Recent progress in Asymptotic Safety

Based on *Symmetry* 15 (2023) 8, 1497 & *Phys.Rev.D* 109 (2024) 6, 065030 & 2410.????

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The Standard Model works

Fields

Gauge fields + fermions + scalars

Interactions

$SU(3) \times SU(2) \times U(1)$ at EW scale

Yukawa: Fermion masses/Flavour

Scalar self-interaction

Culprit: Higgs

Gauge-Yukawa theories

$$L = -\frac{1}{2}F^2 + i\bar{Q}\gamma_\mu D^\mu Q + y(\bar{Q}_L H Q_R + h.c.)$$

Yukawa

$$+\mathrm{Tr}[DH^\dagger DH] - \lambda_u \mathrm{Tr}[(H^\dagger H)^2] - \lambda_v \mathrm{Tr}[H^\dagger H]^2$$

Gauge

Scalar self-interactions

4D: Standard Model, dark matter, ...

3D: Condensed matter, phase transitions

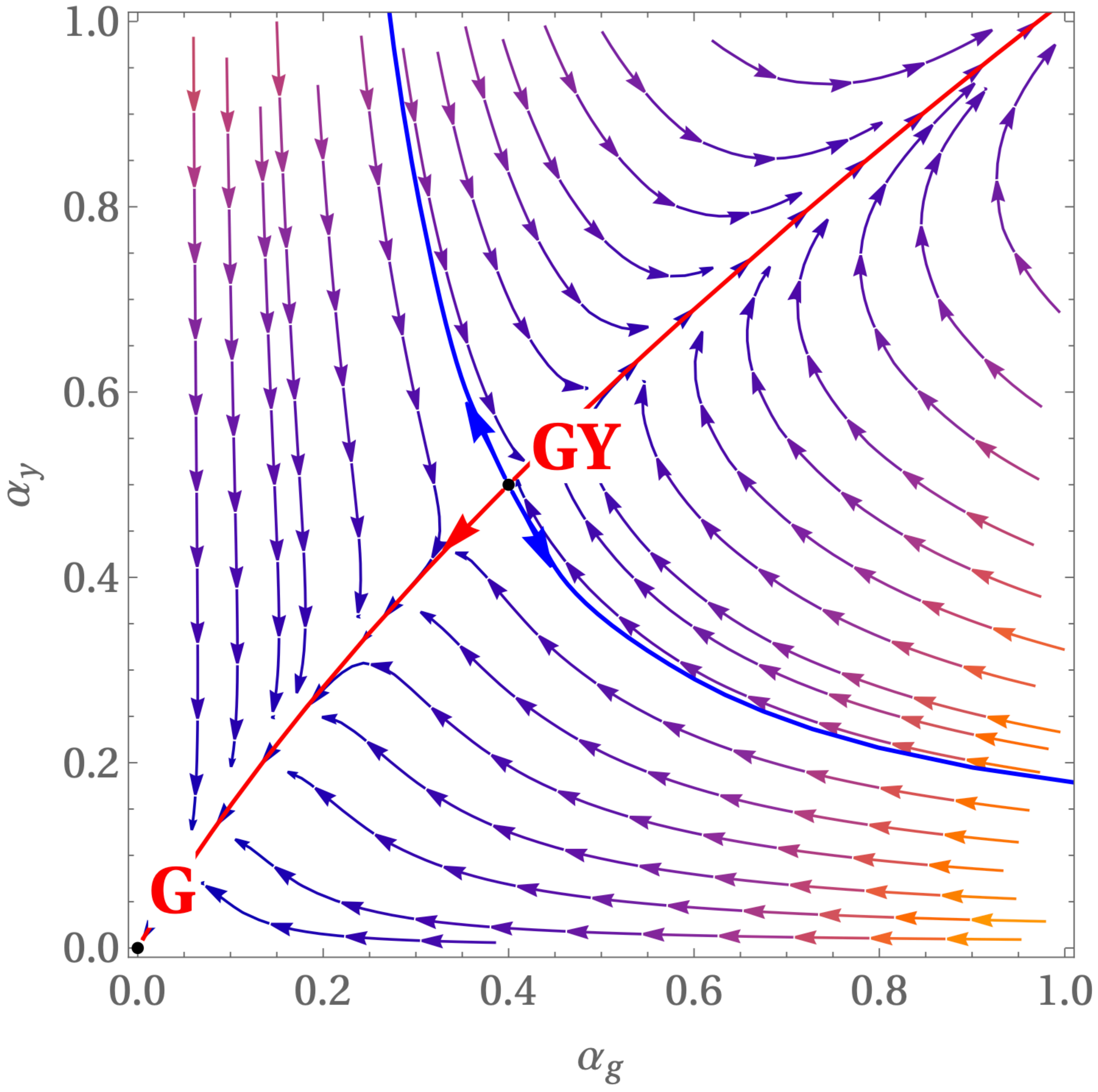
2D: Graphene, ...

4plusD: extra dimensions, string theory, ...

Fundamental theory

Wilson: A fundamental theory has a UV fixed point

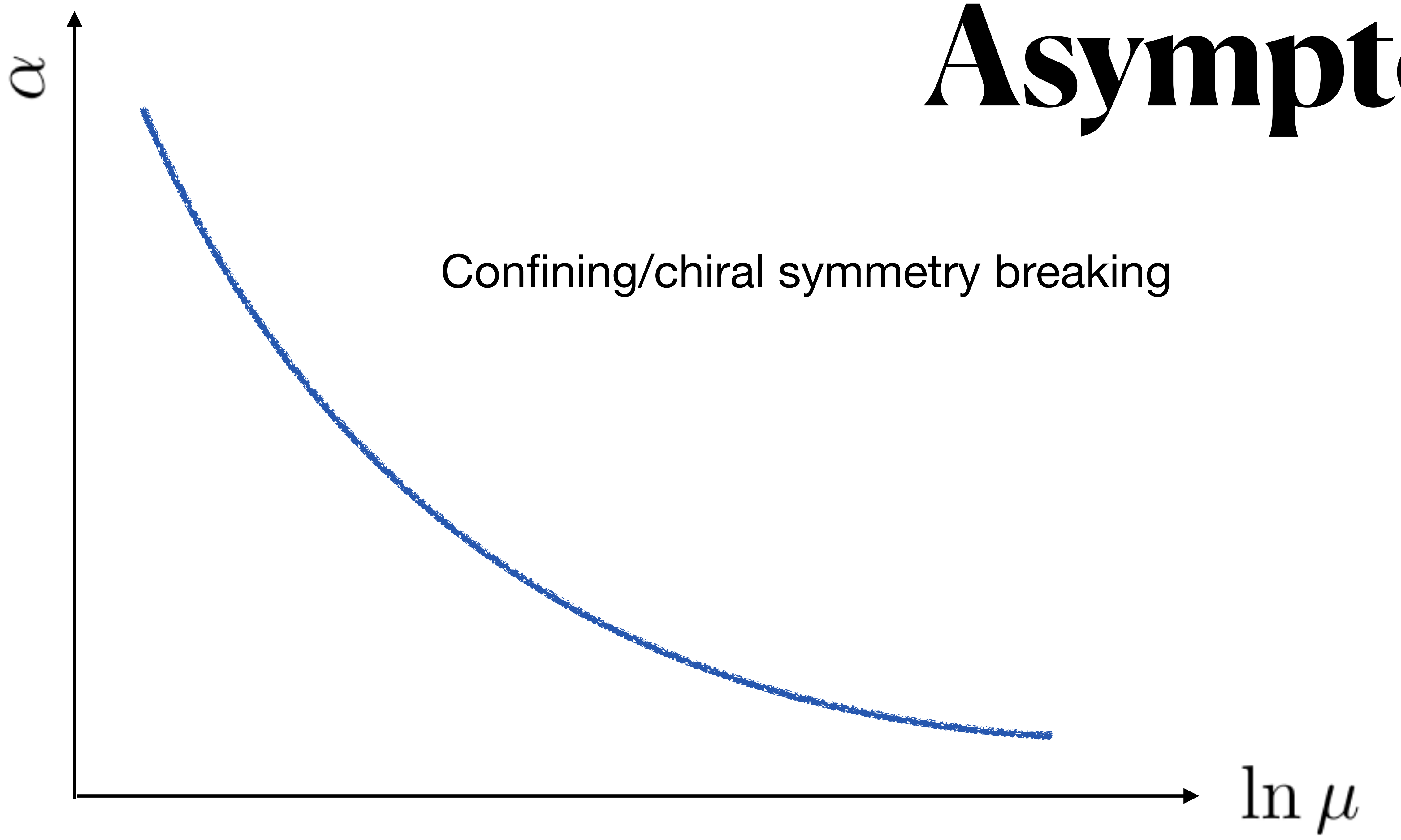
- Short distance conformality
- Continuum limit well defined
- Complete UV fixed point
- Smaller critical surface dim. = more IR predictiveness
- Mass operators relevant only for IR

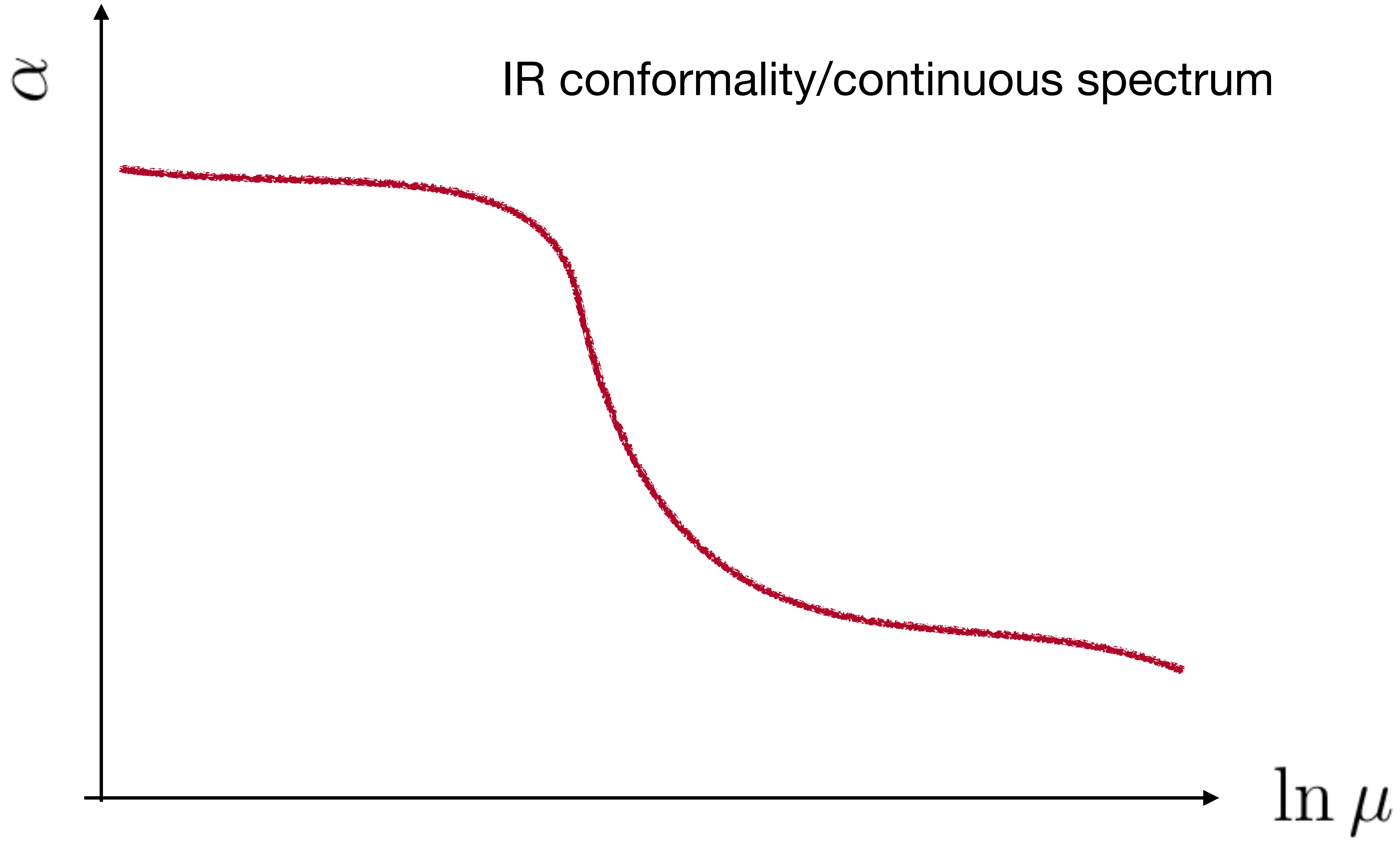


The Standard Model is not a fundamental theory

Bednyakov, Mukhaeva'23

Asymptotic Freedom





Trivial Fixed point

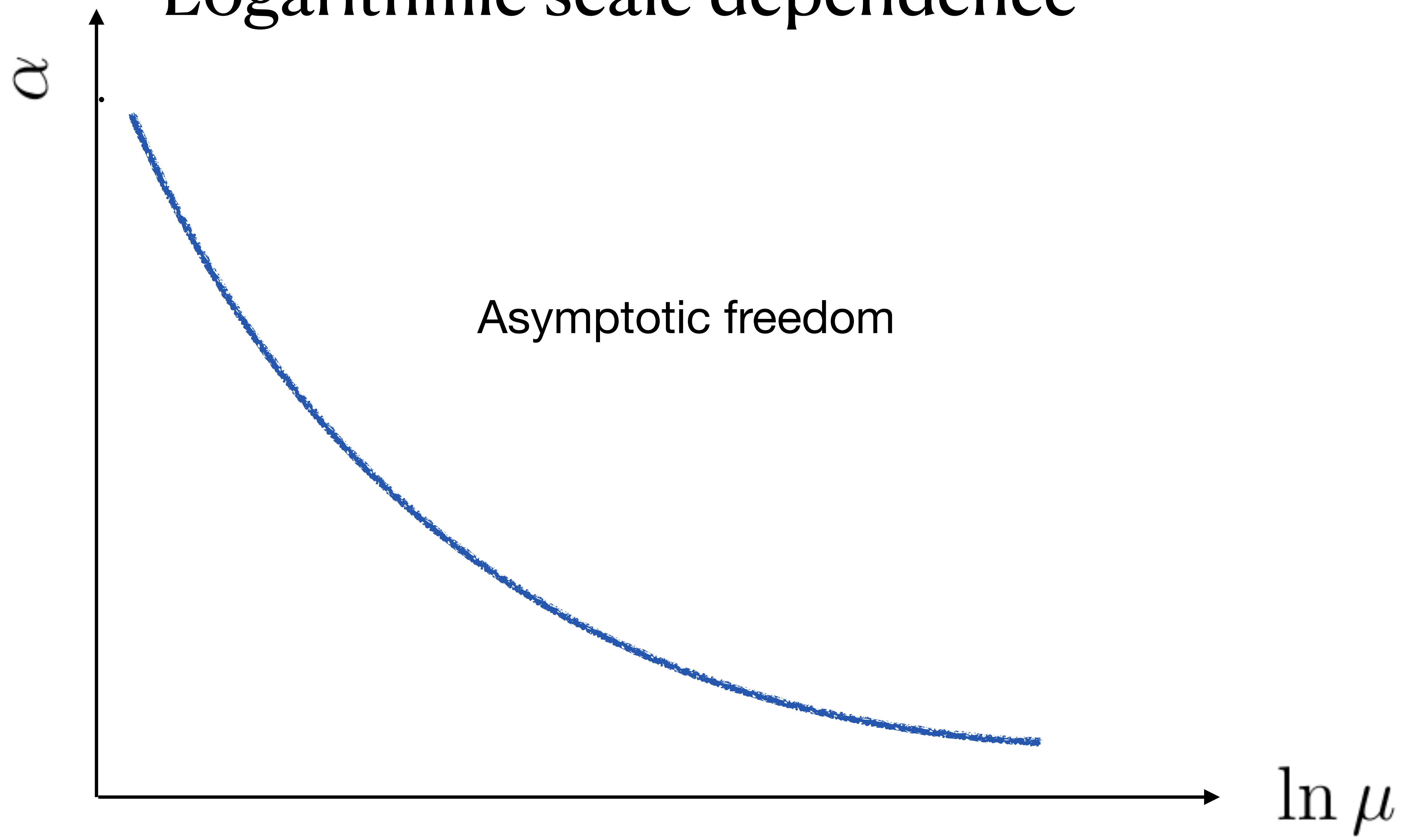
- Non-interacting in the UV
- UV logarithmic approach
- Perturbation theory in UV
- IR conformal or dyn. scale

Asymptotic Safety

Wilson: A fundamental theory has a UV fixed point

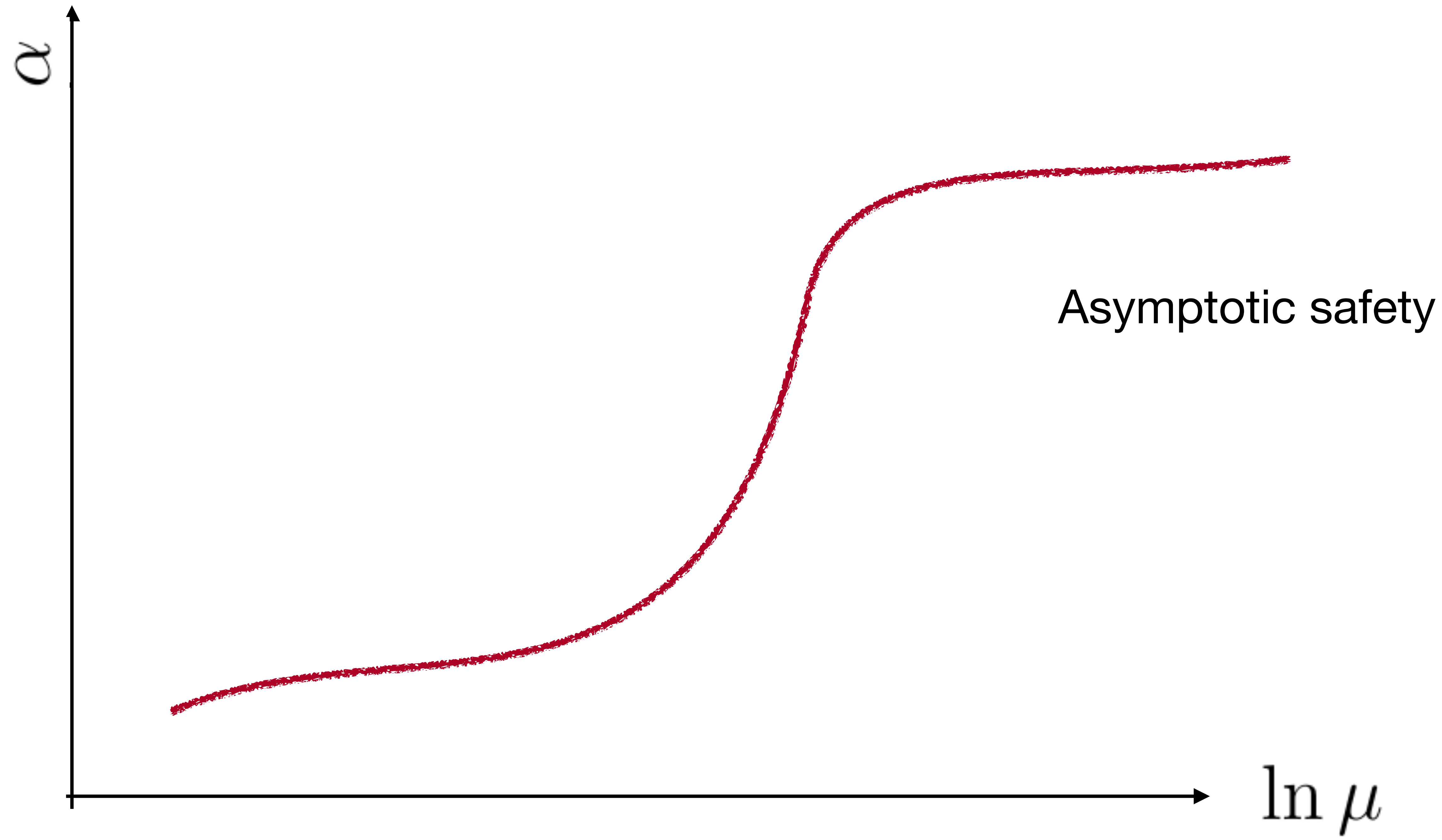
Trivial Fixed point

- Non-interacting in the UV
- Logarithmic scale dependence



Interacting Fixed point

- Integrating in the UV
- Power-law



Does a theory like this exist?

Litim–Sannino Model

field	$SU(N_c)$	$U_L(N_f)$	$U_R(N_f)$
ψ_L	N_c	N_f	1
ψ_R	N_c	1	N_f
H	1	N_f	$\bar{N}_f$

gauge sector (QCD)

$$\mathcal{L} = -\frac{1}{4}F^{A\mu\nu}F^{A}_{\mu\nu} + \mathcal{L}_{gf} + \mathcal{L}_{gh} + \mathrm{Tr}(\bar{\psi} i \hat{D} \psi)$$

$$+ \mathrm{Tr}(\partial^\mu H^\dagger \partial_\mu H) - y\,\mathrm{Tr}[\bar{\psi}(H\mathcal{P}_R + H^\dagger \mathcal{P}_L)\psi]$$

Yukawa (chiral)

$$- m^2\,\mathrm{Tr}(H^\dagger H) - u\,\mathrm{Tr}((H^\dagger H)^2) - v(\mathrm{Tr}(H^\dagger H))^2$$

single trace

double trace

- interacting fixed points under perturbative control

Litim, Sannino (2014)

Perturbative control

- Veneziano limit: $N_{f,c} \to \infty$ but $N_f/N_c = \text{const}$

- 't Hooft couplings:

$$\alpha_g = \frac{N_c\, g^2}{(4\pi)^2} \qquad \alpha_y = \frac{N_c\, y^2}{(4\pi)^2} \qquad \alpha_u = \frac{N_f\, u}{(4\pi)^2} \qquad \alpha_v = \frac{N_f^2\, v}{(4\pi)^2}$$

- Small expansion parameter: $\epsilon = \frac{N_f}{N_c} - \frac{11}{2}$ $\qquad -\frac{11}{2} < \epsilon < \infty \qquad |\epsilon| \ll 1$
- 1 loop gauge coefficient $\beta_g = \alpha_g^2\, \left[\frac{4}{3}\epsilon + \mathcal{O}(\alpha^1)\right]$
- Conformal expansion: $\alpha^* = \epsilon\, a_{\text{LO}} + \epsilon^2\, a_{\text{NLO}} + \epsilon^3\, a_{\text{NNLO}} + \ldots$

211	322	433	433
Litim, Sannino (2014)	Litim, Bond, Medina, Steudtner (2017)	Litim, Riyaz, Stamou, Steudtner (2023)	Bednyakov, Mukhaeva (2023)

n loop gauge
m loop Yukawa
l loop scalar
$\equiv (n, m, l)$

Conformal Window

How to probe the UV conformal window

I. *Directly from beta functions* $\beta_{g,y,u,v}$ ϵ_{strict}

- fixed point values $\alpha^*_{g,y,u,v}(\epsilon)$ from $\beta_{g,y,u,v} = 0$
  - coupling $0 < |\alpha^*| \lesssim 1$ [Weinberg, 1978]
  - vacuum stability $\alpha^*_u > 0$ and $\alpha^*_u + \alpha^*_v > 0$ [A.J. Paterson, 1980]
- critical exponents ϑ_i as eigenvalues of stability matrix $M_{xx'} = \left.\frac{\partial \beta_x}{\partial \alpha_{x'}}\right|_{\alpha=\alpha^*}$

$$\beta_i = \sum_j M_{ij}\,(\alpha_j - \alpha_j^*) + \text{subleading}$$

Relevant

$$\vartheta_1 < 0 < \vartheta_{2,3,4}$$

Irelevant

II. ϵ*-expansion of* $\alpha_i^*(\epsilon)$ *and* $\vartheta_i(\epsilon)$ *(series is exact up to third order)*

ϵ_{subl}

III. *Strong coupling constraints* $\alpha_x^* \gtrsim 1$

Investigation of UV conformal window

Bednyakov, Mukhaeva'24

- Gauge coupling

$$\alpha_g^* = 0.456\epsilon + 0.781\epsilon^2 + 6.610\epsilon^3 + O(\epsilon^4),$$
$$\alpha_y^* = 0.211\epsilon + 0.508\epsilon^2 + 3.322\epsilon^3 + O(\epsilon^4),$$
$$\alpha_u^* = 0.200\epsilon + 0.440\epsilon^2 + 2.693\epsilon^3 + O(\epsilon^4),$$
$$\alpha_v^* = -0.137\epsilon - 0.632\epsilon^2 - 4.313\epsilon^3 + O(\epsilon^4)$$

$$\epsilon_{strict} \sim (0.117 - 0.457)$$
$$\epsilon_{subl} \sim (0.117 - 0.363)$$

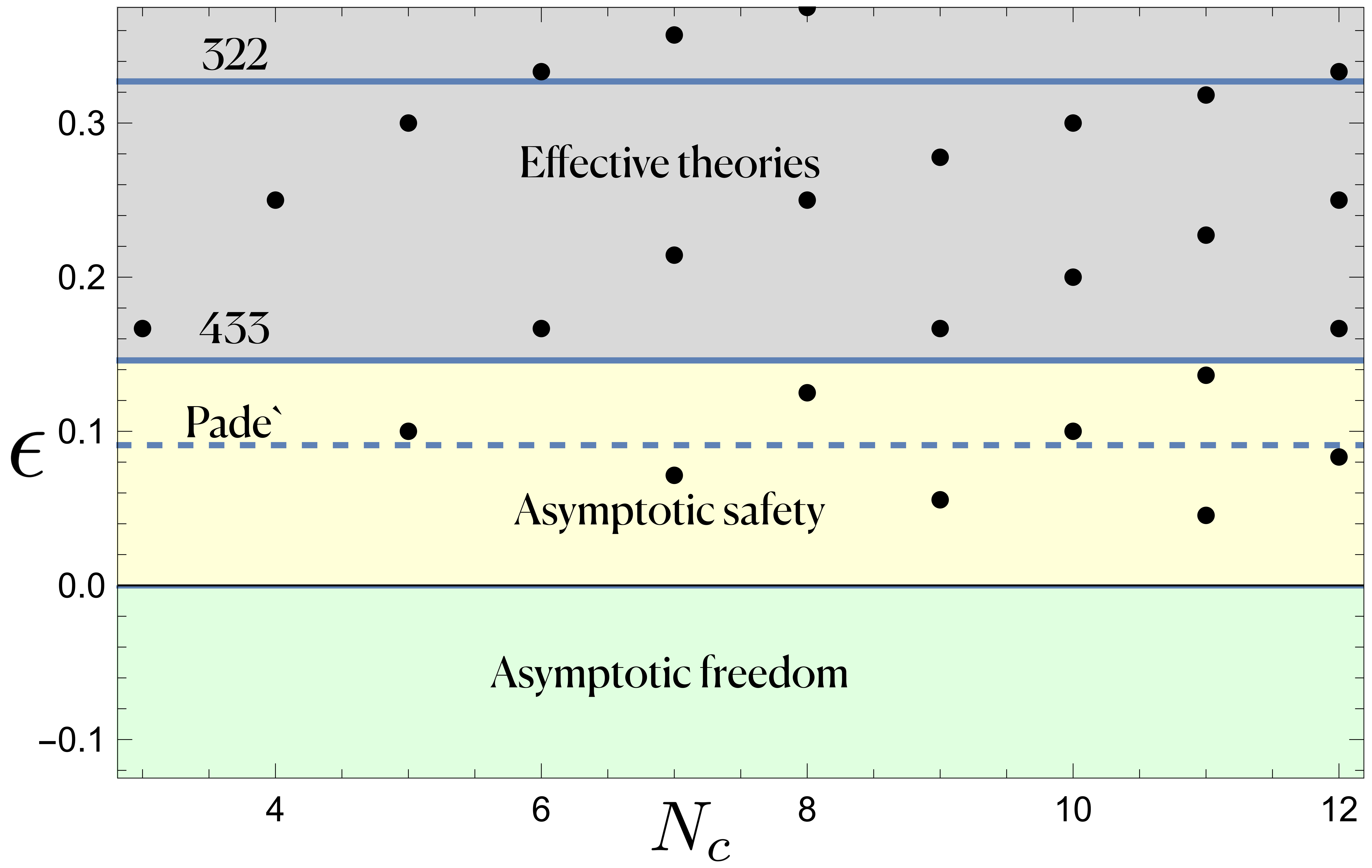
- Scaling exponents

$$\theta_1 = -0.608\epsilon^2 + 0.707\epsilon^3 + 6.947\epsilon^4 + O(\epsilon^5)$$
$$\theta_2 = 2.737\epsilon + 6.676\epsilon^2 + 22.120\epsilon^3 + O(\epsilon^4)$$
$$\theta_3 = 2.941\epsilon + 1.041\epsilon^2 + 5.137\epsilon^3 + O(\epsilon^4)$$
$$\theta_4 = 4.039\epsilon + 9.107\epsilon^2 + 38.646\epsilon^3 + O(\epsilon^4)$$

$$\epsilon_{strict} \sim (0.091 - 0.249)$$
$$\epsilon_{subl} \sim (0.091 - 0.234)$$

$N_c = 2n \qquad N_f = 11n + j \qquad \epsilon = \frac{j}{N_c}$

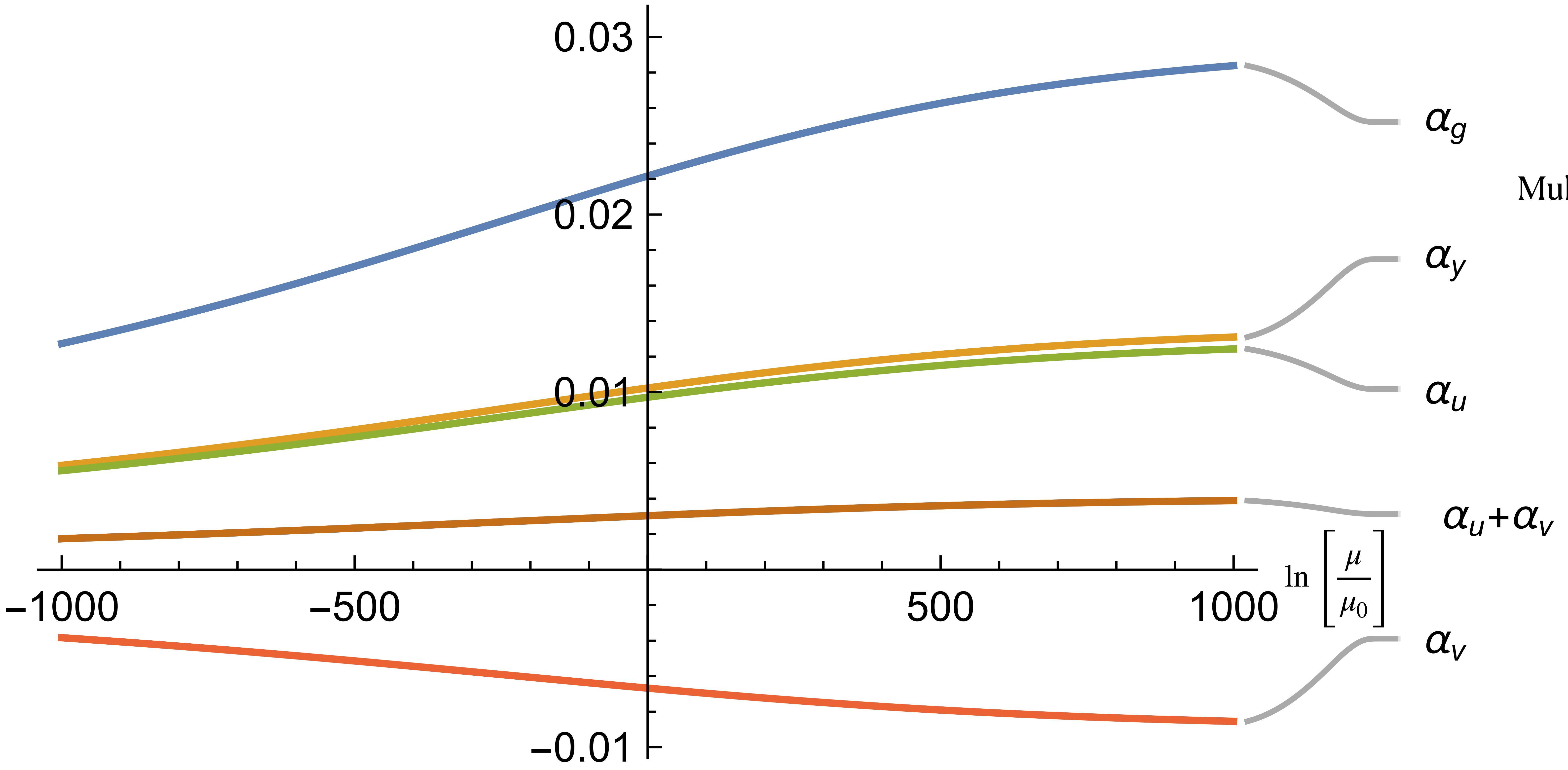
$N_c = 2m+1 \quad N_f = 11m + 5 + j \quad \epsilon = \frac{j - \frac{1}{2}}{N_c}$



Safe QFTs: (Nc,Nf) =(5,28), (7,39), (8,45), (9,50), (10,56), (11,61), (11,62), (12,67),....

Complete asymptotic safety

Gauge + fermion + scalars theories can be fundamental at any energy scales



Mukhaeva, in prep.

Bond, Litim'16

Scalars are needed to make the theory fundamental

Summary & Outlook

- Computed all β_x in 433-scheme
- Computed α^* and θ_i up to $O(\epsilon^3)$
- Updated conformal window
- set $\epsilon_{\rm max} \approx 0.09 \pm 0.01$

- Tree-level vacuum stability → effective potential?
- What about 544?

Thanks for attention!