \omega^2}{2m^2} \right) \arctan \left( \frac{2m}{N\omega} \right) - \frac{N\omega}{m} \right] \right)$$

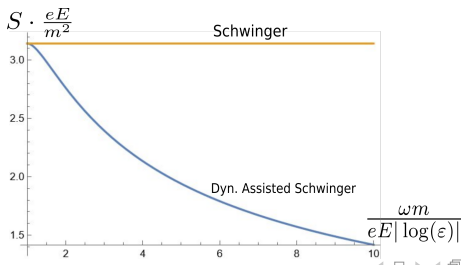
# Dynamical assistance by a plane wave

Which term gives the maximal contribution?

$$\text{saddle point: } \left. \frac{d(\varepsilon^{2N} P_N)}{dN} \right|_{N=N_*} = 0$$

$$1 - \frac{N_* \omega}{2m} \arctan \frac{2m}{N_* \omega} = \frac{|\log \varepsilon|}{m\omega/eE} \equiv \frac{\gamma_{crit}}{\gamma}$$

$$\begin{aligned} \gamma \ll \gamma_{crit} &\rightarrow N_* = 0, & P &\sim e^{-\frac{\pi m^2}{eE}}, \\ \gamma \gg \gamma_{crit} &\rightarrow N_* = \frac{4m}{\omega} \sqrt{\frac{3m\omega/eE}{|\log \varepsilon|}}, & P &\sim e^{-8 \frac{m^2}{eE} \sqrt{\frac{|\log \varepsilon|}{3m\omega/eE}}} \end{aligned}$$

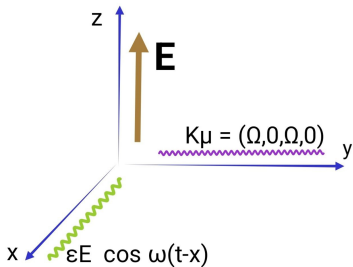


# Double assistance by a plane wave and photon

$$\vec{E} = E_0 \vec{e}_z + \varepsilon E_0 \cos \omega(t-x) \vec{e}_z$$

$$\vec{K} = (\Omega, 0, \Omega, 0)$$

$$\omega \ll m, \quad \Omega \sim m$$



$$P = \sum_{N=1}^{\infty} \varepsilon^{2N} P_N,$$

$$P_N : \omega_{\Sigma} = N\omega + \Omega, \quad k_{\Sigma} = \sqrt{N^2\omega^2 + \Omega^2}$$

$$\varepsilon^{2N} P_N = \exp \left( 2N \log \varepsilon - \frac{m^2}{eE} S_N \right)$$

$$S_N = \left[ \left( 2 + \frac{k_{\Sigma}^2}{2m^2} \right) \arctan \frac{\sqrt{4m^2 - \omega_{\Sigma}^2 + k_{\Sigma}^2}}{\omega_{\Sigma}} - \frac{\omega \sqrt{4m^2 - \omega_{\Sigma}^2 + k_{\Sigma}^2}}{2m^2} \right]$$



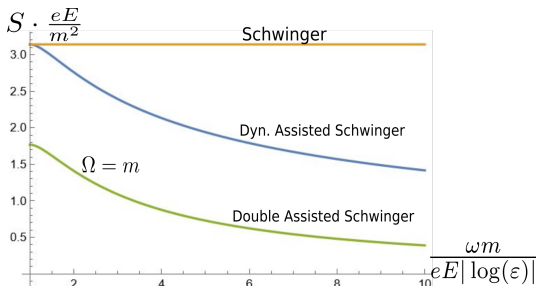
# Double assistance by a plane wave and photon

saddle point: 
$$-\frac{m}{2\omega} \cdot \frac{dS_N}{dN} \Big|_{N=N_*} = \frac{|\log \varepsilon|}{m\omega/eE} \equiv \frac{\gamma_{crit}}{\gamma}$$

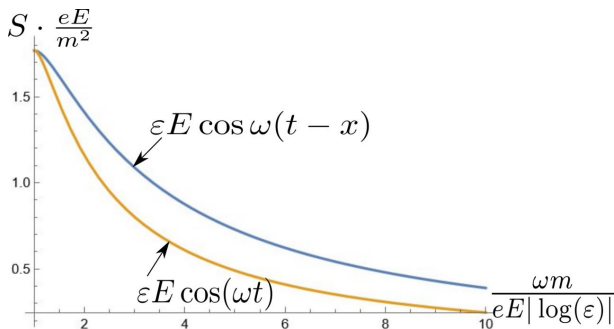
$$\sqrt{1 - \frac{N_*\omega\Omega}{2m^2}} - \frac{N_*\omega}{2m} \arctan \frac{\sqrt{4m^2 - 2N_*\omega\Omega}}{N_*\omega + \Omega} = \frac{\gamma_{crit}}{\gamma}$$

$$\gamma \ll \gamma_{crit} \rightarrow N_* = 0, \quad P \sim e^{-\frac{m^2}{eE} \left[ \left(2 + \frac{\Omega^2}{2m^2}\right) \arctan\left(\frac{2m}{\Omega}\right) - \frac{\Omega}{m} \right]}$$

$$\gamma \gg \gamma_{crit} \rightarrow N_* = \frac{2m^2}{\omega\Omega} \left(1 - \frac{\gamma_{crit}^2}{\gamma^2} \frac{(2m^2 + \Omega^2)^2}{\Omega^4}\right) \quad P \sim e^{-\frac{4m^2}{eE} \frac{\gamma_{crit}}{\gamma}}$$



# Plane wave vs time-dependent electric field



Double assistance with plane wave is less effective

# Conclusions

- 3 components of EM field increase the pair production probability compared to 2
- Intermediate plane wave - worse than time-dependent field / standing wave
- Experimental convenience for plane//standing wave for X-ray laser?
- other approaches - comparison

Mahlin Villalba-Chavez Mueller 2023

Thank you for your attention!