National Research Nuclear University MEPhI (Moscow Engineering Physics Institute)

ATROPOS

An Improved String Hadronization Model

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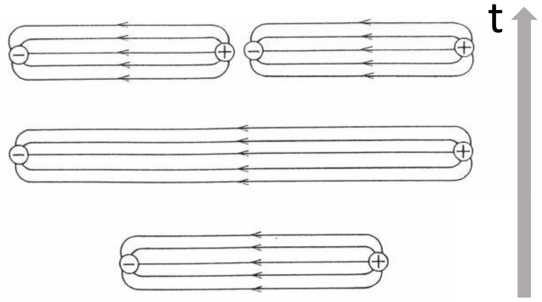
Introduction. String models of hadronization

- **Preconfinement** approach: partons after production stay bound together as colorless systems
- It is believed that QCD field between partons compresses into a flux **tube** due to gluons self-interaction
- To simplify the theoretical description we neglect the transverse size of the tube ➡ **strings**
- Strings fragment via **pair production** and light strings are identified as **hadrons**

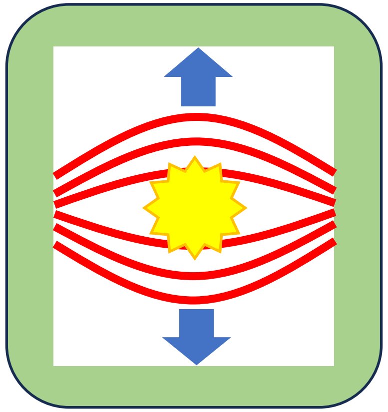
Field lines between a quark and an antiquark squeeze into a tube

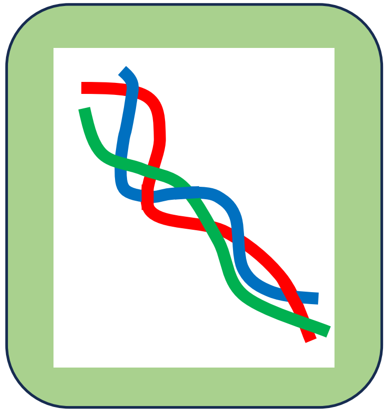


Meson production: $q - \bar{q}$ pair
Baryon production: $qq - \overline{qq}$ pair

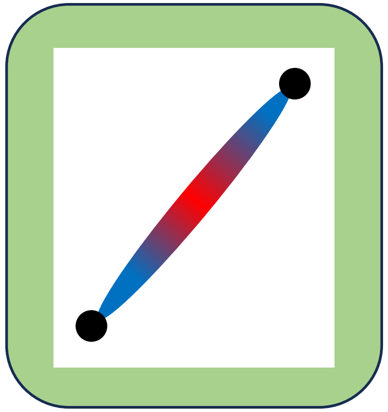
Challenges of modern string models

String interactions: shoving, rope hadronization, etc.

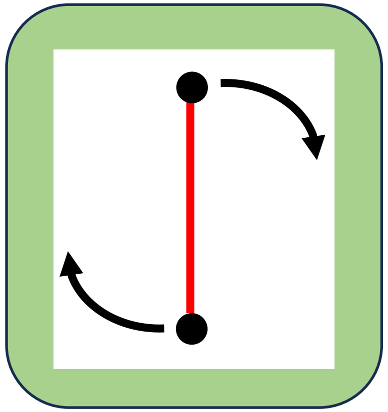




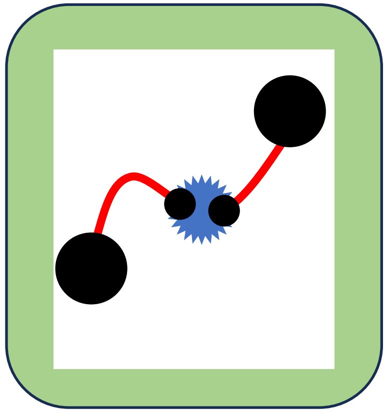
Non-constant string tension



Angular momentum conservation



Heavy quarks fragmentation



… and many more!

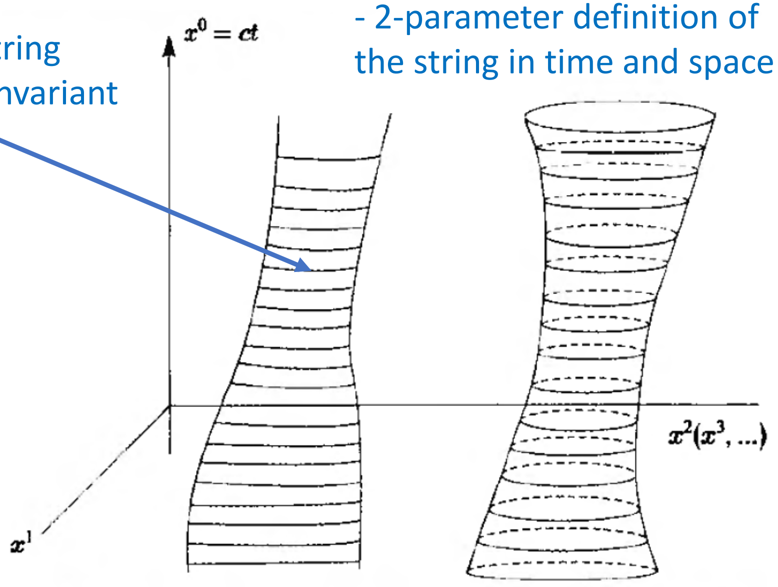
ATROPOS-v1.0: proof of concept

➢ Can we use a **detailed** Monte-Carlo simulation of relativistic string fragmentation to produce hadrons?

- PYTHIA: use LUND fragmentation model
  - Only the simplest case of the string is derived from the initial postulates
  - Fragmentation apparatus (de-facto) not applicable for any non-symmetric picture: non-zero quark masses, initial extension of the string, multi-gluon string, non-zero angular momentum …
  - String dynamics is not calculated microscopically

$x_\mu(\tau, \sigma)$
- 2-parameter definition of the string in time and space

World sheet of the string
Use its area A as an invariant



ATROPOS:

- Derive string equations of motion directly from action (modified Nambu-Goto, see below)
- Govern fragmentation process by Artru-Mennessier Area Decay Law:

$$\frac{dP_{\text{break}}}{dA} = P_0 = \text{const}$$

- Define the string **properly:** conserve energy, momentum and angular momentum + more …
- Use the exact analytical solutions to calculate invariant area and string characteristics at the break point

Relativistic string with masses at its ends

Regular Nambu-Goto action

$$S_{\text{string}} = -\kappa \int_{\tau_1}^{\tau_2} d\tau \int_{\sigma_1(\tau)}^{\sigma_2(\tau)} d\sigma \sqrt{(\dot{x}x')^2 - \dot{x}^2 x'^2} - \sum_{i=1}^{2} m_i \int_{\tau_1}^{\tau_2} d\tau \sqrt{\left(\frac{dx_\mu(\tau,\sigma_i(\tau))}{d\tau}\right)^2}$$

Term to describe heavy quarks at string ends

From the action follow the equations of motion

$$\ddot{x}^\mu - x''^\mu = 0$$

and boundary conditions

$$\frac{m_1}{\kappa}\frac{d}{d\tau}\left(\frac{\dot{x}_\mu}{\sqrt{\dot{x}^2}}\right) = x'_\mu, \qquad \sigma = 0,$$
$$\frac{m_2}{\kappa}\frac{d}{d\tau}\left(\frac{\dot{x}_\mu}{\sqrt{\dot{x}^2}}\right) = -x'_\mu, \qquad \sigma = \pi.$$

linearization →

$$\ddot{x}_\mu(\tau,0) = q_1 x'_\mu(\tau,0),$$
$$\ddot{x}_\mu(\tau,\pi) = -q_2 x'_\mu(\tau,\pi),$$
$$q_1 = \frac{\kappa}{m_1^2}, \qquad q_2 = \frac{\kappa}{m_2^2}$$

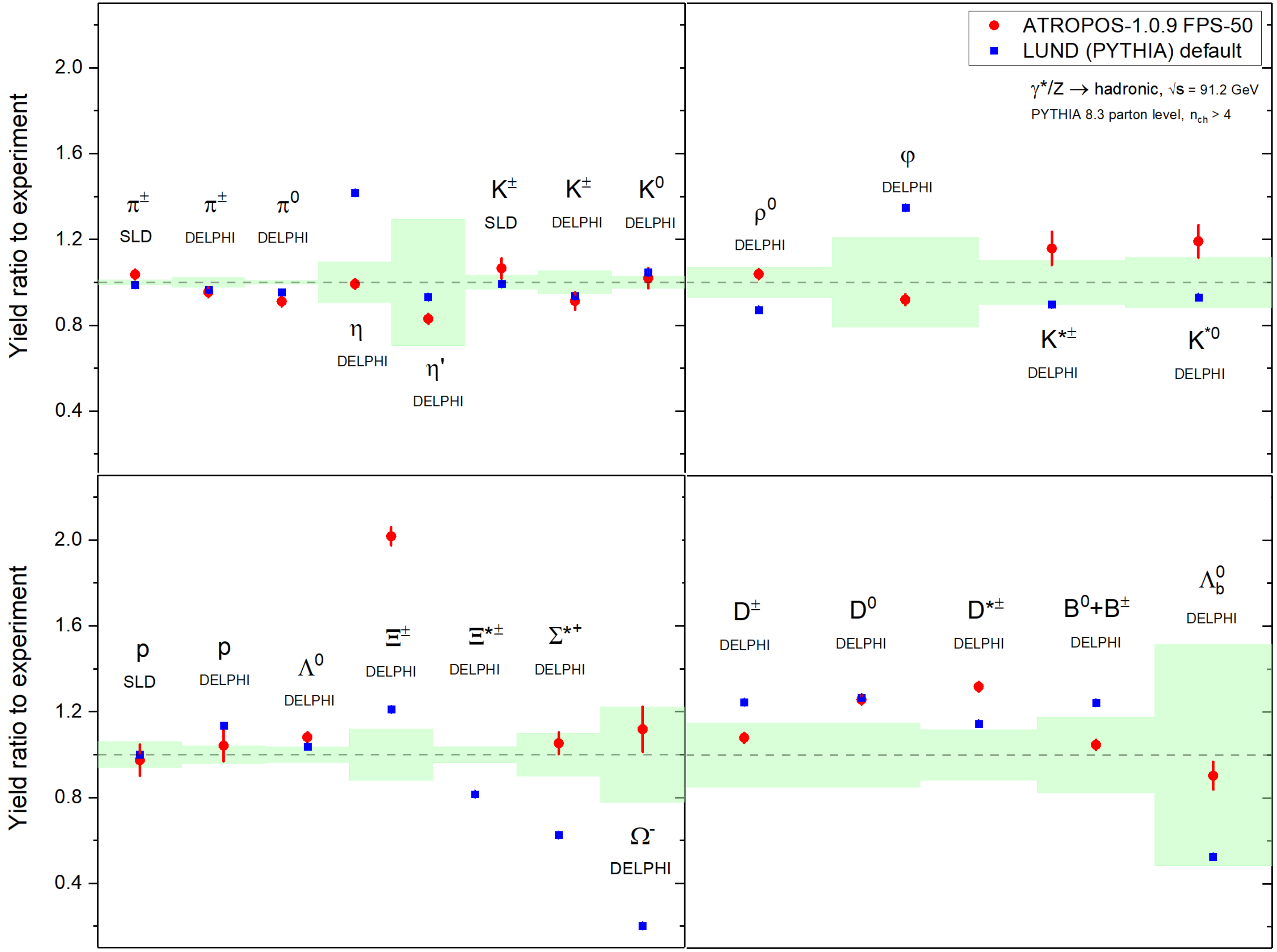
To linearize the initial conditions the following condition is used (it restricts the type of string motion):

$$\dot{x}^2(\tau,0) = m_1^{-2}, \qquad \dot{x}^2(\tau,\pi) = m_2^{-2}$$

The solution to the Cauchy problem on the string movement can be written as the Fourier series:

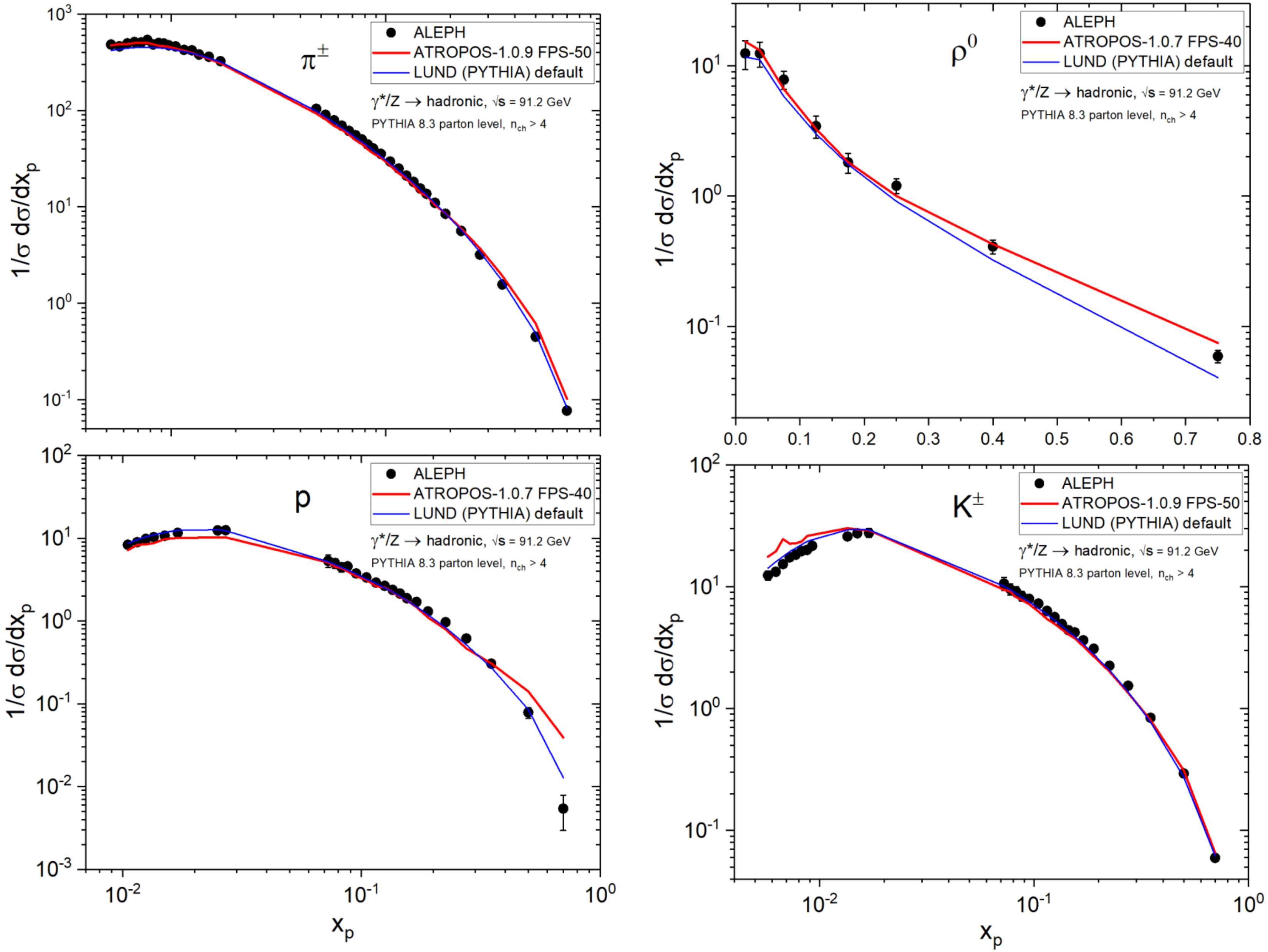
$$x^\mu(\tau,\sigma) = C_0^\mu \tau + D_0^\mu + \sum_{n=1}^{+\infty}\left[C_n^\mu \sin(\omega_n\tau) + D_n^\mu \cos(\omega_n\tau)\right]u_n(\sigma)$$

First results… Particles yields



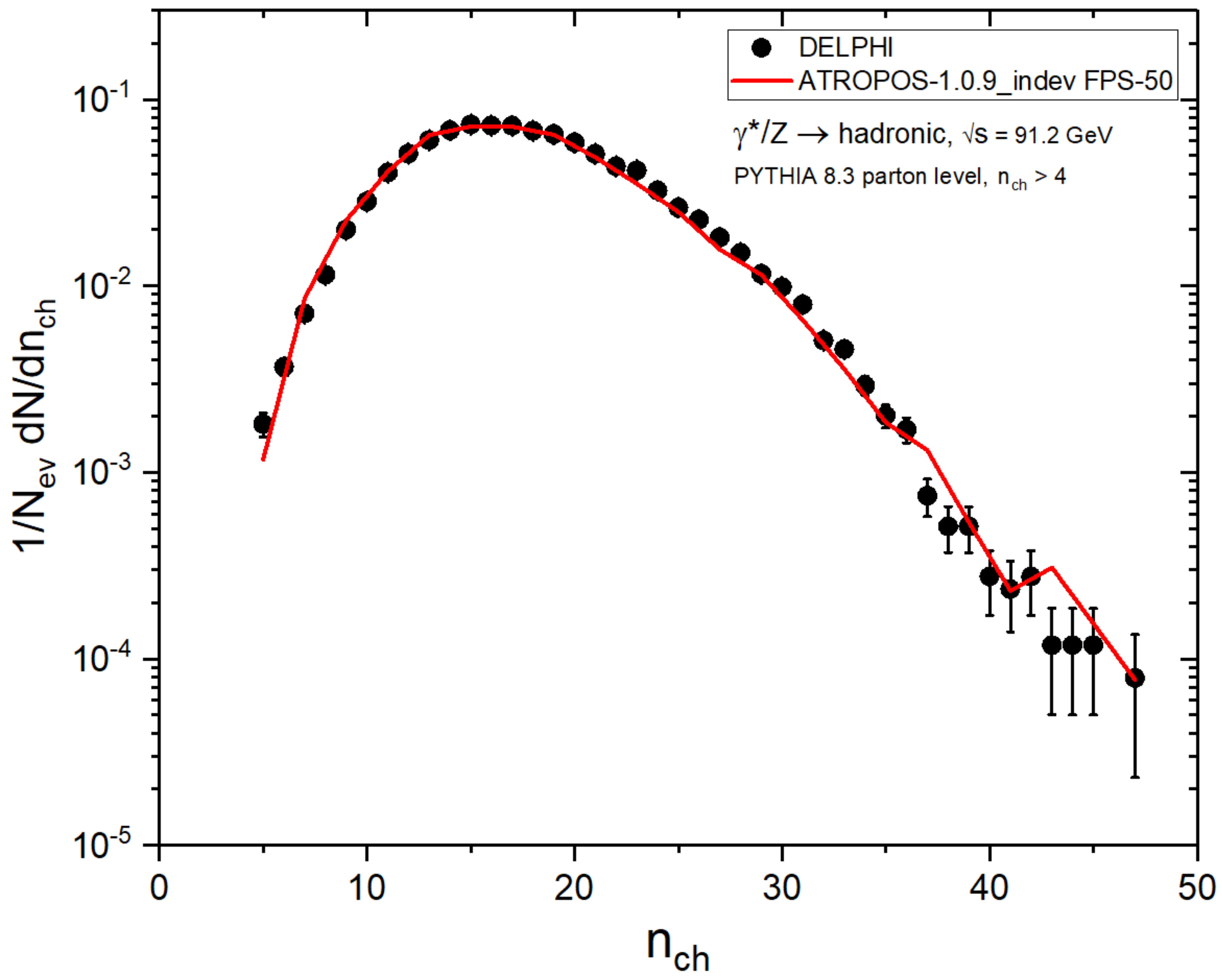
- Yield of η-mesons is well described (not specifically tuned though)
- The yield of light vector mesons and Σ-, Ω-hyperons is described (somewhat) better then in PYTHIA
- Good agreement on the multiplicity of c- and b-particles
- Few problems: to many Ξ-hyperons and some vector resonances (to be improved ☺)

First results… Momentum spectra



- $\pi^{\pm}$-mesons and $K^{\pm}$-mesons spectra are well reproduced
- The spectrum of ρ^0-mesons is in good agreement with data
- Proton spectrum too high at large momenta: need further tuning
- Note:
  No special parameters used to tune vector/charm/bottom/baryon production

Multiplicity of charged particles



- Good description overall, however:
  1. Much of the available LEP data was corrected with old PYTHIA simulations to fill the experimental gaps
  2. Raw data description can be achieved with different tunes (selection criteria leave much space for ambiguous model description)
  3. The behavior at wide energy range was not yet tested
  4. No p-p collisions were simulated, as this version of the model is itself not complete (and thus would likely fail at p-p interactions)

- Remains the problem of proper string definition, so …

ATROPOS-v1.1: towards the proper and reliable string definition

So where does the problem stand?

$x_\mu(\tau, \sigma)$ is a 2-parameter definition of the string world sheet in time and space

$$\dot{x}_\mu \equiv \frac{\partial x_\mu(\tau,\sigma)}{\partial \tau}$$

$$x'_\mu \equiv \frac{\partial x_\mu(\tau,\sigma)}{\partial \sigma}$$

If we consider the usual string action

$$S_{\text{string}} = -\kappa \int_{\sigma_1}^{\sigma_2} d\sigma \int_{\tau_1(\sigma)}^{\tau_2(\sigma)} d\tau \sqrt{(x'\dot{x})^2 - x'^2\dot{x}^2}$$

the equations of motion will actually take the form

$$\frac{\partial}{\partial \tau}\left(\frac{(\dot{x}x')x'_\mu - x'^2\dot{x}_\mu}{\sqrt{(\dot{x}x')^2 - \dot{x}^2x'^2}}\right) + \frac{\partial}{\partial \sigma}\left(\frac{(\dot{x}x')\dot{x}_\mu - \dot{x}^2x'_\mu}{\sqrt{(\dot{x}x')^2 - \dot{x}^2x'^2}}\right) = 0.$$

Does not look promising, right? Likely, the Nambu-Goto action is reparameterization-invariant, so a specific relation between τ and σ can be chosen:

$$\boldsymbol{\dot{x}^2 + x'^2 = 0,\ \dot{x}x' = 0.}$$

This is called orthonormal gauge. Only with that we get the simple wave equation to describe string movement:

$$\ddot{x}_\mu - x''_\mu = 0.$$

Virasoro conditions for the initial data

Lets substitute the solution to the free string Cauchy problem into the orthonormal gauge:

$$x_\mu(\tau,\sigma) = Q_\mu + P_\mu \frac{\tau}{\pi\kappa} + \frac{i}{\sqrt{\pi\kappa}} \sum_{\substack{n=-\infty \\ n\neq 0}}^{+\infty} e^{-in\tau} \frac{\alpha_{n\mu}}{n} \cos(n\sigma)$$

$$\begin{cases} \dot{x}^2 + x'^2 = 0 \\ \dot{x}x' = 0. \end{cases}$$

- Here Q_μ are the coordinates of the string center-of-mass in the initial moment of time, P_μ is a 4-vector of total string momentum

What we get are the Virasoro conditions:

$$\Rightarrow \quad \sum_{m=-\infty}^{+\infty} \boldsymbol{\alpha_{n-m}\,\alpha_m = 0}, \qquad \boldsymbol{n = 0, \pm 1, \pm 2, \ldots}$$

Here $\alpha_{n\mu}$ are the Fourier amplitudes:

$$\alpha_{n\mu} = \sqrt{\frac{\kappa}{\pi}} \int_0^\pi d\sigma \cos(n\sigma) \left(v_\mu(\sigma) - in\rho_\mu(\sigma) \right), \qquad n \neq 0,$$

$$\alpha_{0\mu} = \frac{P_\mu}{\sqrt{\kappa\pi}}.$$

Functions $\boldsymbol{v_\mu(\sigma)}$ and $\boldsymbol{\rho_\mu(\sigma)}$ define the velocity and form of the string in the initial moment of time. So, Virasoro conditions restrict the way this functions may be defined.

The FOEE-method to define the initial conditions of the string

Existing models like EPOS, NEXUS, Caltech-II all use the same ansatz for initial conditions:

$$\rho_\mu \equiv 0,$$
$$v_\mu = const = \frac{E}{p}.$$

One can check, though, that when substituted into Virasoro conditions, these functions yield the condition: $\boldsymbol{M^2 = 0}$!

- To find the functions that can satisfy the Virsosro conditions and would not require the string to be massless turns out to be quit a challenge…
- In addition, angular momentum conservation adds 6 more equations to satisfy!
- A new approach was suggested:
  - ➢ Define the initial data in the form of **Final-Order Eigenfunction Expansion (FOEE)**

$$v_\mu(\sigma) = a_{0\mu} + \sum_{k\neq 0} a_{k\mu}\cos(k\sigma)$$
$$\rho_\mu(\sigma) = b_{0\mu} + \sum_{k\neq 0} b_{k\mu}\cos(k\sigma)$$

The FOEE-method to define the initial conditions of the string

- It turns out, the minimal order for the FOEE system to have enough variables is 2
- So, we define initial data in the form

$$v_\mu(\sigma) = a_\mu + b_\mu \cos(\sigma) + c_\mu \cos(2\sigma)\,,$$
$$\rho_\mu(\sigma) = d_\mu + e_\mu \cos(\sigma) + f_\mu \cos(2\sigma)\,.$$

This eventually yields the following system of equation:

Virasoro conditions

$$\begin{cases} b^2 + c^2 + e^2 + 4f^2 + \dfrac{P^2}{2(\kappa\pi)^2} = 0 \\ bc + 2ef + \dfrac{bP}{\kappa\pi} = 0 \\ 2bf - ce + \dfrac{eP}{\kappa\pi} = 0 \\ b^2 - e^2 + \dfrac{2cP}{\kappa\pi} = 0 \\ be + \dfrac{2fP}{\kappa\pi} = 0 \\ bc - 2ef = 0 \\ ce + 2bf = 0 \\ c^2 - 4f^2 = 0 \\ cf = 0 \\ d_\mu P_\nu - d_\nu P_\mu + \dfrac{\kappa\pi}{2}\left(e_\mu b_\nu - e_\nu b_\mu + f_\mu c_\nu - f_\nu c_\mu\right) = M_{\mu\nu}. \end{cases}$$

Angular momentum conservation

20 variables, 15 equations

➢ Additional assumptions to be made

No analytical solution is known:

- Use numerical methods instead
- However, the complexity of the system does not allow usual methods to succeed

➢ Need for better method (or more brute force computation power)

The first solutions?

String with $P_\mu = \{P_0, P_x, P_y, P_z\} = \{10, 1, 2, 3\}$ GeV, no rotation

Zero, as the string has no rotation

"Solution" 1 (accuracy 10^{-5}):

$$a_\mu = \begin{pmatrix} 15.92 \\ 1.59 \\ 3.18 \\ 4.77 \end{pmatrix}, \quad b_\mu = \begin{pmatrix} -0.63 \\ -9.48 \\ 1.6 \\ 0 \end{pmatrix}, \quad c_\mu = \begin{pmatrix} 1.95 \\ -0.2 \\ -1.94 \\ 0 \end{pmatrix}, \quad d_\mu = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad e_\mu = \begin{pmatrix} 0.27 \\ 4.06 \\ -0.69 \\ 0 \end{pmatrix}, \quad f_\mu = \begin{pmatrix} -1.02 \\ 0.1 \\ 1.02 \\ 0 \end{pmatrix}$$

"Solution" 2 (accuracy 10^{-7}):

$$a_\mu = \begin{pmatrix} 15.92 \\ 1.59 \\ 3.18 \\ 4.77 \end{pmatrix}, \quad b_\mu = \begin{pmatrix} -0.07 \\ -3.08 \\ 1.91 \\ 0 \end{pmatrix}, \quad c_\mu = \begin{pmatrix} -3.34 \\ -1.7 \\ -2.88 \\ 0 \end{pmatrix}, \quad d_\mu = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad e_\mu = \begin{pmatrix} -0.2 \\ 8.32 \\ -5.16 \\ 0 \end{pmatrix}, \quad f_\mu = \begin{pmatrix} -1.43 \\ -0.73 \\ -1.23 \\ 0 \end{pmatrix}$$

"Solution" 3 (accuracy 10^{-12}):

$$a_\mu = \begin{pmatrix} 15.92 \\ 1.59 \\ 3.18 \\ 4.77 \end{pmatrix}, \quad b_\mu = \begin{pmatrix} 1.34 \\ 6.04 \\ 3.67 \\ 0 \end{pmatrix}, \quad c_\mu = \begin{pmatrix} -0.47 \\ 0.16 \\ -0.44 \\ 0 \end{pmatrix}, \quad d_\mu = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad e_\mu = \begin{pmatrix} 1.5 \\ 6.78 \\ 4.13 \\ 0 \end{pmatrix}, \quad f_\mu = \begin{pmatrix} 2.01 \\ -0.7 \\ 1.88 \\ 0 \end{pmatrix}$$

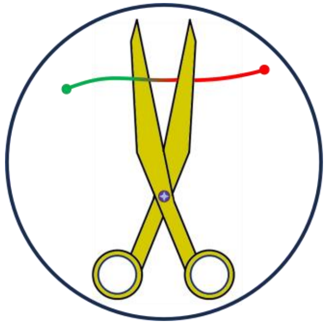
Note:
Even the simplest case of the massive relativistic string requires non-zero extension coefficients!

No solutions for rotating strings found yet… urgent need for search algorithms optimization!

Conclusion

- ATROPOS is a state-of-the-art string hadronization model that allows the full-scale detailed Monte-Carlo simulation of the string fragmentation process and hadron production.
- ATROPOS is **the first** model to introduce an approach to conserve angular momentum of the system during hadron production
- The use of modified Nambu-Goto action makes possible to treat the heavy c-, b-hadron production using the same technique, as for the light quarks
- The promising first results encourage to further improve such approach and continue the development of the model
- The consideration of the restrictions imposed by Virasoro conditions is done for the first time for the string hadronization model
- It is shown, that a very non-trivial ways need to be taken to **properly** define the non-massless relativistic string

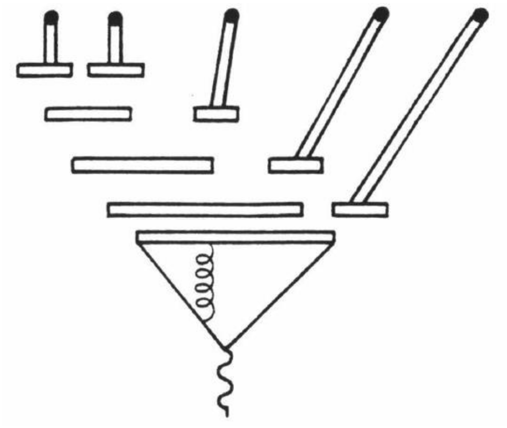
- **Much more improvements and results are upcoming!**



Thank you for your attention!

String models of hadronization

LUND

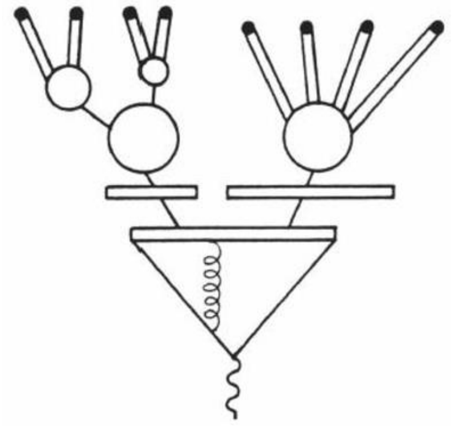


String fragmentation functions:

$$f(z) \propto \frac{(1-z)^a}{z} e^{-\frac{bm^2}{z}}$$

- Quark masses violate initial postulates of the model
- Scaling-invariant
- Bowler modification for heavy quarks

Caltech-II (Area Decay Law)



$$S_{\text{string}} = -\kappa \int_{\sigma_1}^{\sigma_2} d\sigma \int_{\tau_1(\sigma)}^{\tau_2(\sigma)} d\tau \sqrt{(\dot{x}x')^2 - \dot{x}^2 x'^2}$$

Area law:

$$\frac{dP_{\text{break}}}{dA} = P_0 = \text{const}$$

- Approximation: initially point-like strings
- The simplest ansatz for initial conditions
- Non-physical segments on the strings to describe heavy quarks

Observed discrepancies with experimental data :

- π^0 cross-sections (especially for SoftQCD minimum-bias generators)
- Production of ρ^0- mesons in forward direction in hadron-nucleus interactions (important for extensive air showers)
- The Muon Puzzle: ratio $R = \frac{E_{\pi^0}}{E_{\text{hadr}}}$
  - ➢ Strange particles production
  - ➢ Resonance enhancement
  - ➢ Baryon enhancement

Both collective and non-collective effects matter!

T. Pierog, K. Werner, DOI: 10.22323/1.444.02300

- No angular momentum conservation!

Fragmentation algorithm

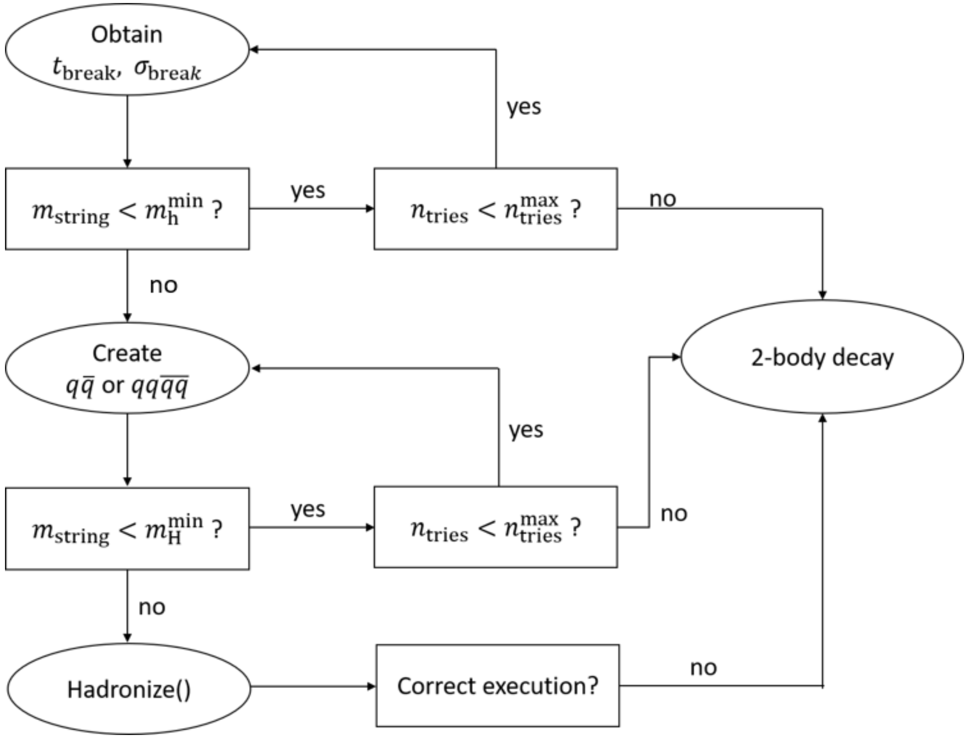
$$A(\tau) = \int_0^{\pi} d\sigma \int_0^{\tau} d\tau' \, \dot{x}^2(\tau', \sigma) + N_{\text{massive}} \tau$$

Only with orthonormal gauge we obtain such simple form

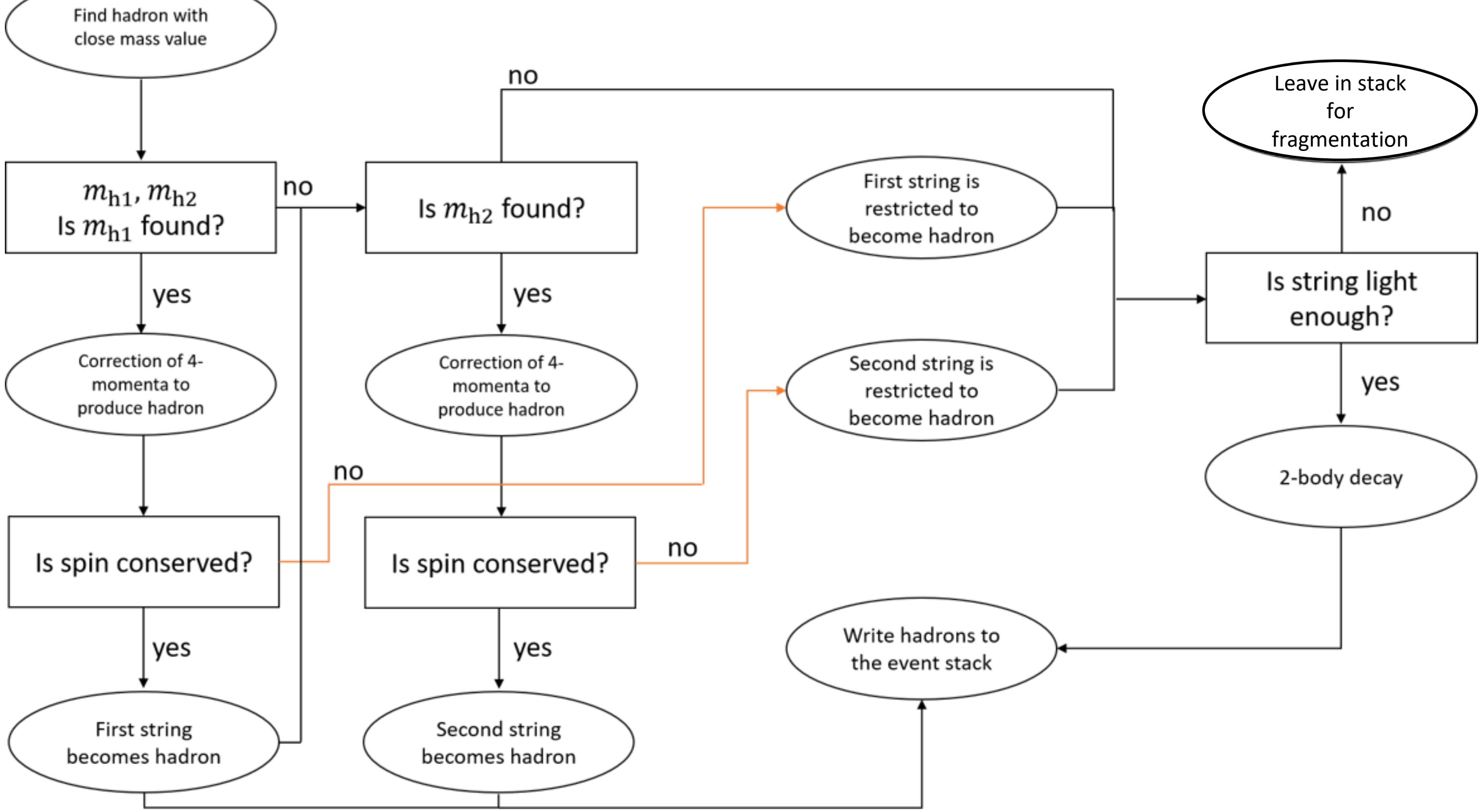
$$P_{\text{alive}}(\tau) = e^{-P_0 A(\tau)}$$

Sample break point using are decay law

Fragmentation cycle



String-to-hadron transition algorithm



Conservation of 4-momentum at string-to-hadron transition: «hadron-string shoving» mechanism

Free parameters of the model. FPS-50

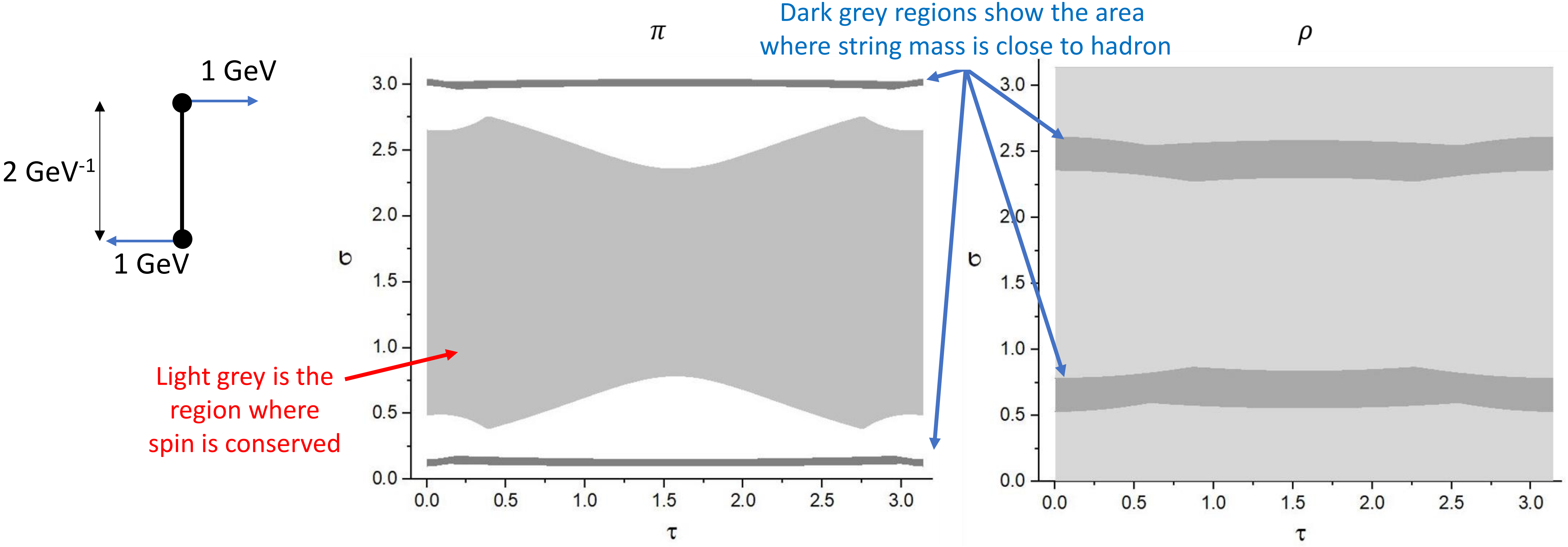
Parameter	Value	Description
P_0	0.5 GeV^2	Constant probability of string breaking per unit area of the world sheet
$P_{u\bar{u}} = P_{d\bar{d}}$	0.3075	Relative probability of formation of a quark-antiquark pair
$P_{uu\overline{uu}} = P_{dd\overline{dd}} = P_{ud\overline{ud}}$	0.06	Relative probability of formation of a diquark-antidiquark pair
$P_{s\bar{s}}$	0.145	Relative probability of formation of a quark-antiquark pair
$P_{us\overline{us}} = P_{ds\overline{ds}}$	0.02	Relative probability of formation of a diquark-antidiquark pair
$P_{ss\overline{ss}}$	0.02	Relative probability of formation of a diquark-antidiquark pair
ε	0.2	Mass tolerance at string-hadron transition (SHMT)
κ	0.2 GeV^2	String tension
MUT	1.5	Limit on string mass for further fragmentation (Mass Upper Threshold)

Restrictions on hadron production

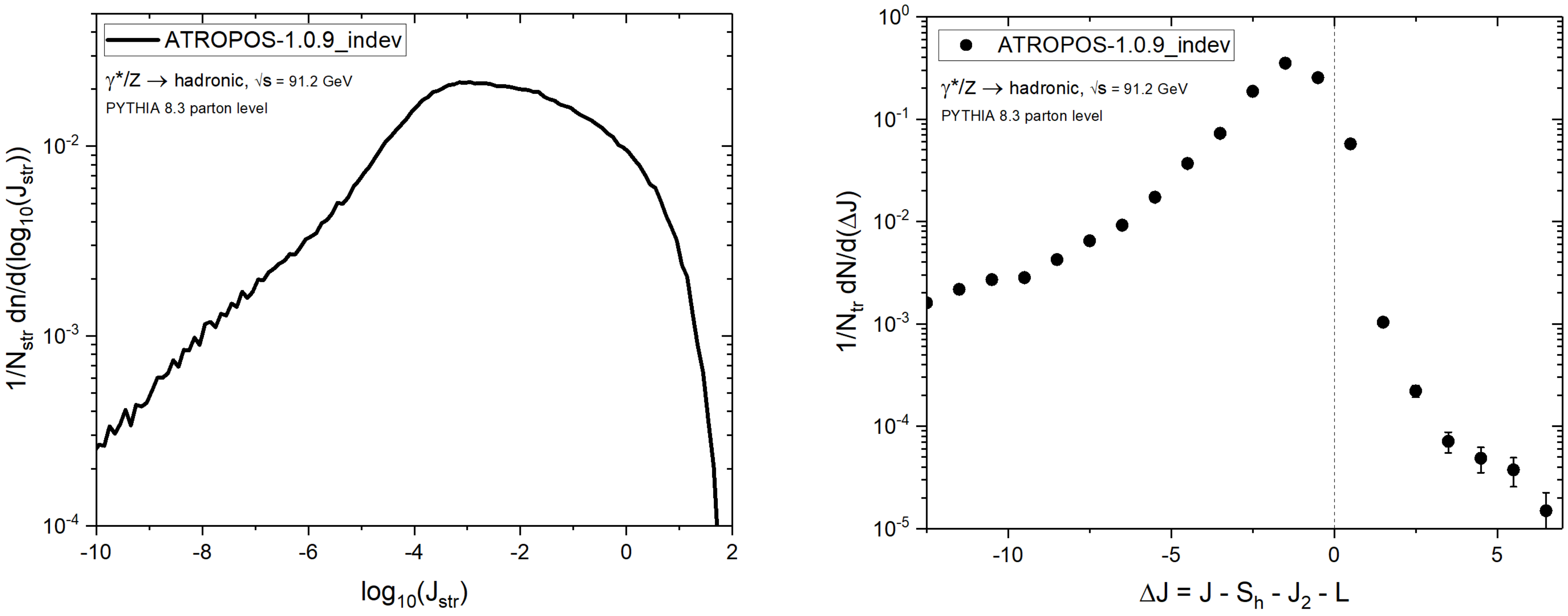
- The total angular momentum of the string must be conserved after the transition of its fragment into hadron
- System's total angular momentum is defined by the spin of the remaining fragment $\vec{J}_{\text{frag}}$, the spin of the hadron $\vec{S}_{\text{hadr}}$ and the orbital angular momentum of their relative movement $\vec{L}$
- As we don't know, how exactly the transition happens, let us define the rule, that angular momentum can be conserved, if the maximum value of $\vec{J}_{\text{frag}}+\vec{S}_{\text{hadr}}+\vec{L}$ is larger than initial string spin $\vec{J}$:

$$J_{\text{frag}} + S_{\text{hadr}} + L \geq J$$

- We can now calculate the area in τ, σ – space where this rule is fulfilled for different break points

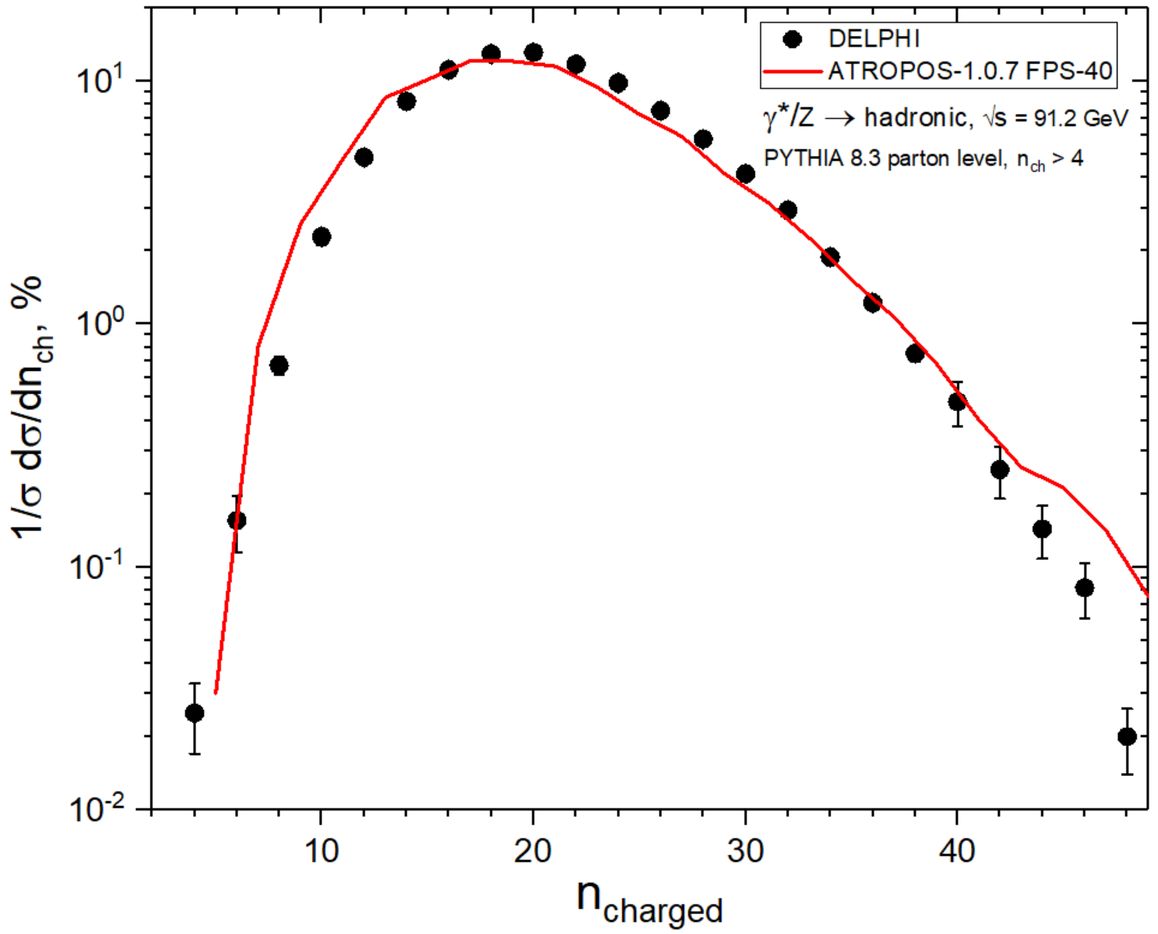


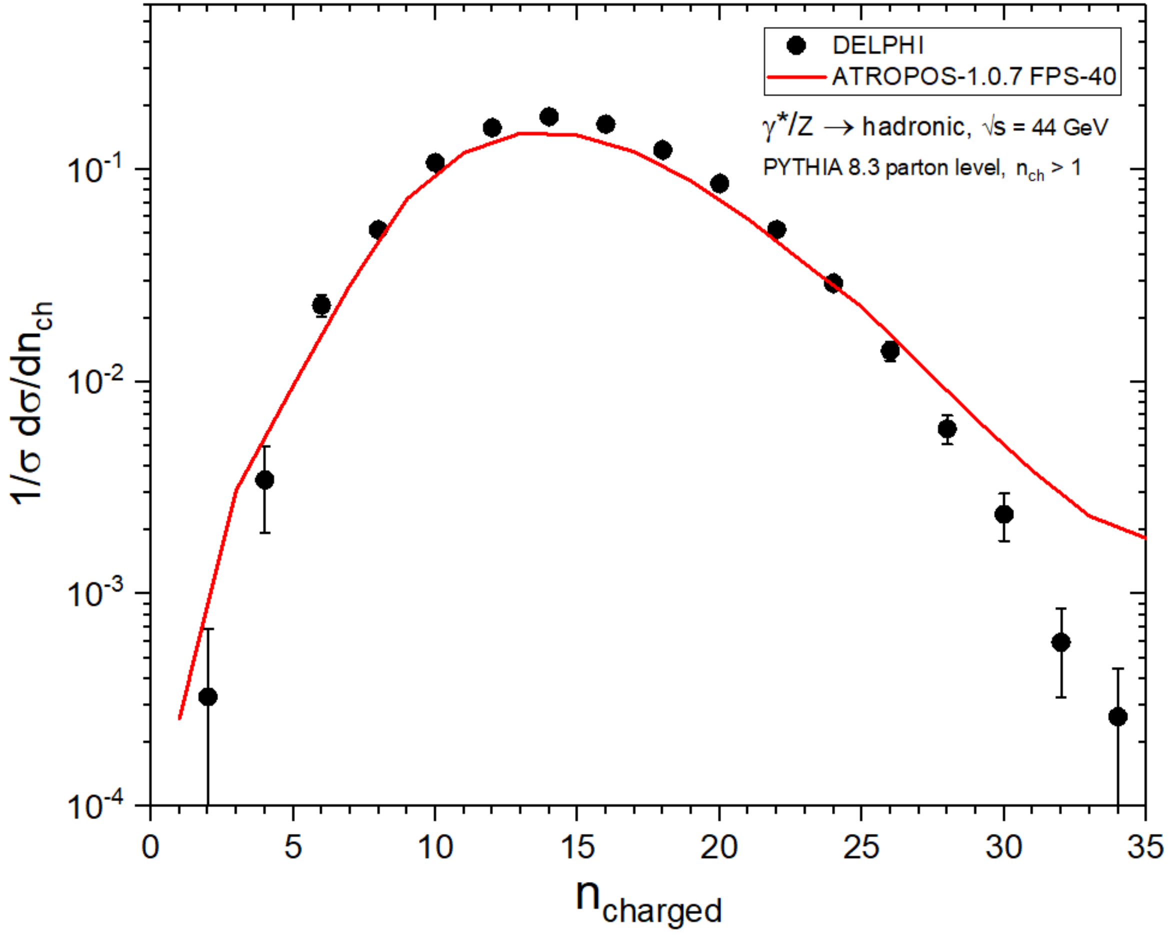
Angular momentum of relativistic strings



Note, that even in electron-positron interactions there is a significant fraction of strings with high angular momentum due to the transverse gluons. The fraction of restricted string-to-hadron transitions according to angular momentum conservation (when $\Delta J > 0$) is around 6%. This is, however, the most simple and least strict way to impose such law.

Multiplicity of charged particles (free strings approximation)

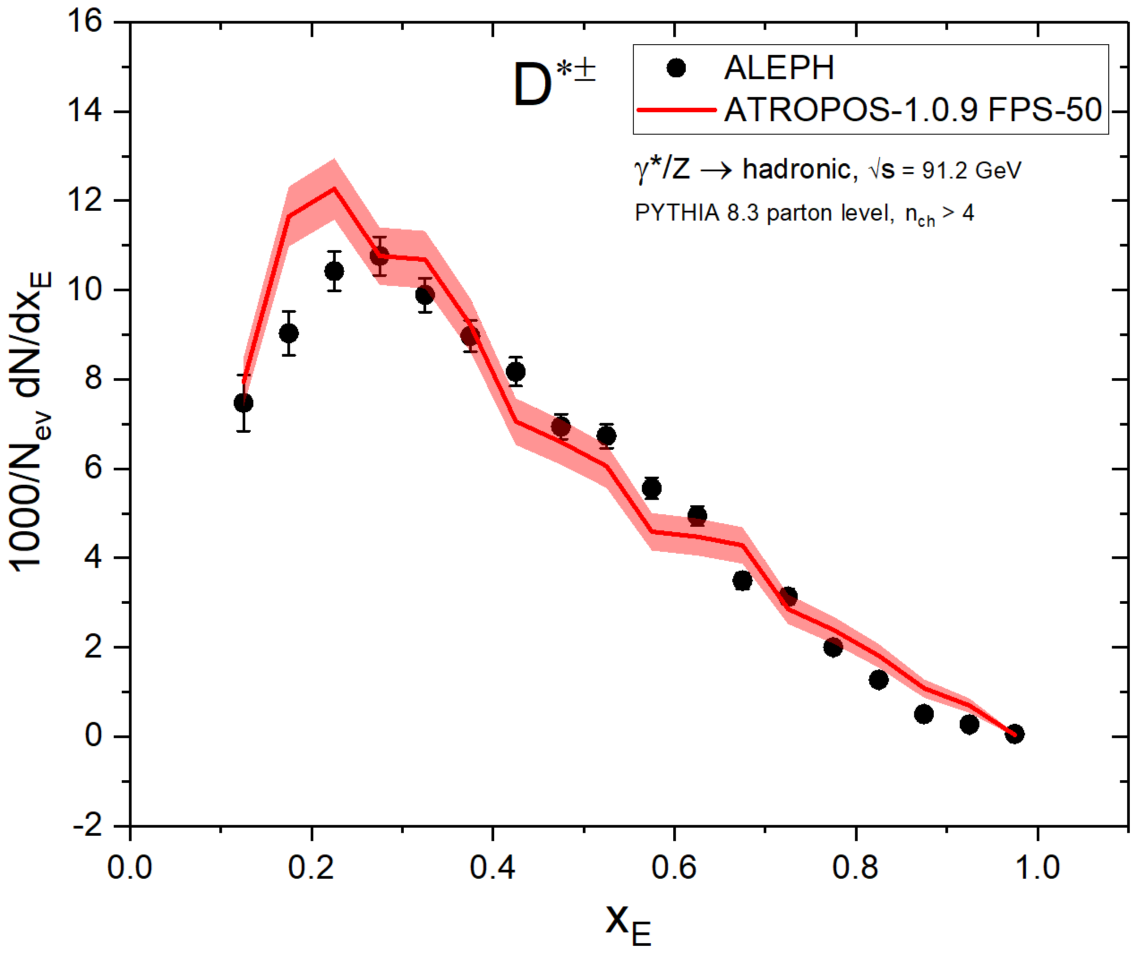




- There are difficulties with the tail of the distributions - most likely due to an incorrect description of strings with heavy c- and b-quarks
- Small multiplicities – deviation (probably) as the result of correction of experimental data using JETSET

D-mesons spectrum

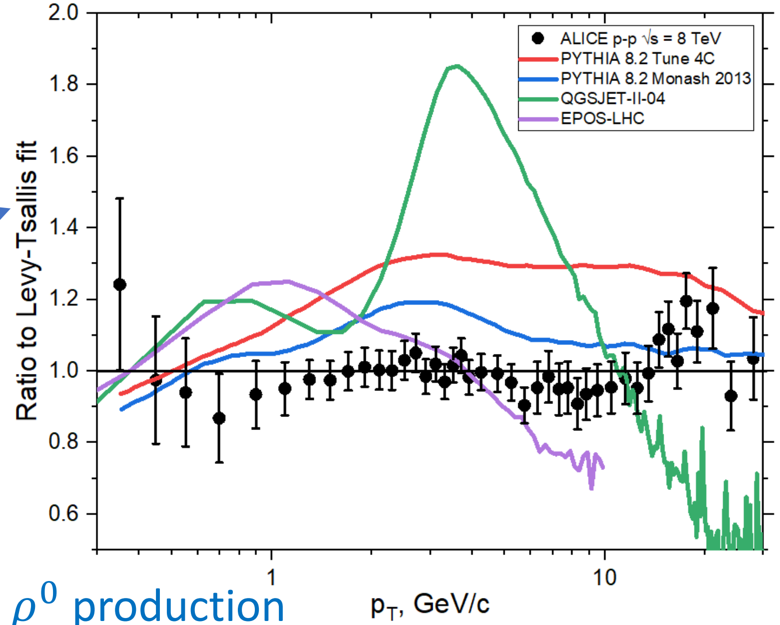
- Preliminary result!
- The description of the fragmentation of systems with heavy quarks is introduced for the first time from the first principles of the theory (starting from the action of the string)
- Additional parameterizations may be necessary



Problems with hadronization

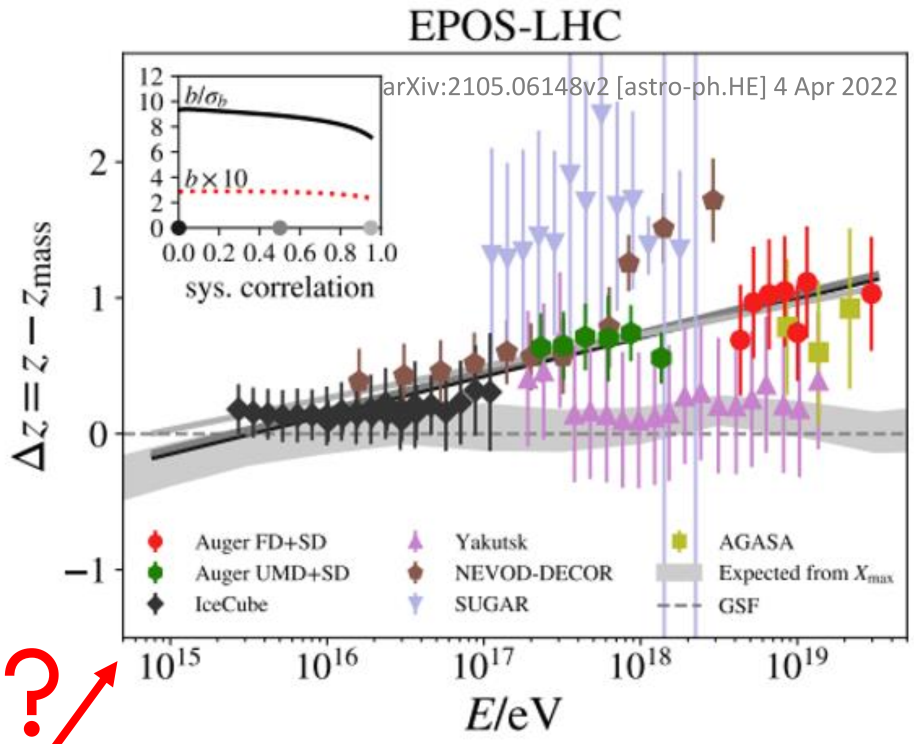
- Hadronization is a process of final state hadron production from quarks and gluons (partons)
- Common mistake is to believe that there is a complete and consistent way of hadronization description

Cross-sections for π^0 production at LHC energies

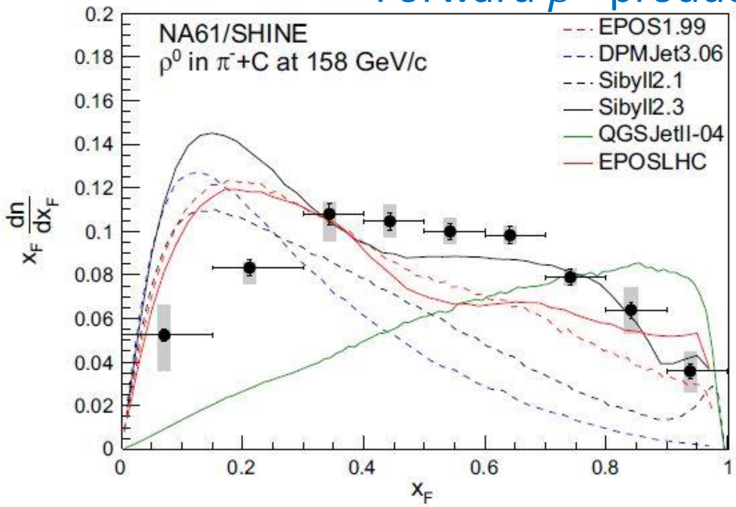


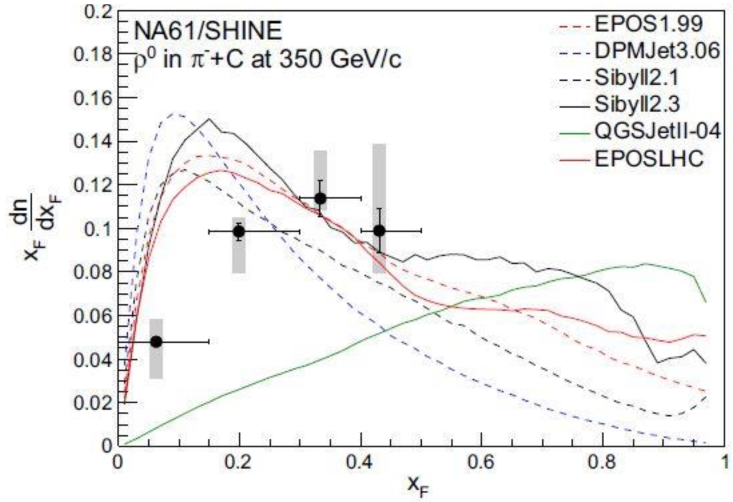
Same quark composition for π^0 and ρ^0, so only hadronization scheme matters

The Muon Puzzle



Forward ρ^0 production



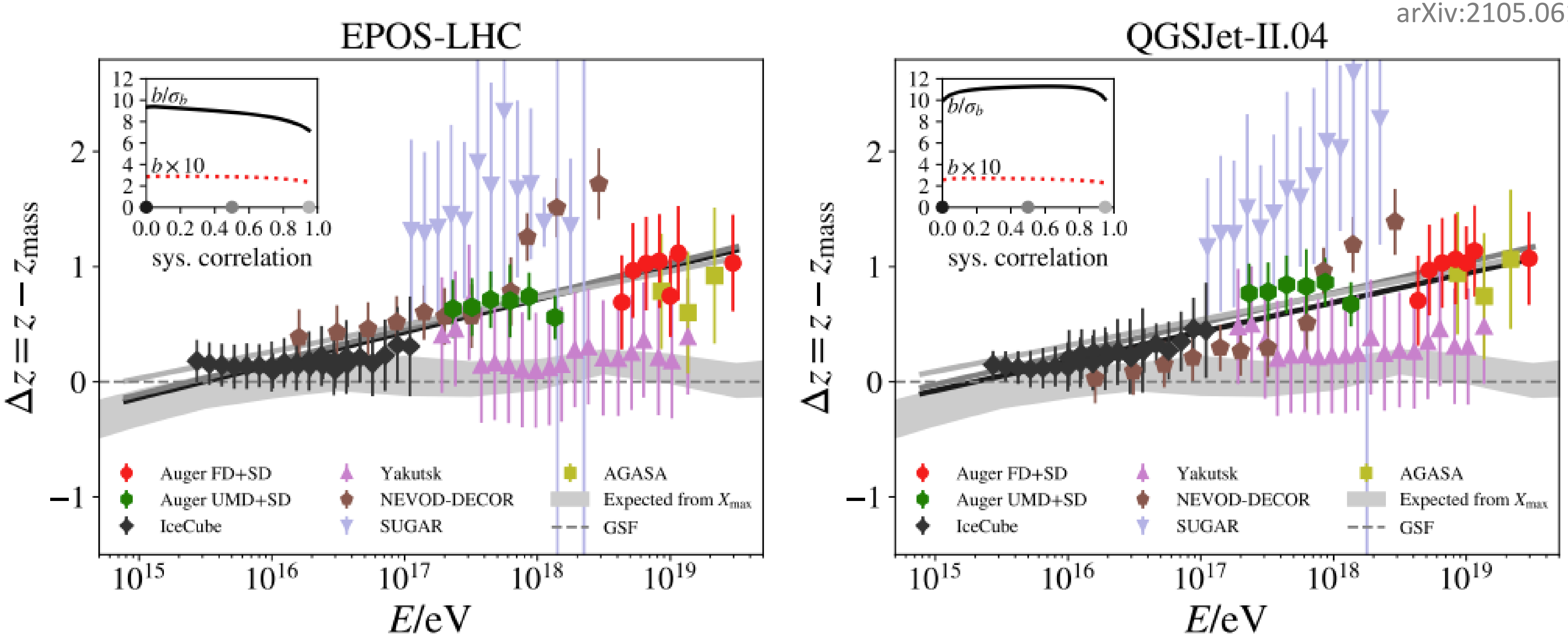


Both collective and non-collective effects matter!

T. Pierog, K. Werner, DOI: 10.22323/1.444.02300

The muon puzzle

arXiv:2105.06148v2 [astro-ph.HE] 4 Apr 2022



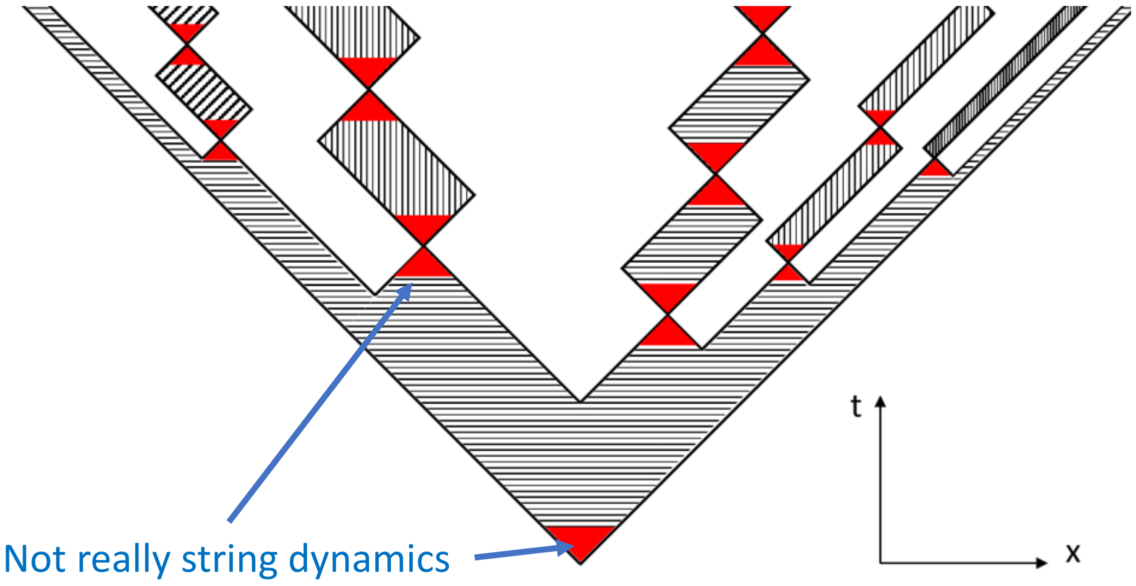
$$z = \frac{ln\langle N_\mu^{det}\rangle - ln\langle N_{\mu,p}^{sim}\rangle}{ln\langle N_{\mu,Fe}^{sim}\rangle - ln\langle N_{\mu,p}^{sim}\rangle}$$

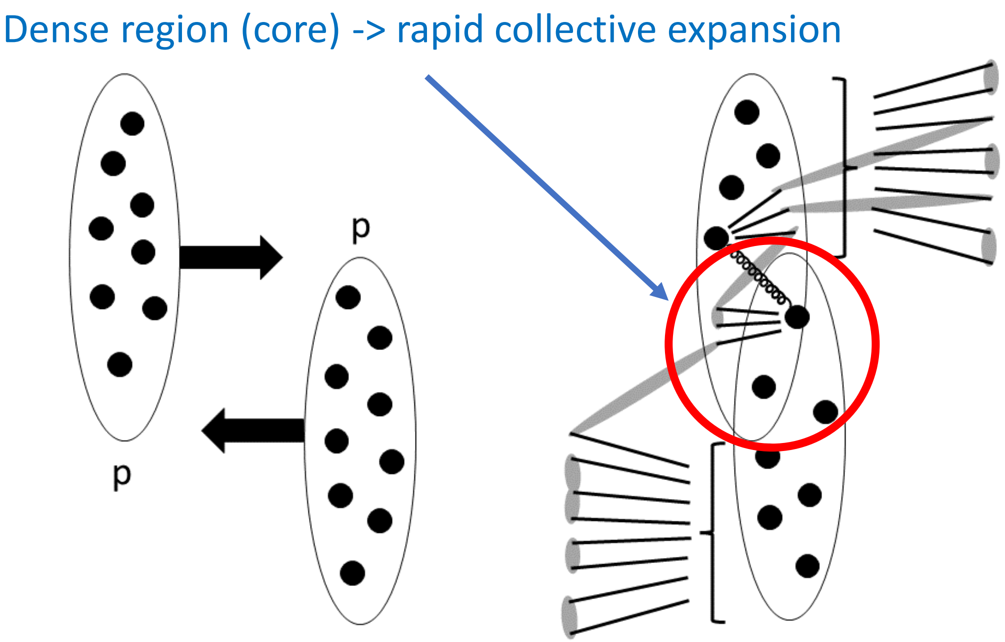
The value of z parameter should be strictly between 0 for pure proton composition of PCR an 1 for pure iron

Reduced parameter Δz is obtained by subtracting from calculated z the z_{mass}, which is based mainly on X_{max} measurements, Global Spline Fit. Significant non-zero slope!

Around 30% - 60% more muons than expected according to models at high energies

Initially-extended strings





The question of the importance of consideration of initial extension of the quark-gluon strings arises.

What properties of hadron production this might change?

Gauge and initial conditions (for ATROPOS-v1.0)

- How to define initial conditions for a string made out of produced partons?

Initial momentum of the string (per unit σ) $\vec{\boldsymbol{p}}(\sigma)$ is constructed from partons momentums as piecewise-linear functions:

=1 for quarks and 0.5 for gluons as they loose energy for two pieces of string

$$\tilde{p}_i^{\mu}(\sigma) = \frac{2(N-1)}{\pi}\left(\frac{p_{i+1}^{\mu}\xi_{i+1} - p_i^{\mu}\xi_i}{\sigma_{i+1}-\sigma_i}(\sigma-\sigma_i) + p_i^{\mu}\xi_i\right), \qquad \sigma \in [\sigma_i, \sigma_{i+1}];$$

$q: p_x = 2$ GeV
$g: p_x = 1$ GeV
$\bar{q}: p_x = -2$ GeV

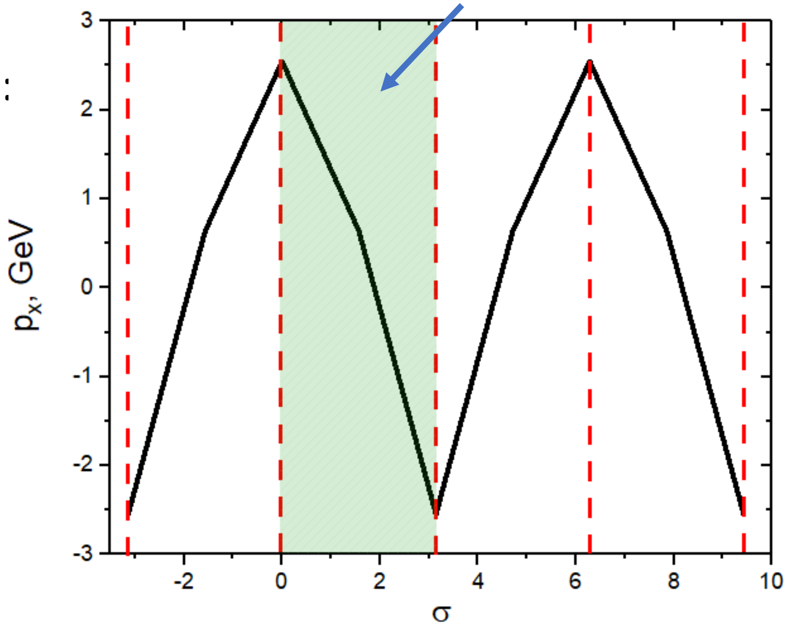
But for energy the initial extension also must be taken into account:

$$p_{i+1}^{\mu} = p_{\text{parton},i}^{\mu} + p_{\text{QCD},i}^{\mu}$$

Energy stored in QCD field

- This gauge leads to quadratic relation between time t and parameter τ: $t \sim \tau^2$

Main interval



Extended evenly over the $[0, \pi]$ interval to use D'Alamber solution