Exact calculation of photon polarization observables in Bethe–Heitler process

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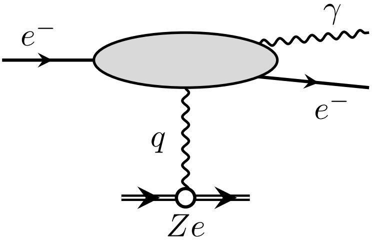
Bethe–Heitler process

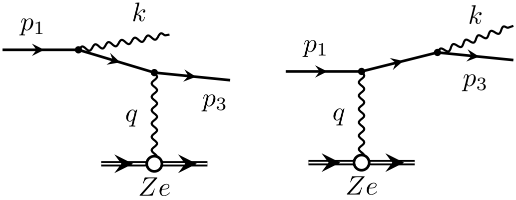
Bethe–Heitler process is the process of bremsstrahlung during lepton scattering off the Coulomb center with charge Ze:



First correct calculation and comprehensive analysis of this process was done in paper:
Hans Bethe and Walter Heitler,
"On the Stopping of fast particles and on the creation of positive electrons",
Proc. Roy. Soc. Lond. A , Vol. 146, p. 83-112, (1934).

Earlier there were many attempts by others:
Bethe'1930, Møller'1932, Bloch'1933, Heitler'1933, ...

Bethe–Heitler process

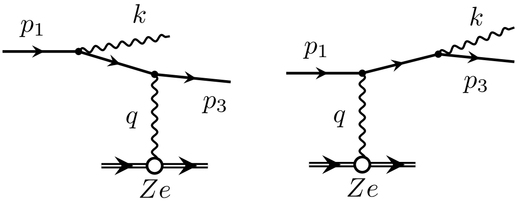


The calculation was done in the lowest order of perturbation theory assuming nucleus to be infinitely heavy. The cross section then reads as:

$$d\sigma = \frac{Z^2 e^4}{137 \cdot 2\pi} \frac{d\omega}{\omega} \frac{|\mathbf{p_3}|}{|\mathbf{p_1}|} \frac{\sin\theta_1 \ \sin\theta_3 \, d\theta_1 \, d\theta_3 \, d\phi}{q^4} \times$$
$$\times \left\{ \frac{|\mathbf{p_1}|^2 \sin^2\theta_1}{(E_1 - |\mathbf{p_1}|\cos\theta_1)^2} \left(4E_1^2 - q^2\right) + \frac{|\mathbf{p_3}|^2 \sin^2\theta_3}{(E_3 - |\mathbf{p_3}|\cos\theta_3)^2} \left(4E_3^2 - q^2\right) - \right.$$
$$- \frac{2\,|\mathbf{p_1}|\,|\mathbf{p_3}|\sin\theta_1 \sin\theta_3 \cos\phi}{(E_1 - |\mathbf{p_1}|\cos\theta_1)\,(E_3 - |\mathbf{p_3}|\cos\theta_3)} \left(4E_1E_3 - q^2 - 2\omega^2\right) +$$
$$\left. + \frac{2\omega^2 \left(|\mathbf{p_1}|^2 \sin^2\theta_1 + |\mathbf{p_2}|^2 \sin^2\theta_2\right)}{(E_1 - |\mathbf{p_1}|\cos\theta_1)\,(E_3 - |\mathbf{p_3}|\cos\theta_3)} \right\}, \qquad (1)$$

where ω is the photon energy, θ_1, θ_3 are angles between photon 3-momentum $\mathbf{k}$ and $\mathbf{p_1}$ or $\mathbf{p_3}$ correspondingly. Angle ϕ is the photon momentum azimuthal angle.

Thus 4-momenta has the form: $p_1 = (E_1, \mathbf{p_1})$, $p_3 = (E_3, \mathbf{p_3})$, $k = (\omega, \mathbf{k})$.



After integration over orientations of bremsstrahlung photon in ultra-relativistic approximation (i.e., for $E_{1,3} \gg m$, $\omega \gg m$) one gets:

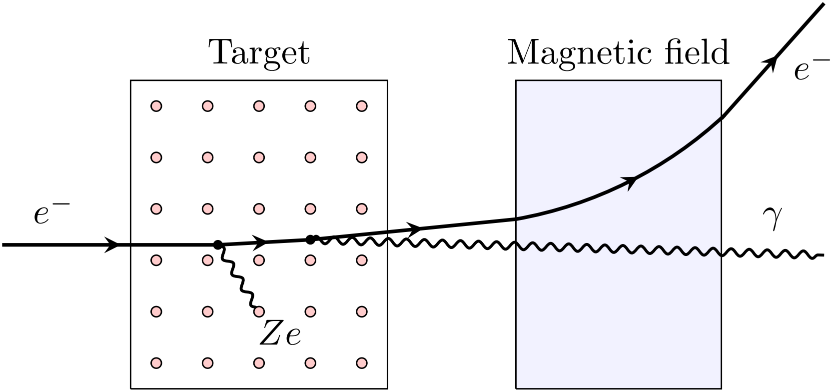
$$d\sigma = \frac{Z^2 e^4}{137} \left(\frac{e^2}{m}\right)^2 \frac{d\omega}{\omega} \frac{4}{E_1^2} \left(E_1^2 + E_3^2 - \frac{2}{3} E_1 E_3\right) \left(\ln\left(\frac{2E_1 E_3}{\omega m}\right) - \frac{1}{2}\right), \tag{2}$$

where large logarithm goes mostly from collinear emission regions, when

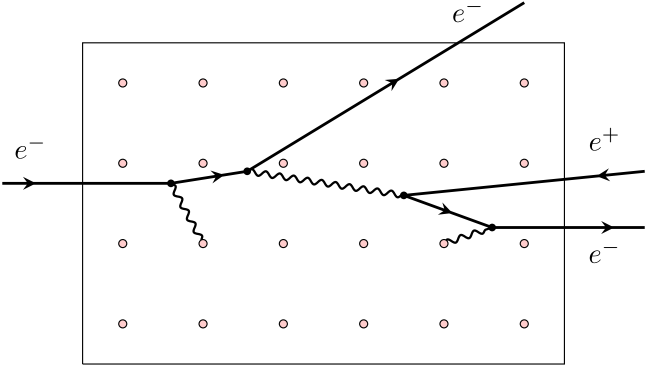
$$\theta_{1,3} \sim \frac{m}{2E_1} \ll 1. \tag{3}$$

This opens an opportunity to make a dense source of bremsstrahlung photons.

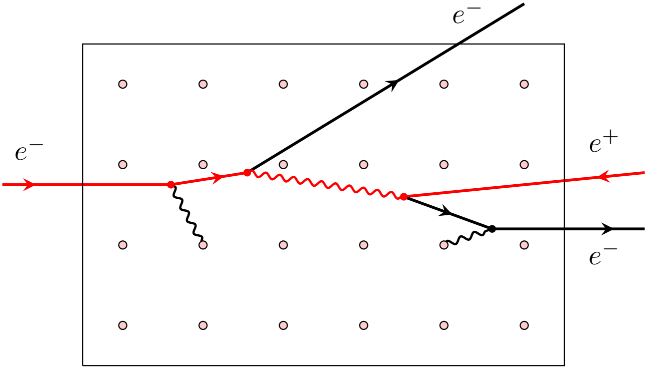
Source of bremsstrahlung photons



Production of electron–positron pairs



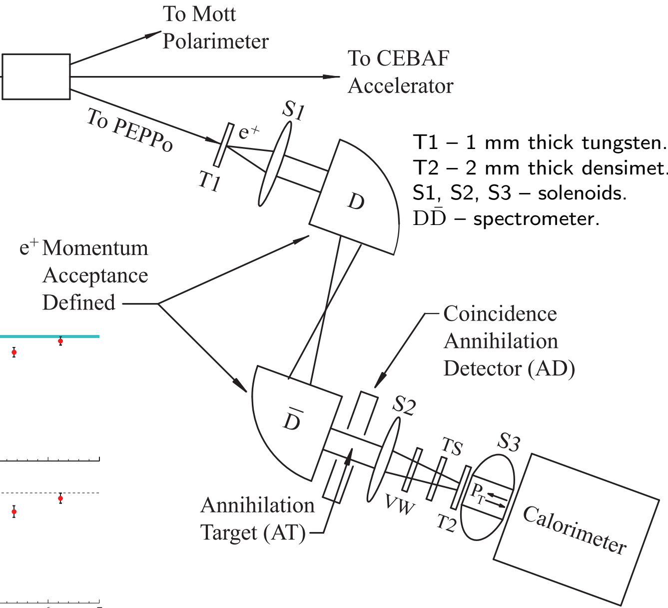
Production of polarized positrons

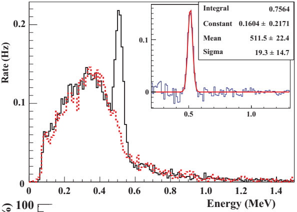


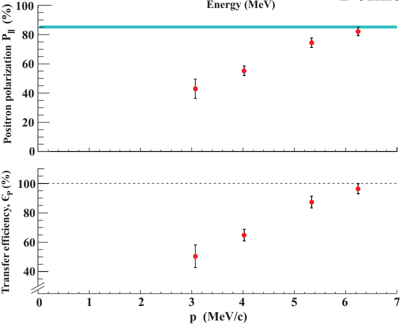
The positron yield per one initial electron with $E_1 = 50$ MeV is about $\Delta N_+ \approx 1.4 \times 10^{-2}$. The positron yield with average polarization more then 60 % of initial electron polarization is about $\Delta N_+ \approx 2 \times 10^{-3}$.

A. P. Potylitsin, Nucl. Instrum. Methods Phys. Res., Sect. A **398**, 395 (1997).

PEPPo Experiment (2012 at CEBAF injector of JLab)





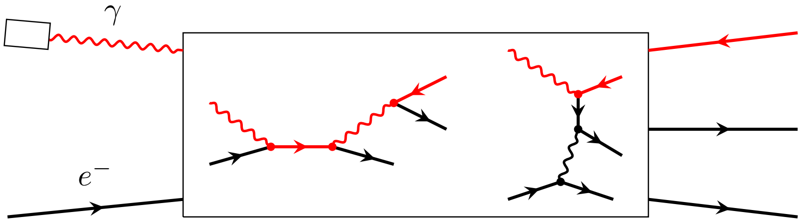


T1 – 1 mm thick tungsten.
T2 – 2 mm thick densimet.
S1, S2, S3 – solenoids.
$D\bar{D}$ – spectrometer.

Pictures are from PRL **116**, 214801 (2016).

Other methods of polarized positron production

Laser with energy about ~ 10 MeV or
undulator radiation of electrons
with energy $E \sim 100$ GeV.

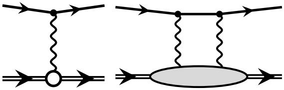


$$E_1 = 10...250 \text{ GeV}$$

V.E. Balakin, A.A. Michaihchenko, Preprint INP 79-85, 1979,
Yung Su Tsai, Preprint SLAC-PUB-5924, 1992,
V.N. Baier, R. Chehab, V.V. Katkov, Nucl. Instr. and Meth. A **338** (1994) 156.

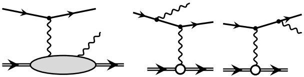
1. **Electromagnetic form factors**

   It was suggested that mechanisms associated with Two Photon Exchanges (TPE) may reconciliate the measurements of the ratio of the proton form factors G_E/G_M as obtained from Rosenbluth separation or polarization transfer experiments.



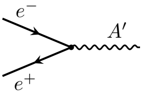
2. **Generalized parton distributions (GPDs)**

   The comparison between electron and positron cross sections separate the interference contributions which can be further decomposed taking advantage of the beam polarization sensitivity. Thus the comparison between polarized lepton beams of opposite charges allows for a full separation of 4 unknown amplitudes involving GPDs.

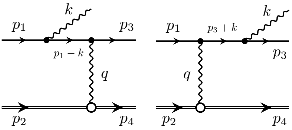


3. **Tests of the Standard Model**

   The search for Light Dark Matter (LDM) through the production of a feebly interacting boson, the so-called A'-dark photon, is possible in annihilation channel. This mechanism is different from the A'-strahlung and may be thought more sensitive to LDM if responsible of the anomalies observed in cosmic radiations.



Brief details of our calculation

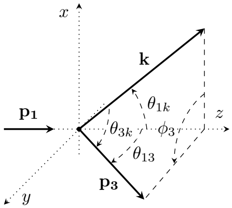


Cross section has a standard form:

$$d\sigma_{ij} = \frac{1}{4F}\overline{\sum_{s}}\mathcal{M}_i\mathcal{M}_j^+\, d\Phi_3,$$

where amplitude square reads as:

$$\overline{\sum_{s}}\mathcal{M}_i\mathcal{M}_j^+ = \frac{(4\pi\alpha)^3\, Z^2}{4(q^2)^2} S_{ij}.$$



Traces are the following:

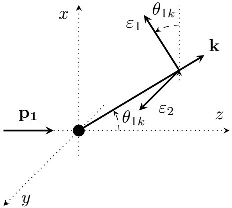
$$S_{ij} = \varepsilon_i^{\alpha}\left(\varepsilon_j^{\beta}\right)^* \mathsf{Sp}\left[(\hat{p}_1 + m_e)\left(1+\lambda_1\gamma_5\right)\widetilde{O}_{\nu\beta}(\hat{p}_3 + m_e)O_{\mu\alpha}\right] \mathsf{Sp}\left[(\hat{p}_2 + M_A)\gamma^{\nu}(\hat{p}_4 + M_A)\gamma^{\mu}\right],$$

where $O_{\mu\nu}$ describes the emission from electron line:

$$O_{\mu\nu} = \frac{\gamma_\mu(\hat{p}_1 - \hat{k} + m_e)\gamma_\nu}{-2(p_1k)} + \frac{\gamma_\nu(\hat{p}_3 + \hat{k} + m_e)\gamma_\mu}{2(p_3k)}.$$

And we also need pure polarization states $\varepsilon_{1,2}$ of photon:

$$\varepsilon_1 = \left(0, \cos\theta_{1k}, 0, -\sin\theta_{1k}\right), \qquad \varepsilon_2 = (0, 0, 1, 0).$$



Brief details of our calculation: Stokes parameters

The polarization of the bremsstrahlung photon is parameterized by Stokes parameters $P_{1,2,3}$. In order to get them we calculate cross sections $d\sigma_{ij}$ with pure polarization of final photon ε_i and then we arrange the following combinations:

$$d\sigma_I = d\sigma_{11} + d\sigma_{22}, \tag{4}$$

$$d\sigma_{P_1} = d\sigma_{11} - d\sigma_{22}, \tag{5}$$

$$d\sigma_{P_2} = d\sigma_{12} + d\sigma_{21}, \tag{6}$$

$$d\sigma_{P_3} = i\,(d\sigma_{21} - d\sigma_{12})\,. \tag{7}$$

After that we can define Stokes parameters as:

$$d\sigma_{P_i} = P_i\ d\sigma_I, \qquad\qquad i = 1, 2, 3. \tag{8}$$

where cross section $d\sigma_I$ is the unpolarized cross section, i.e. intensity of produced photon beam.

[W. McMaster, American Journal of Physics **22**, 351 (1954)]

Brief details of our calculation: screening and Coulomb corrections

We must take into account the possible screening of nucleus field by atomic electrons. That is done by standard replacement of virtual photon propagator:

$$\frac{1}{q^2} \quad \longrightarrow \quad \frac{1-F(q^2)}{q^2} = \sum_{i=1}^{3} \frac{\alpha_i}{\beta_i^2 - q^2}, \tag{9}$$

where $F(q^2)$ is the atomic form factor obtained within the Molière approach, which is based on Thomas–Fermi atomic model. The values of the parameters are

$$\alpha_1 = 0.1, \qquad \alpha_2 = 0.55, \qquad \alpha_3 = 0.35, \tag{10}$$

$$b_1 = 6.0, \qquad b_2 = 1.2, \qquad b_3 = 0.3, \tag{11}$$

where $\beta_i = \left(Z^{\frac{1}{3}}/121\right) m_e b_i$.

Another delicate effect is the *Coulomb corrections* which appear in the kinematical region where particles move with small relative velocity ($\sim e^2/\hbar v$) and multiple rescattering can contribute significantly. This is very complicated mechanism and it can be taken into account in the very specific cases. See for example [A. Tarasov, O. Voskresenskaya, *An Improvement of the Molière–Fano Multiple Scattering Theory*, arXiv: 1107.5018 [hep-ph]]. In our case we chase for precise results and we do not dare to estimate this effect with the proper accuracy. Thus we systematically neglect the effect of Coulomb corrections in our estimations.

Etalon calculations

Since Bethe–Heitler calculation in 1934 there were many attempts to estimate bremsstrahlung cross section from electron–nucleus scattering. The most well-known one is:

H. Olsen and L. Maximon, Phys. Rev. **114**, 887 (1959),

which is often used for programming of Monte Carlo generators code in modelling software packages. There is also a book:

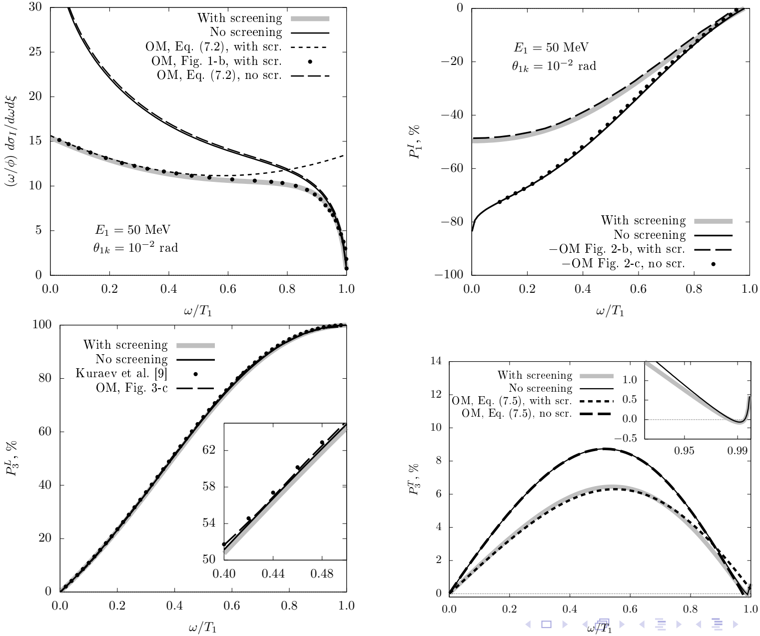
E. Haug and W. Nakel, *The elementary process of bremsstrahlung*,
(World Scientific, Singapore, 2004),

which comprehensively considers the bremsstrahlung process and usually is recommended as manual for everyone who would like to describe bremsstrahlung process. There are also excellent paper unknown for wide community of physicist:

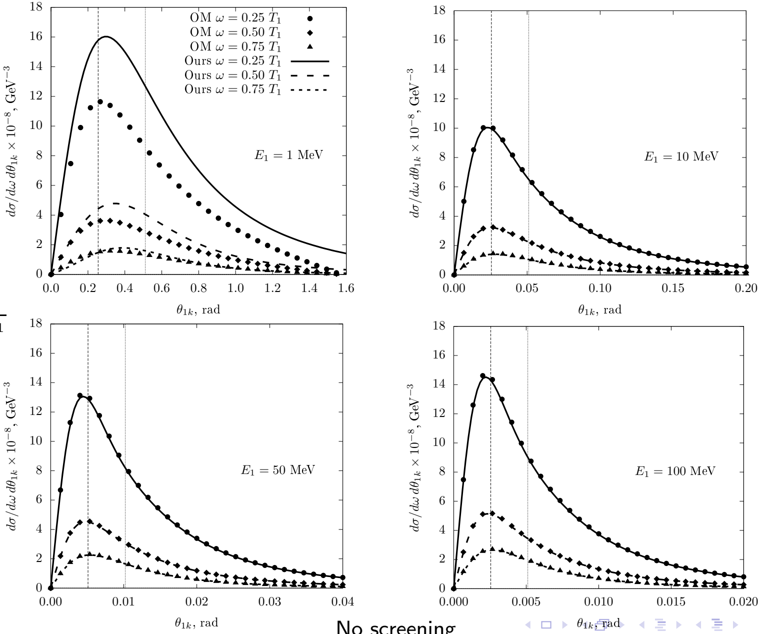
J. Asai, H.S. Caplan, D.M. Skopik, W. Del Bianco, L.C. Maximon,
Can. J. Phys., **66**, 1079 (1988),

which presents bremsstrahlung cross section in great details and convenient kinematical variables.

Comparison witn Maximon–Olsen'1959 at $E_1 = 50$ MeV



Comparison witn Maximon–Olsen'1959 at different beam energies E_1

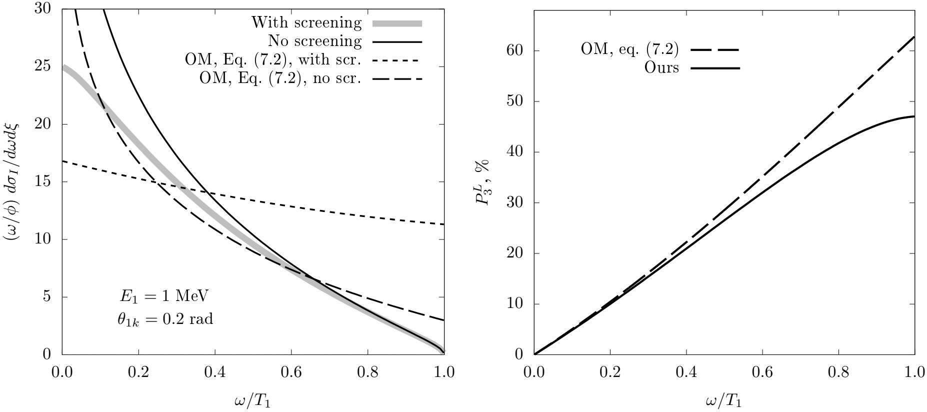


Peak at $\theta_{1k} \approx \frac{m_e}{2E_1}$

Cone width $\frac{m_e}{E_1}$

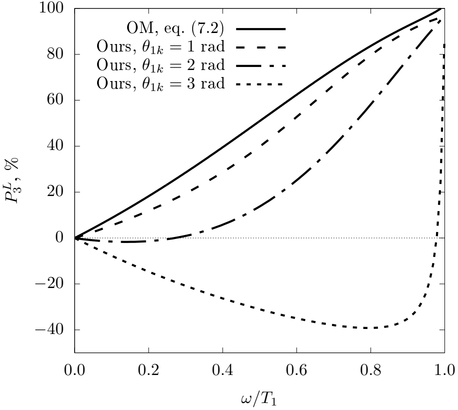
No screening

Comparison witn Maximon–Olsen'1959 at $E_1 = 1$ MeV



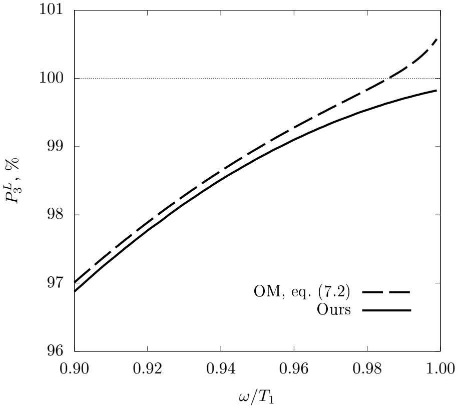
Problems with polarized cross section

No diagonal approximation



$E_1 = 3$ MeV

Probability $> 100\%$



$E_1 = 10$ MeV

Conclusions

- The cross section of the bremsstrahlung in electron–nucleus collision is calculated keeping the electron mass. The polarized initial state of the electron beam and the polarization of the final photon are considered.
- It is shown that at rather low energies the approximations of paper of Olsen–Maximon–1959 becomes not valid and must be dropped in favour of precise calculation.
- In case of polarization transfer from initial particles to the final ones this invalidity appears at higher energies and can produce surprises in Monte Carlo modelling.

We're grateful to Eric Voutier (IJCLab, Orsay) for drawing our attention to the problem and for useful discussions at the initial stage of the work.