Small-p_T production of J/ψ mesons in the Soft Gluon Resummation approach

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Outline

- Theoretical hadronisation model: NRQCD
- Soft Gluon Resummation approach
- J/ψ production at low-p_T
- Conclusion

Hadronisation model: NRQCD

- J/ψ wave function as a series with respect to relative constituent quarks velocity v:

$$|J/\psi\rangle = \mathcal{O}(v^0)\,|c\bar{c}[{}^3S_1^{(1)}]\rangle + \mathcal{O}(v^1)\,|c\bar{c}[{}^3P_J^{(8)}]g\rangle + \mathcal{O}(v^2)\,|c\bar{c}[{}^3S_1^{(1,8)}]gg\rangle + \\ + \mathcal{O}(v^2)\,|c\bar{c}[{}^1S_0^{(8)}]g\rangle + \mathcal{O}(v^2)\,|c\bar{c}[{}^1D_J^{(1,8)}]gg\rangle + \dots$$

- Approximate v-scaling due to $v^2 \approx 0.2$
- Hard cross section factorisation:

$$d\hat{\sigma}(ab \to \mathcal{C}X) = \sum_n d\hat{\sigma}(ab \to c\bar{c}[n]X)\langle \mathcal{O}^{\mathcal{C}}[n]\rangle$$

- Nonperturbative (hadronisation) factors:

$\langle \mathcal{O}^{\mathcal{C}}[n]\rangle$ – nonperturbative matrix elements (NME):

color singlet NMEs — potential models, data for leptonic decay

color octet NMEs — lattice QCD calculation or experimental data fitting

General remarks on our approximations in calculations of prompt J/ψ production

- Direct production: $g + g \to J/\psi + X$, feed-down contributions from $\psi(2S) \to J/\psi + X$ and $\chi_{cJ} \to J/\psi + \gamma$

  Prompt = Direct + Feed-down contributions

- We study here the direct production & the P-wave feed-down. At $\sqrt{s} = 200$ GeV (PHENIX data), feed-down contribution is about 30 %

- We study here gluon-gluon fusion & quark-antiquark annihilation, quark-antiquark contribution may be about 10% for the total cross section at $\sqrt{s} = 200$ GeV [Saleev, Chernyshev, 2022]

- Our preliminary calculations were done in the LO approximation of the pQCD in α_s

TMD factorisation and initial parton transverse momenta

- **Transverse Momentum Dependent (TMD) factorization**: $q_T, k_T \ll \mu_F \sim M$,
- TMD parton distribution functions $F(x, \boldsymbol{q_T}, \mu_F, \zeta) \Rightarrow$ two-scale **Collins-Soper** equations,

$$q_1^\mu = x_1 p_1^\mu + y_1 p_2^\mu + q_{1T}^\mu, \qquad q_2^\mu = x_2 p_2^\mu + y_2 p_1^\mu + q_{2T}^\mu,$$

- Preserving $\mathcal{O}(q_T/M)$ terms, neglecting $\mathcal{O}(q_T^2/M^2)$ terms and, therefore, assuming $y_{1,2} \to 0$:

$$q_1 \approx \left(\frac{x_1\sqrt{s}}{2}, \boldsymbol{q_{1T}}, \frac{x_1\sqrt{s}}{2} \right), \quad q_2 \approx \left(\frac{x_2\sqrt{s}}{2}, \boldsymbol{q_{2T}}, -\frac{x_2\sqrt{s}}{2} \right)$$

- Relevant processes only $2 \to 1$, intermediate $c\bar{c}$-states can be
  - color-octet ${}^1S_0^{(8)}$ and ${}^3P_{0,2}^{(8)}$ for J/ψ
  - color-octet ${}^3S_1^{(8)}$ and color-sinlget ${}^3P_{0,2}^{(1)}$ for χ_{cJ}

TMD factorisation and TMD PDFs

- General formula of TMD factorization [TMD Handbook, arXiv:2304.03302]:

$$d\sigma(J/\psi) \sim \int dx_1\, dx_2 \int d\boldsymbol{q_{1T}}\, d\boldsymbol{q_{2T}}\, f(x_1, \boldsymbol{q_{1T}}, \mu_F, \zeta_1)\, f(x_2, \boldsymbol{q_{1T}}, \mu_F, \zeta_2)\, \delta(\boldsymbol{q_{1T}} + \boldsymbol{q_{2T}} - \boldsymbol{k_T}) d\hat{\sigma}$$

- To implement **CS** evolution, the transfer to impact parameter $\boldsymbol{b_T}$ space by 2D Fourier transform is done:

$$d\sigma(J/\psi) \sim \int dx_1\, dx_2 \int d\boldsymbol{b_T}\, e^{i\boldsymbol{p_T}\boldsymbol{b_T}}\, \tilde{f}(x_1, \boldsymbol{b_T})\, \tilde{f}(x_2, \boldsymbol{b_T})\, d\hat{\sigma}(x_1, x_2, s)$$

  - $d\hat{\sigma}(x_1, x_2, s)$ is calulated as series in small α_S

Soft Gluon Resummation Approach

- Soft and collinear gluon resummation approach by [J. Collins, D. Soper, 1981]:

$$d\sigma(J/\psi) \sim \int_0^\infty db_T\, b_T\, J_0(p_T b_T)\ e^{-S_P(b_T,\mu_F,Q)}\ e^{-S_{NP}(b_T)}\ \hat{F}(x_1,\mu'_{b^*},b_T^*)\, \hat{F}(x_2,\mu'_{b^*},b_T^*)\, d\hat{\sigma}$$

  Note, $\hat{F}(x,\mu'_{b^*},b_T^*)$ is not conventional TMD PDF !

- Sudakov factor in LL–LO perturbative calculations [J. Collins, D. Soper (1982)]:

$$S_P(b_T,\mu_F,Q) = \frac{C_A}{\pi}\int_{\mu_b^2}^{Q^2} \frac{d\mu'^2}{\mu'^2}\alpha_s(\mu')\left[\ln\frac{Q^2}{\mu'^2} - \left(\frac{11-2N_f/C_A}{6}\right)\right] + \mathcal{O}(\alpha_s)$$

- Sudakov factor expression is valid only on region $b_0/Q \leqslant b_T \leqslant b_{T,\,\mathsf{max}}$ which is being controlled with [D. Boer, W. J. den Dunnen (2014); J. Collins, D. Soper, G. Sterman (1985)]

$$\mu_b \to \mu'_b = \frac{Qb_0}{Qb_T+b_0} \quad \text{and} \quad b_T^*(b_T) = \frac{b_T}{\sqrt{1+(b_T/b_{T,\,\mathsf{max}})^2}}$$

Soft Gluon Resummation approach

- Master formula for soft gluon resummation:

$$d\sigma(J/\psi) \sim \int_0^\infty db_T\, b_T\, J_0(p_T b_T)\; e^{-S_P(b_T,\mu_F,Q)}\; e^{-S_{NP}(b_T)}\; \hat{F}(x_1,\mu'_{b^*},b_T^*)\, \hat{F}(x_2,\mu'_{b^*},b_T^*)\, d\hat{\sigma}$$

- **Nonperturbative** quark factor obtained in SIDIS data fitting: [S. Aybat, T. Rogers (2011)]:

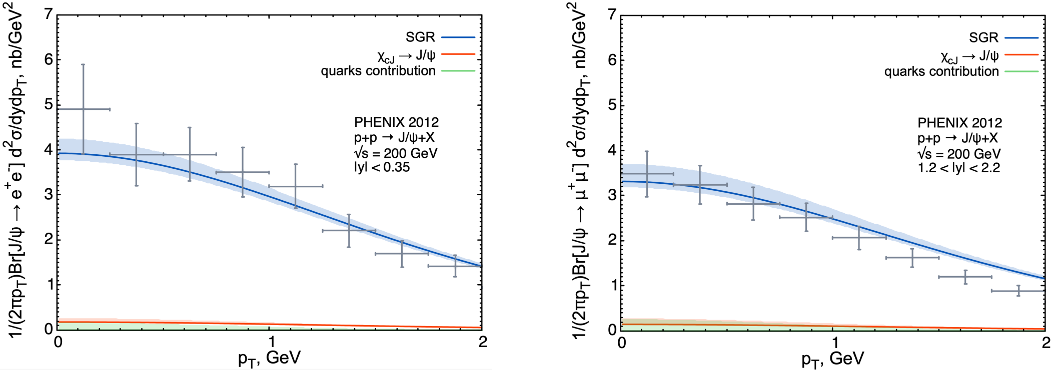
$$S_{NP}(b_T,Q) = \frac{C_A}{C_F}\left[g_1 \ln\frac{Q}{2Q_{NP}} + g_2\left(1 + 2g_3 \ln\frac{10xx_0}{x_0+x}\right)\right] b_T^2$$

  - it should be scaled by C_A/C_F for gluons

- In the leading order in α_S, the perturbative tail of TMD PDF is expressed with collinear PDF:

$$\hat{F}(x,b_T^*) = f(x,\mu'_{b^*}) + \mathcal{O}(\alpha_s)$$

Extraction of CO NME in the Soft Gluon Resummation approach at $p_T < 1$ GeV



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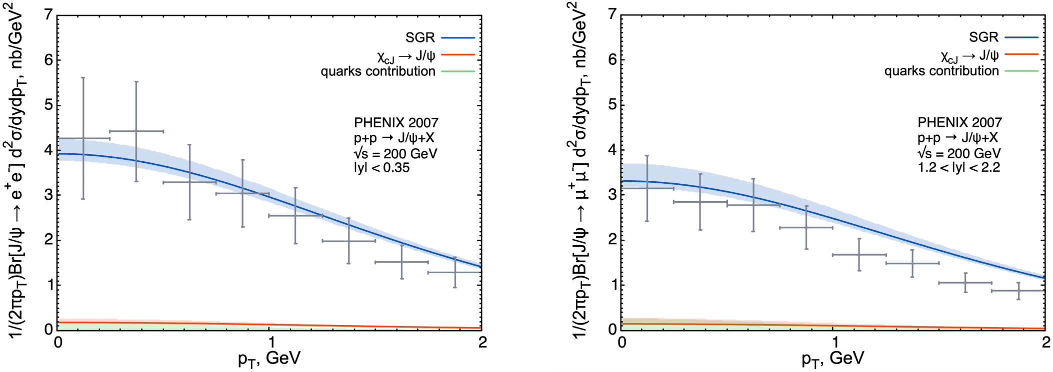

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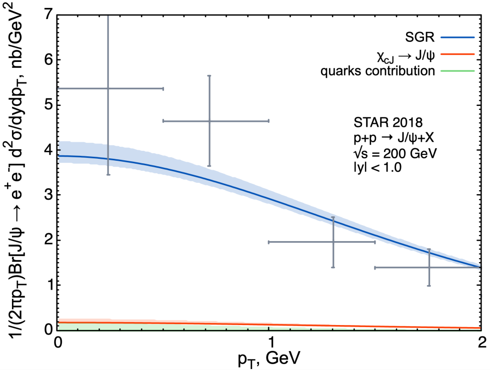


At $\sqrt{s} = 200$ GeV:

- quark annihilation contribution $< 7\%$
- χ_{cJ} decays contribution $\approx 5 - 10\%$

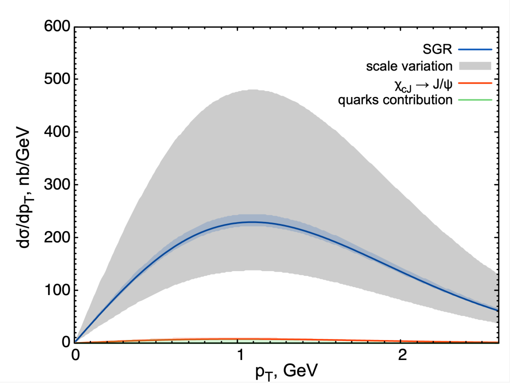
Extraction of CO NME in the Soft Gluon Resummation approach at $p_T < 1$ GeV

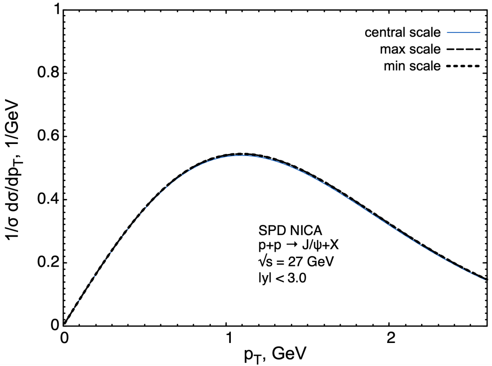
- $M_7^{J/\psi} = \langle \mathcal{O}[^1S_0^{(8)}] \rangle + 7 \cdot \langle \mathcal{O}[^3P_0^{(8)}] \rangle / m_c^2 = (1.39 \pm 0.06) \cdot 10^{-1}$ GeV^3,
- $\langle \mathcal{O}^{J/\psi}[^3S_1^{(8)}] \rangle = (0.00 \pm 3.15) \cdot 10^{-3}$ GeV^3
- $\langle \mathcal{O}^{\chi_{c0}}[^3S_1^{(8)}] \rangle = (0.00 \pm 3.59) \cdot 10^{-3}$ GeV^3

χ^2/d.o.f. $= 0.44$

CO LDME	LO CPM [Cho, Leibovich (1996)]	NLO CPM [Butenschön, Kniehl (2011)]	NLO CPM [Ma, Wang, Chao (2011)]
$M_3^{J/\psi}$	$(6.6 \pm 1.5) \cdot 10^{-2}$ GeV^3	$(1.83 \pm 0.56) \cdot 10^{-2}$ GeV^3	$(-1.18 \pm 2.94) \cdot 10^{-2}$ GeV^3
$\langle \mathcal{O}^{J/\psi}[^3S_1^{(8)}] \rangle$	$(6.6 \pm 2.1) \cdot 10^{-3}$ GeV^3	$(1.68 \pm 0.46) \cdot 10^{-3}$ GeV^3	$(8.86 \pm 3.91) \cdot 10^{-2}$ GeV^3
$\langle \mathcal{O}^{\chi_{c0}}[^3S_1^{(8)}] \rangle$	$(3.3 \pm 0.5) \cdot 10^{-3}$ GeV^3	–	–

Predictions for SPD NICA using the Soft Gluon Resummation approach





At $\sqrt{s} = 27$ GeV:

- quark annihilation contribution $< 5\%$
- χ_{cJ} decays contribution $\approx 5\%$

Conclusion

- We have analysed the Soft Gluon Resummation approach to calculate low-p_T J/ψ production in the TMD factorisation
- CO LDMEs of NRQCD are necessary to describe J/ψ production using the TMD factorisation, where they are major contributions
- Soft Gluon Resummation approach for gluon and quark TMD PDF satisfyingly describe experimental data of unpolarized J/ψ production at $\sqrt{s} = 200$ GeV in the TMD domain
- However, quite biased result for CO NME fitting may be a sign of a narrow fitting region for TMD and a necessity to include collinear factorisation for mutual NME fitting
- We estimate the perspective region for the extraction of TMD PDF in the J/ψ production at the SPD NICA experiment as $p_T \leqslant 1$ GeV