Multiplicity fluctuations in the string model in pp collisions at LHC energies

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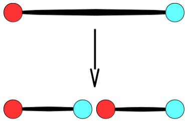
The string model

A. Capella, U. Sukhatme, Chung-I Tan, J. Tran Thanh Van, Phys. Lett. B 81, 68 (1979).
A.B. Kaidalov, Phys. Lett. B 116, 459 (1982).
A.B. Kaidalov, K.A.Ter-Martirosyan, Phys. Lett., 117B (1982) 247.
A. Capella, U. Sukhatme, Chung-I Tan, J. Tran Thanh Van, Phys. Rep. 236 (1994) 225.

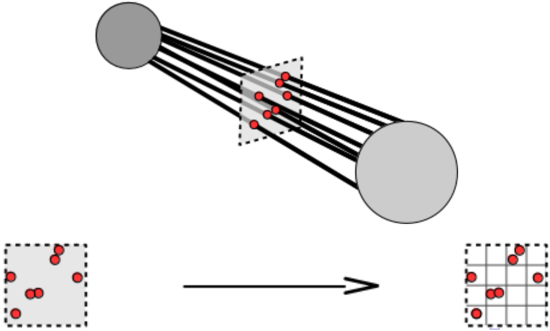
First stage: colour quark-gluon strings (colour flux tubes) are formed
Second stage: hadronization of these strings produces the observed hadrons



The model with string fusion

- pp → pA → AA - the increase of the string density in transverse plain
  M.A. Braun, C. Pajares, Phys.Lett. **B287**, 154 (1992); Nucl. Phys. **B390**, 542 (1993).

  ⇒ Reduction of multiplicity, increase of transverse momenta.
- *Vechernin V., Kolevatov R.S.*, hep-ph/0304295; hep-ph/0305136
  Braun M.A., Kolevatov R.S., Pajares C., V.Vechernin Eur.Phys.J. C32 (2004) 535.
  V.Vechernin, Kolevatov R.S. Phys.of Atom.Nucl. **70** (2007) 1797; 1858.



Strongly intensive variable Σ

M.I. Gorenstein, M. Gazdzicki, Phys.Rev.C 84 (2011) 014904:

$$\Sigma(A,B) \equiv \frac{\langle A\rangle\ \omega_B + \langle B\rangle\ \omega_A - 2\,\mathrm{cov}(A,B)}{\langle A\rangle + \langle B\rangle}\ , \tag{1}$$

Andronov, E.V., Theor Math Phys 185, 1383–1390 (2015).

$$\Sigma(n_F,\ n_B) \equiv \frac{\langle n_F\rangle\,\omega_{n_B} + \langle n_B\rangle\,\omega_{n_F} - 2\mathrm{cov}(n_F,\ n_B)}{\langle n_F\rangle + \langle n_B\rangle}, \tag{2}$$

The case of equal-width symmetrically located rapidity observation windows and symmetrical reaction:

$$\delta y_F = \delta y_B = \delta y,\ \langle n_F\rangle = \langle n_B\rangle = \langle n\rangle\,,\ \omega_{n_F} = \omega_{n_B} = \omega_n. \tag{3}$$

Then (2) can be rewritten in a simpler form:

$$\Sigma(n_F,\ n_B) = \frac{\left\langle n^2\right\rangle - \langle n_F\ n_B\rangle}{\langle n\rangle}. \tag{4}$$

Let us introduce the variable $\Sigma_\eta(\mu_F, \mu_B)$ for a string cluster of η strings in the i-th cell, which will be completely determined by the properties of this cluster:

$$\Sigma_\eta(\mu_F, \mu_B) \equiv \frac{\left\langle \mu^2 \right\rangle_\eta - \left\langle \mu_F \ \mu_B \right\rangle_\eta}{\left\langle \mu \right\rangle_\eta}. \quad (5)$$

Then Σ can be rewritten as:

$$\Sigma(n_F, n_B) = \sum_{\eta=1}^{\infty} \alpha(\eta)\, \Sigma_\eta(\mu_F, \mu_B) \ . \quad (6)$$

$$\alpha(\eta) \equiv \langle n \rangle_\eta \,/\, \langle n \rangle \quad (7)$$

Model of independent identical strings

$$\Sigma^{no\ fusion}[n_F, n_B] = \Sigma_1[\mu_F, \mu_B]$$

Scaled variance ω_n

Similarly, one can obtain for ω_n

$$\omega_n = \frac{\left\langle n^2 \right\rangle - \left\langle n \right\rangle^2}{\left\langle n \right\rangle} \tag{8}$$

Let us introduce the variable ω_μ^η for a string cluster of η strings in the i-th cell, which will be completely determined by the properties of this cluster:

$$\omega_\mu^\eta \equiv \frac{\left\langle \mu^2 \right\rangle_\eta - \left\langle \mu \right\rangle_\eta^2}{\left\langle \mu \right\rangle_\eta}. \tag{9}$$

Then ω_n can be rewritten as:

$$\omega_n = \sum_{\eta=1}^{\infty} \alpha(\eta)\, \omega_\mu^\eta + \sum_{C_\eta} P(C_\eta) \frac{\left\langle n \right\rangle_{C_\eta}^2}{\left\langle n \right\rangle} - \left\langle n \right\rangle . \tag{10}$$

The practical advantage of formulas

$$\Sigma(n_F, n_B) = \sum_{\eta=1}^{\infty} \alpha(\eta)\, \Sigma_\eta(\mu_F, \mu_B)$$

and

$$\omega_n = \sum_{\eta=1}^{\infty} \alpha(\eta)\, \omega_\mu^\eta + \sum_{C_\eta} P(C_\eta) \frac{\langle n \rangle_{C_\eta}^2}{\langle n \rangle} - \langle n \rangle$$

is that for MC calculations in pp collisions it is sufficient to limit ourselves to generating only the emerging string configurations.

We will characterize the properties of the new string through the one-particle and two-particle distribution functions of particles formed during the fragmentation of the string:

$$\lambda_\eta(y) = \frac{dN}{dy}, \;\; \lambda_2^\eta(y_1, y_2) = \frac{d^2 N}{dy_1\, dy_2}. \tag{11}$$

The two-particle (pair) correlation function is defined standardly in terms of these distribution functions

$$\Lambda_\eta(y_1, y_2) \equiv \frac{\lambda_2^\eta(y_1, y_2)}{\lambda_\eta(y_1)\lambda_\eta(y_2)} - 1 \;, \tag{12}$$

In this case it is possible to rewrite $\Sigma_\eta(\mu_F, \mu_B)$ and ω_μ^η for the string cluster of η strings as

$$\Sigma_\eta(\mu_F, \mu_B) = 1 + \langle\mu\rangle_\eta \left[J_{FF}^\eta - J_{FB}^\eta\right] , \omega_\mu^\eta = 1 + \langle\mu\rangle_\eta \, J_{FF}^\eta \tag{13}$$

where

$$J_{FB}^\eta \equiv \frac{1}{\langle\mu_F\rangle_\eta \langle\mu_B\rangle_\eta} \int_{\delta y_F} dy_1 \int_{\delta y_B} dy_2 \;\lambda_\eta(y_1)\lambda_\eta(y_2)\Lambda_\eta(y_1, y_2), \tag{14}$$

$$J_{FF}^\eta \equiv \frac{1}{\langle\mu_F\rangle_\eta^2} \int_{\delta y_F} dy_1 \int_{\delta y_F} dy_2 \;\lambda_\eta(y_1)\lambda_\eta(y_2)\Lambda_\eta(y_1, y_2). \tag{15}$$

The two-particle (pair) correlation function is chosen in its simplest form

$$\Lambda_\eta(\Delta y) = \Lambda_0^{(\eta)} e^{-\frac{|\Delta y|}{y_{corr}^{(\eta)}}}, \tag{16}$$

$y_{corr}^{(\eta)}$— characteristic correlation rapidity length, $y_{corr}^{(\eta)} = \frac{y^{(1)}}{\sqrt{\eta}}$, $\Lambda_0^{(\eta)} = const$.
Then integrals can be calculated analyticity

$$J_{FF}^{\eta} = \frac{2\Lambda_0^{(\eta)}}{(\delta y)^2} y_{corr}^{(\eta)} \left(\delta y - y_{corr}^{(\eta)} \left(1 - e^{-\frac{\delta y}{y_{corr}^{(\eta)}}} \right) \right), \tag{17}$$

$$J_{FB}^{\eta} = \frac{\Lambda_0^{(\eta)} (y_{corr}^{(\eta)})^2}{(\delta y)^2} e^{-\frac{\Delta y}{y_{corr}^{(\eta)}}} \left(e^{-\frac{\delta y}{y_{corr}^{(\eta)}}} + e^{\frac{\delta y}{y_{corr}^{(\eta)}}} - 2 \right). \tag{18}$$

$$\mu_0 \Lambda_0^{(1)} = 0.56, \;\; y_{corr}^{(1)} = 2.7 \tag{19}$$

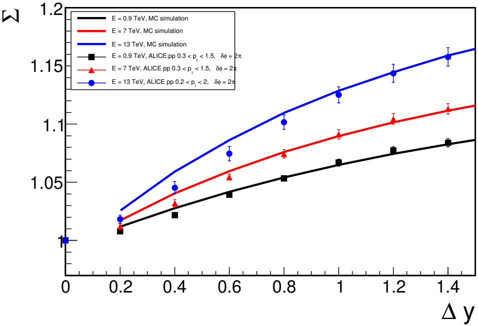
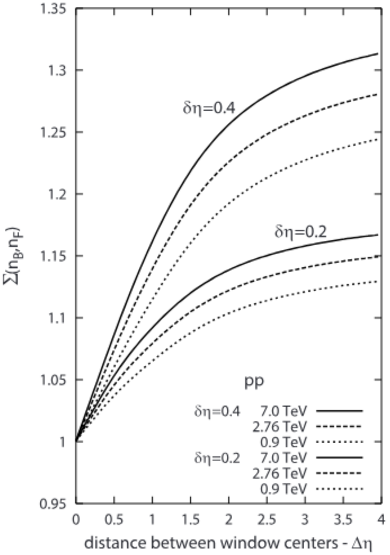


Figure: The $\Sigma(n_F, n_B)$ for windows of width $\delta y = 0.2$ for initial energies 0.9, 7 and 13 TeV. Points - experimental values obtained in [A.Erokhin (for the ALICE Collaboration) reported at IS2021]

V.Vechernin, SB, Theoretical and Mathematical Physics, 2023, Volume 216, Issue 3, Pages 1299–1312 DOI: 10.1134/S0040577923090052

$$\Sigma^{no\ fusion}[n_F, n_B] = \Sigma_1[\mu_F, \mu_B]$$



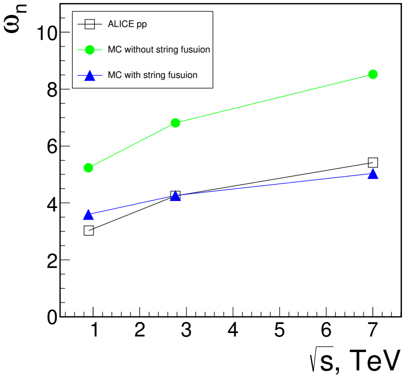
$\sqrt{s}$, TeV	0.9	2.76	7.0
$\mu_0\Lambda_0$	0.73	0.83	0.93
η_{corr}	1.52	1.43	1.33

V.Vechernin, EPJ Web Conf., 191 (2018) 04011 DOI: 10.1051/epjconf/201819104011

The value of the parameters for the two-particle correlation function of a string obtained by a fitting of the experimental pp ALICE data on forward-backward correlations between multiplicities

The string parameters occur dependent on initial energy (!?)

Hint on string fusion and formation of string clusters - the sources with new properties.



$\sqrt{s}$, TeV	0.9	2.76	7
$\mu_0\Lambda_0$	0.73	0.83	0.93
y_{corr}	1.52	1.43	1.33
μ_0	0.29	0.26	0.24

Figure: Results of calculations of the scaled variance ω_n as functions of the energy for rapidity observation window $\delta y = 1.6$ for pp collisions with and without taking into account the string fusion compared with the ALICE experimental data from [The ALICE collaboration, J. High Energ. Phys. 2015, 97 (2015). arXiv:1502.00230].

Conclusion

- Strongly intensive variable Σ and scaled variance ω_n were studied in the framework of the string fusion model and model of independent identical strings.
- Comparison with experimental data shows that the results obtained taking into account the fusion of strings and the formation of string clusters are in good agreement with the experimental values, in contrast to the case that does not take into account the processes of string fusion.

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Backup

Distribution of strings in the transverse plane

pp interactions

$$w_{str}(\vec{s},\vec{b}) \sim T(\vec{s}-\vec{b}/2)\,T(\vec{s}+\vec{b}/2)/\sigma_{pp}(b) \qquad (20)$$

$\sigma_{pp} = \int \sigma_{pp}(b)\; d^2\vec{b}$ - non-diffractive pp cross section

$T(\vec{s}) = \int\limits_{-\infty}^{+\infty} \rho(\vec{s},z)\; dz$ - parton profile function of nucleon

$$\rho(r) = \frac{1}{\pi^{3/2}\alpha^3} e^{-r^2/\alpha^2}\,, \qquad T(s) = \frac{e^{-s^2/\alpha^2}}{\pi\alpha^2}\,, \qquad (21)$$

$$w_{str}(\vec{s},\vec{b}) \sim e^{-(\vec{s}+\vec{b}/2)^2/\alpha^2} e^{-(\vec{s}-\vec{b}/2)^2/\alpha^2}/\sigma_{pp}(b) = e^{-2s^2/\alpha^2} e^{-b^2/2\alpha^2}/\sigma_{pp}(b)$$

b–s factorization $\Rightarrow$

$$\langle N_{str}(b)\rangle \sim e^{-b^2/2\alpha^2}/\sigma_{pp}(b) \qquad (22)$$

Event-by-event fluctuations of the number of cut pomerons

$$P(N,b) = e^{-\overline{N}(b)}\overline{N}(b)^N/N!$$ -Poisson, $P(0,b) = e^{-\overline{N}(b)}$

$$\widetilde{P}(N,b) = P(N,b)/[1-P(0,b)]$$ -modified Poisson, $\sum_{N=1}\widetilde{P}(N,b) = 1$

$$\langle N(b)\rangle = \sum_{N=1} N\,\widetilde{P}(N,b) = \overline{N}(b)/[1-P(0,b)] \qquad (23)$$

$$\sigma_{pp}^{ND}(b) = 1 - P(0,b) = 1 - e^{-\overline{N}(b)} \qquad (24)$$

$$\langle N(b)\rangle = \overline{N}(b)/\sigma_{pp}^{ND}(b)$$

$$\overline{N}(b) = N_0 e^{-b^2/2\alpha^2}$$ $$\langle N(b)\rangle = \overline{N}(b)/[1-\exp(-\overline{N}(b))]$$

$N_{str} = 2N$, N - the number of cut pomerons in a given event

$$\langle N(b)\rangle = N_0 e^{-b^2/2\alpha^2}/\sigma_{pp}(b)$$

Probability to have N cut pomerons in a non-diffractive pp collision

Integration over the impact parameter b leads to

$$w_N = \frac{2\pi\alpha^2}{\sigma_{pp} N} \left[1 - e^{-N_0} \sum_{k=0}^{N-1} N_0^k / k!\right] = \frac{\sigma_N}{\sigma_{pp}^{ND}}$$

where we have introduced the σ_N by

$$\sigma_N \equiv \frac{2\pi\alpha^2}{N} \left[1 - e^{-N_0} \sum_{k=0}^{N-1} N_0^k / k!\right]$$

$$\sum_{N=1}^{\infty} \sigma_N = 2\pi\alpha^2 [E_1(N_0) + \gamma + \ln N_0] = \sigma_{pp}^{ND}$$

where σ_{pp}^{ND} is the non-diffractive pp cross section.

$$E_1(z) = \int_1^{\infty} e^{-zt} \frac{dt}{t} \,, \qquad \gamma = 0.577...$$

Comparison with quasi-eikonal and Regge approaches

Now we see that our formula for the σ_N coincides with the well known result for the cross-section σ_N of N cut-pomeron exchange, obtained in the quasi-eikonal and Regge approaches:

$$\sigma_N = \frac{4\pi\lambda}{C\,N}\left[1 - e^{-z}\sum_{k=0}^{N-1} z^k/k!\right]$$

where

$$z = \frac{2\gamma C}{\lambda}\exp(\Delta\xi)\ , \quad \lambda = R^2 + \alpha'\xi\ , \quad \xi = \ln(s/1GeV^2)\ .$$

Here Δ and α' are the residue and the slope of the pomeron trajectory. The parameters γ and R characterize the coupling of the pomeron trajectory with initial hadrons. The quasi-eikonal parameter C is related to the small-mass diffraction dissociation of incoming hadrons.

K.A. Ter-Martirosyan Phys. Lett. B **44**, 377 (1973).
A.B. Kaidalov, K.A. Ter-Martirosyan Yad. Fiz. **39**, 1545 (1984); **40**, 211 (1984).
V.A. Abramovsky, V.N. Gribov, O.V. Kancheli Yad. Fiz. **18**, 595 (1973).

Comparison with the Regge approach

This enables to connect the parameters N_0 and α of our model with the parameters of the pomeron trajectory and its couplings to hadrons. Comparing we have

$$N_0 = z = \frac{2\gamma C}{\lambda}\exp(\Delta\xi)\ , \quad \alpha = \sqrt{\frac{2\lambda}{C}}\ , \quad \lambda = R^2 + \alpha'\xi \qquad (25)$$

Our values of the parameters:

$$\Delta = 0.2, \quad \alpha' = 0.05\ GeV^{-2},$$
$$\gamma_{pp} = 1.035\ GeV^{-2}, \quad R^2 = 3.3\ GeV^{-2}, \quad C = 1.5\ .$$

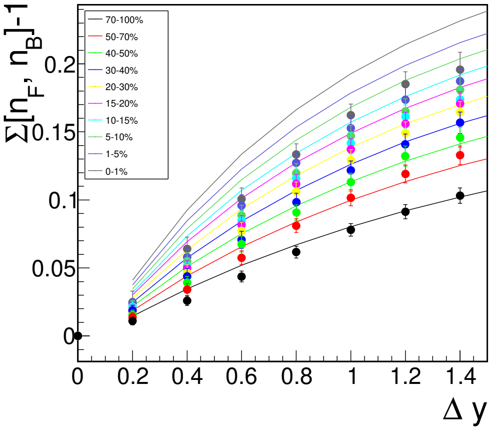

Figure: The $\Sigma(n_F, n_B)$ for different pp-collision centrality classes at initial energy 13 TeV, Experimental points from [A.Erokhin (for the ALICE Collaboration) reported at IS2021]. Curves - our results in the model with the formation of string clusters.