Müeller–Navelet dijet production in the High–Energy Factorization approach

A. Chernyshev[†,⋆,1] & V. Saleev[†,*]

[†]Bogoliubov Laboratory of Theoretical Physics, JINR

[⋆]Moscow State U., Dubna Branch

[*]Samara National Research U.

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[1]Email: chernyshev@theor.jinr.ru

Motivation: «small x» activities at LHC

Two most important kinematical regimes:

i. **Bjorken limit:** $(-\hat{t}) \sim Q^2 \to \infty$ while $\hat{s} \sim (-\hat{t}) \Rightarrow$ DGLAP logs $\alpha_s^n \ln^n \left((-\hat{t})/\Lambda^2\right)$.

ii. **Regge limit:** $\hat{s} \gg (-\hat{t}) \Rightarrow$ BFKL logs $\alpha_s^n \ln^n (\hat{s}/(-\hat{t}))$.

Search for BFKL evolution manifestations:

- **Müeller–Navelet dijets**[Müeller, Navelet '87]**:**
  - Experimental data at large Y[CMS '16,20];
  - Theoretical studies using *collinear factorization*[Szymanowski *et.al.*; Papa *et.al.*; Sabio–Vera *et.al.*; Kim *et.al.*];
- Higgs boson+jet production, see[Papa *et.al.*].

There is an approach *beyond collinear factorization*–High–Energy Factorization (HEF).

In this talk: Müeller–Navelet dijet production in the HEF with BFKL *via gluodynamics*.

High–Energy Factorization approach: I

Consider some process:

$$p\,(P_1) \,+\, p\,(P_2) \,\to\, \mathcal{Y}\,(p^{\pm}, \mathbf{p}_T) \,+\, X,$$

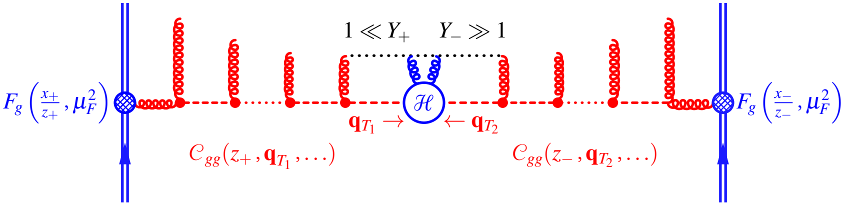
where $P_{1,2} = \left(P^{\pm}/2\right) n_{\mp}$ with $P^{\pm} = \sqrt{S}$. We use Sudakov light–cone basis vectors $n_{\pm}$: $(n_{\pm}, n_{\mp}) = 2$, $p^{\pm} = (p, n_{\pm})$, so $y(p) = (1/2)\ln(p^{+}/p^{-})$. Introduce $x_{\pm} = p^{\pm}/P^{\pm}$, then:

$$\sigma^{(\mathrm{CPM})} = \int\limits_{x_+}^{1} \frac{dz_+}{z_+}\, F\left(\frac{x_+}{z_+}, \mu_F^2\right) \int\limits_{x_-}^{1} \frac{dz_-}{z_-}\, F\left(\frac{x_-}{z_-}, \mu_F^2\right) \times H\left(z_{\pm}, \alpha_S(\mu_R^2)\right) \,+\, \mathcal{O}\left(\frac{\Lambda^{\#}}{\mu_F^{\#}}\right),$$

here $F(x, \mu_F^2) = x\, f(x, \mu_F^2)$. We assume $\mu_F \simeq \mu_R \simeq \mu$.

- **Motivation:** resum already in LO large radiative corrections enhanced by logs $\ln(1/z_{\pm})$.
- **Resummation formalism:** High–Energy Factorization *or k_T–factorization*, uses properties of hard scattering amplitudes **Reggeization** in the limit $\boxed{z_{\pm} \ll 1}$, see[Gribov, Levin, Ryskin ’84; Collins, Ellis ’91, 94; Catani, Hautman ’94].

High–Energy Factorization approach: II



Factorization formula in the HEF approach[Collins, Ellis '91; Catani, Hautman '94] PROVEN UP TO NLLA:

$$\sigma^{(\mathrm{HEF})} = \int \frac{dx_1}{x_1}\int \frac{d^2\mathbf{q}_{T_1}}{\pi}\,\Phi_g(x_1,\mathbf{q}_{T_1}^2,\mu^2)\int \frac{dx_2}{x_2}\int \frac{d^2\mathbf{q}_{T_2}}{\pi}\,\Phi_g(x_2,\mathbf{q}_{T_2}^2,\mu^2)\times\mathcal{H}\left(x_{1,2},\mathbf{q}_{T_{1,2}}^2,\ldots\right)$$

- PROCESS INDEPENDENT ***Unintegrated PDF (uPDF)*** is a convolution of PDF with *resummation factor* $\mathcal{C}_{gg}$:

$$\Phi_g(x,\mathbf{q}_T^2,\mu^2) = \int\limits_x^1 \frac{dz}{z}\,F_g\left(\frac{x}{z},\mu^2\right)\times\mathcal{C}_{gg}(z,\mathbf{q}_T,\mu^2)$$

- PROCESS DEPENDENT ***Hard Scattering Coefficient (HSC)*** $\mathcal{H}$ is calculated in the approximation of *Multi–Regge Kinematics (MRK)*, so it is gauge–invariant.

A sketch of the EFT: I

The **gauge–invariant Lipatov's EFT** for the MRK processes in QCD$_{\text{[Lipatov '95]}}$ is formulated in terms of Yang–Mills gluon fields $v_\mu^{(i)}(x)$ and *Reggeon fields* $A(x)$ local in the rapidity:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{kin}}(A_+(x), A_-(x)) + \sum_{i \in \text{rap.int.}} \left\{ \mathcal{L}_{\text{QCD}}\left(v_\mu^{(i)}(x)\right) + \mathcal{L}_{\text{ind}}\left(v_\mu^{(i)}(x), A_+(x), A_-(x)\right) \right\},$$

where $v_\mu(x) = -iT_a v_\mu^a(x)$ and $A(x) = -iT_a A^a(x)$. Due to the MRK constraints:

$$\partial_+ A_-(x) = \partial_- A_+(x) = 0 \qquad \Rightarrow \qquad A_+(x) = A_+(x_+, \mathbf{x}_T) \text{ and } A_-(x) = A_-(x_-, \mathbf{x}_T),$$

where $\partial_\pm \equiv n_\pm^\mu \partial_\mu$. The kinetic part

$$\mathcal{L}_{\text{kin}}(A_+(x), A_-(x)) = 4\text{tr}\left[A_+(x)\, \partial_T^2 A_-(x)\right]$$

leads to the Reggeon propagator in the form:

$$D^{(\pm)}_{a_1 a_2}(q) = \frac{i\delta_{a_1 a_2}}{2\mathbf{q}_T^2}.$$

FOR AN OVERVIEW SEE$_{\text{[Nefedov '19; Hentschinski '20]}}$.

A sketch of the EFT: II

The induced reggeon–gluon interaction lagrangian part may be expanded in g series:

$$\mathcal{L}_{\rm ind} = {\rm tr}\Big[1\left(A_+\partial_T^2 v_-\right) + (-ig)\left(\partial_T^2 A_+\right)\left(v_-\partial_-^{-1}v_-\right) + (-ig)^2\left(\partial_T^2 A_+\right)\left(v_-\partial_-^{-1}v_-\partial_-^{-1}v_-\right)$$
$$+\ (+\leftrightarrow -)\Big]\ +\ \mathcal{O}\left(g^3\right)$$

The **induced vertices** may be obtained from this expansion[Lipatov, Kuraev et.al. '05]:

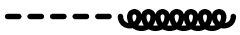
$$\begin{aligned}
\mathcal{O}(g^0):\ R_\pm g \qquad & \Delta^{(\pm)ab_1}_{\mu_1}(q,l_1) &=&\ i\,\mathbf{q}_T^2\left(n^{\mp}_{\mu_1}\right)\delta^{ab_1},\\
\mathcal{O}(g^1):\ R_\pm gg \qquad & \Delta^{(\pm)ab_1b_2}_{\mu_1\mu_2}(q,l_1,l_2) &=&\ -g\,\mathbf{q}_T^2\left(n^{\mp}_{\mu_1}\,n^{\mp}_{\mu_2}\right)\frac{f^{ab_1b_2}}{l_1^{\pm}},\\
\mathcal{O}(g^2):\ R_\pm ggg \qquad & \Delta^{(\pm)ab_1b_2b_3}_{\mu_1\mu_2\mu_3}(q,l_1,l_2) &=&\ i\,g^2\,\mathbf{q}_T^2\left(n^{\mp}_{\mu_1}\,n^{\mp}_{\mu_2}\,n^{\mp}_{\mu_3}\right)\\
& & \times & \sum_{(i_1,i_2,i_3)\in S_3}\frac{{\rm tr}\left[T^aT^{b_{i_1}}T^{b_{i_2}}T^{b_{i_3}}+(i_1\leftrightarrow i_3)\right]}{l_{i_3}^{\pm}\left(l_{i_3}^{\pm}+l_{i_2}^{\pm}\right)}
\end{aligned}$$

etc.

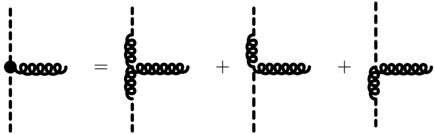
Structure of the RRP and RRPP vertices in the EFT formalism

Structure of the **effective vertices** in the EFT, see$_{\text{[Antonov, Cherednikov, Kuraev, Lipatov '05]}}$:

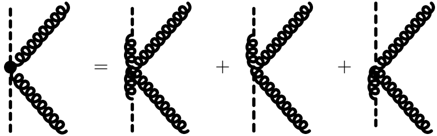
- *Rg* (*induced vertex*)



- *RRg*



- *RRgg*



Effective vertices up to $\mathcal{O}\left(g^4\right)$ implemented in `ReggeQCD`$_{\text{[Nefedov]}}$ for `FeynArts`$_{\text{[Hahn '01]}}$.

Parton Reggeization Approach (PRA)

The PRA based on the *modified MRK* approximation for QCD amplitudes[Nefedov, Saleev '20]:

$$\overline{|\mathcal{M}^{(\text{mMRK})}_{gg\to g\mathcal{Y}g}|^2} = \frac{4g^2}{q_1^2 q_2^2}\frac{P_{gg}(z_1)}{z_1}\frac{P_{gg}(z_2)}{z_2}\times\overline{|\mathcal{A}^{(\text{MRK})}_{\mathcal{Y}}|^2}+\mathcal{O}\left(\frac{\mu^2}{S}\right)$$

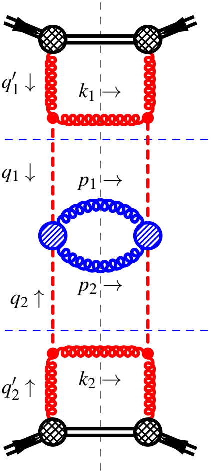
where $z_1 = q_1^+/q_1'^+$ and $z_2 = q_2^-/q_2'^-$. In this approximation:

$$\mathcal{C}^{(\text{LO, mKMRW})}_{gj}(x,z,\mathbf{q}_T^2,\mu^2) = \frac{\alpha_s(\mathbf{q}_T^2)}{2\pi}\frac{T_g(x,\mathbf{q}_T^2,\mu^2)}{\mathbf{q}_T^2}\, zP_{gj}(z)\,\theta\left(\Delta(\mathbf{q}_T^2,\mu^2)-z\right),$$

this is *so called* modified Kimber–Martin–Ryskin–Watt model[KMR '01; MRW '03; Nefedov, Saleev '20]. **Normalization condition:**

$$\int_0^{\mu_F^2} d\mathbf{q}_T^2 \sum_j \mathcal{C}^{(\text{LO, mKMRW})}_{gj}(x,z,\mathbf{q}_T^2,\mu^2) = \delta(1-z),$$

here $j = g, q, \bar{q}$.



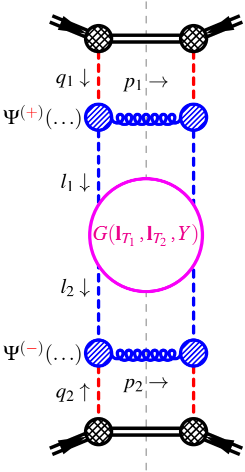
LOGS $\alpha_s \ln\left(\mathbf{q}_T^2/\mu^2\right)$ AND $\alpha_s \ln^2\left(\mathbf{q}_T^2/\mu^2\right)$ ARE RESUMMED IN THE mKMRW uPDFs $\Leftrightarrow$ CONSISTENCY WITH *COLLINS–SOPER–STERMAN APPROACH*[CSS '85; C '11].

BFKL resummation in the PRA

Resummation of logs $\alpha_s^n \ln^n (\hat{s}/(-\hat{t})) \sim \alpha_s^n Y^n$ may be included in the PRA through the **LLA BFKL equation**[BFKL '76–78], SEE ALSO TEXTBOOKS[Ioffe, Fadin, Lipatov '10; Kovchegov, Levin '12]:

$$\frac{d\mathcal{H}^{(\mathrm{BFKL})}}{d^2\mathbf{p}_{T_1}\, dy_1\, d^2\mathbf{p}_{T_2}\, dy_2} = \frac{1}{4x_1x_2S^2} \int \frac{d^2\mathbf{l}_{T_1}}{2\pi} \int \frac{d^2\mathbf{l}_{T_2}}{2\pi}$$

$$\times\, \Psi^{(+)}(\mathbf{l}_{T_1}, \mathbf{p}_{T_1}, y_1) \times G(\mathbf{l}_{T_1}, \mathbf{l}_{T_2}, Y) \times \Psi^{(-)}(\mathbf{l}_{T_2}, \mathbf{p}_{T_2}, y_2)$$



PROCESS DEPENDENT **Impact Factors** describe $RR \rightarrow g$ may be obtained from the PRA $2 \rightarrow 1$ matrix elements projected on the Pomeron channel[Fadin et.al. '99]: $\langle cc' \mid \hat{P}_{\mathbf{1}} \mid 0 \rangle = \delta_{cc'}/\sqrt{N_c^2 - 1}$.

PROCESS INDEPENDENT **Green function** obeys BFKL equation for the Pomeron channel resum logs $\alpha_s^n Y^n$:

$$\frac{\partial G(\mathbf{l}_{T_1}, \mathbf{l}_{T_2}, Y)}{\partial Y} = K^{(\mathbf{1})} \otimes G(\mathbf{l}_{T_1}, \mathbf{l}_{T_2}, Y),$$

with the initial condition: $G(\mathbf{l}_{T_1}, \mathbf{l}_{T_2}, Y = 0) = \delta^{(2)}(\mathbf{l}_{T_1} - \mathbf{l}_{T_2})$.

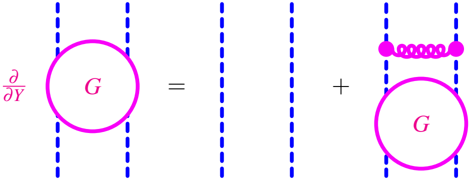
Matching between PRA and BFKL improved PRA

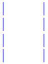
The **matching scheme** between PRA and BFKL improved PRA[He, Kniehl, Nefedov, Saleev '19]:

$$\sigma^{(\mathrm{PRA+BFKL})} = \sigma^{(\mathrm{PRA})} + \sigma^{(\mathrm{BFKL})} - \sigma^{(\mathrm{BFKL},0)},$$

where $\sigma^{(\mathrm{BFKL})}$ is a convolution of the $\mathcal{H}^{(\mathrm{BFKL})}$ with uPDF, $\sigma^{(\mathrm{BFKL},0)} \sim G(\mathbf{l}_{T_1}, \mathbf{l}_{T_2}, Y = 0)$.

- The BFKL equation has the structure:



Reggeon exchange  $\sim \delta^{(2)}(\mathbf{l}_{T_1} - \mathbf{l}_{T_2})$ is the zero BFKL Pomeron approximation already included in the PRA contribution at $Y \to \infty$ due to $\hat{t}$**–channel propagator Reggeization**.

Such contribution should be subtracted $\Rightarrow$ **only term resum $\alpha_s^n Y^n$ remains**.

Blümlein uPDF (JB)

The Blümlein approach[Blümlein '95] based on the Collins–Ellis equation[Collins, Ellis '91] and resummation factor is calculated as a series of α_s.

The LO terms are:

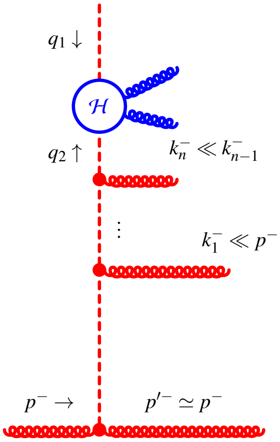
$$\mathcal{C}_{gg}^{(\mathrm{DLA})}\left(z, \mathbf{q}_T^2, \mu^2\right) = \frac{\overline{\alpha}_s(\mu_R^2)}{\mathbf{q}_T^2} \begin{cases} J_0\left(2\sqrt{\overline{\alpha}_s(\mu_R^2)\ \ln\left(\frac{1}{z}\right) \ln\left(\frac{\mu_F^2}{\mathbf{q}_T^2}\right)}\right), \\ I_0\left(2\sqrt{\overline{\alpha}_s(\mu_R^2)\ \ln\left(\frac{1}{z}\right) \ln\left(\frac{\mathbf{q}_T^2}{\mu_F^2}\right)}\right), \end{cases}$$

for $\mathbf{q}_T^2 < \mu_F^2$ and $\mathbf{q}_T^2 > \mu_F^2$ respectively, J_0/I_0 are the Bessel functions of first / second kind.

Normalization condition:

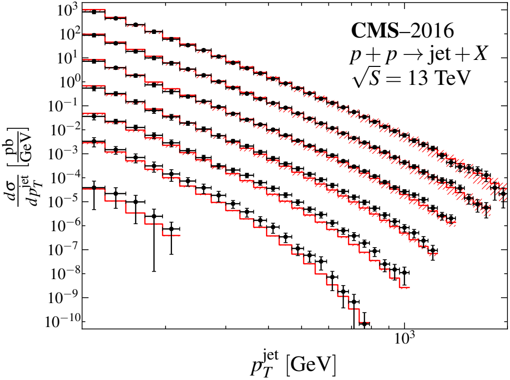
$$\int_0^{\mu_F^2} d\mathbf{q}_T^2\ \mathcal{C}_{gg}^{(\mathrm{DLA})}\left(z, \mathbf{q}_T^2, \mu^2\right) = \delta\left(1 - z\right).$$

► Note that BFKL logs $\ln(1/z)$ are already included in the resummation factor in this approach.



Results: 0 @ incluisve jet at $\sqrt{S} = 13$ TeV

Inclusive jet production in the PRA.



$114 < p_T < 2000$ GeV

$Y_1: \quad |y| < 0.5,$

$Y_2: \quad 0.5 < |y| < 1.0,$

$Y_3: \quad 1.0 < |y| < 1.5,$

$Y_4: \quad 1.5 < |y| < 2.0,$

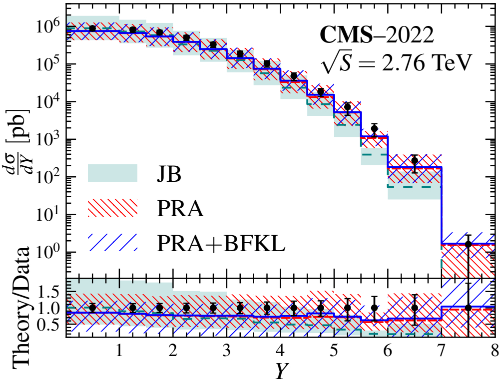
$Y_5: \quad 2.0 < |y| < 2.5,$

$Y_6: \quad 2.5 < |y| < 3.0,$

$Y_7: \quad 3.2 < |y| < 4.7.$

- ME for $R + R \to g$ is already known[Kniehl, Vasin, Saleev '06];
- The dominant LO contribution is $R + R \to g \Rightarrow$ test of *gluodynamics approximation*;
- Good description of data at *small / large* p_T as well as at *small / large* y.

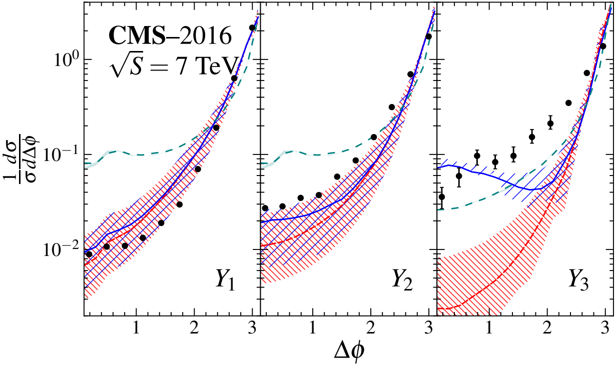
Results: I @ MN dijet, CMS ’22, $\sqrt{S} = 2.76$ TeV



- The PRA based predictions agree with data well up to $Y = 8$;
- Inclusion of the BFKL resummation improves PRA predictions at $Y \gtrsim 4$;
- Predictions with Blümlein uPDFs agree with data at $Y < 4$ and understimate data at $Y > 4$.

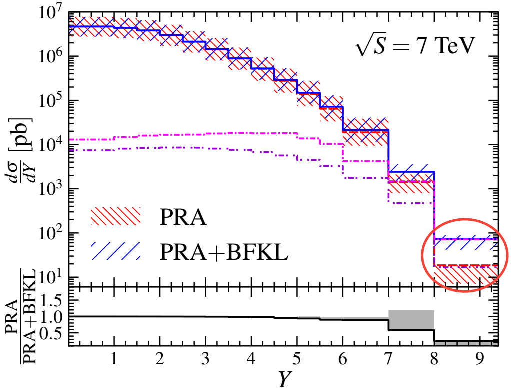
Results: II @ MN dijet, CMS '16, $\sqrt{S} = 7$ TeV

MN dijets production in the 3 **rap.regions**: $Y_1 : |Y| < 3$, $Y_2 : 3 < |Y| < 6$, $Y_3 : 6 < |Y| < 9.4$



- The PRA predictions agree with data well in $Y_{1,2}$, in Y_3 the BFKL resummation is needed;
- There are other sources of uncorrelation: FSR / $DPS_{\text{[Maciula, Szczurek '14]}}$ / ...;
- Calculations based on the Blümlein uPDFs predicts strong uncorrelation.

Results: III @ MN dijet $\sqrt{S} = 7$ TeV



The contributions

$$\text{BFKL} \sim G$$

$$\text{BFKL}^{(0)} \sim \delta^{(2)}$$

are shown separetly.

In the Regge limit:

$$\text{PRA} - \text{BFKL}^{(0)} = 0.$$

- The BFKL contribution becomes dominant at $Y \geq 8$;
- In the region $Y \geq 8$: PRA $-$ BFKL$^{(0)} \simeq 0$;
- The data at $\sqrt{S} \geq 7$ TeV and $Y \geq 8$ is needed.

Conclusions

- Müeller–Navelet dijet production is studied within the framework of the High–Energy Factorization approach in two ways:
  i. BFKL improved Parton Reggeization Approach;
  ii. HEF with Blümlein uPDF;
- The implementation of the BFKL resummation improves Parton Reggeization Approach predictions at large Y;
- We obtain a rather good agreement of our BFKL improved Parton Reggeization Approach predictions with data;
- The BFKL contribution becomes dominant in the Parton Reggeization Approach in the region: $\sqrt{S} \geq 7$ TeV and $Y \geq 8$, so the special study is needed.

Thank you for your attention!

A diagrammatic representation of the PRA amplitude

For derivation of $\overline{|\,\mathcal{M}^{(\text{mMRK})}_{gg\to g(2g)g}\,|^2}$, consider $g\;(q_1')+g\;(q_2')\to g\;(k_1)+(2g)+g\;(k_2)$ subprocess:

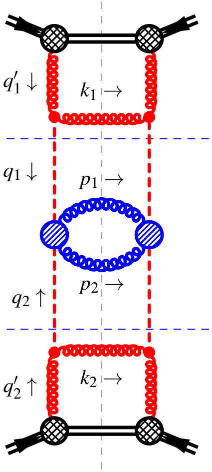
$$\overline{|\,\mathcal{M}^{(\text{mMRK})}_{gg\to g(2g)g}\,|^2} = \left\{\Gamma^{(-)a_1b_1b_2}_{\mu_1\mu_2}(-q_1,-k_1)\,\Gamma^{(-)\mu_1\mu_2}_{a_1b_1b_2}(-q_1,-k_1)\right\} \times\left(\frac{1}{4\,q_1^2q_2^2}\right)^2\overline{|\,\mathcal{A}^{(\text{MRK})}_{(2g)}\,|^2} \times\left\{\Gamma^{(+)a_2b_3b_4}_{\mu_3\mu_4}(-q_2,-k_2)\,\Gamma^{(+)\mu_3\mu_4}_{a_2b_3b_4}(-q_2,-k_2)\right\},$$

keeping exact kinematics up to $---$:

$$q_1 = z_1q_1'^{+}\frac{n_-}{2}-\frac{\mathbf{q}^2_{T_1}}{(1-z_1)q_1'^{+}}\frac{n_+}{2}+q_{T_1},$$

$$q_2 = -\frac{\mathbf{q}^2_{T_2}}{(1-z_2)q_2'^{-}}\frac{n_-}{2}z_2q_2'^{-}\frac{n_+}{2}+q_{T_2}.$$

Rapidity ordering KMR cutoff: $\Delta\left(\mathbf{q}_T^2,\mu^2\right)=\mu/(|\mathbf{q}_T|+\mu)$.



This kind of approximation is actively used[Martin *et.al.* '03; Andersen *et.al.* '09; Nefedov, Saleev '20].