CP-violation and renormalization group effects in the Higgs alignment limit of the MSSM

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The SM of particle physics works extremely well. The discovery of Higgs boson at the LHC by ATLAS and CMS (CERN) in 2012
But it has problems

- 'SM fine-tuning problem'
- sources of CP-violation
- DM candidate ...

SM is an effective theory at low-energy. New physics, non-minimal Higgs sector: the observed Higgs boson is SM-like

- $m = 125.36 \pm 0.14$ GeV
- spin $J = 0$ (99.9% CL)
- CP-even (4σ)

$$h_{125} \to \cos\theta h_{\rm even} + \sin\theta h_{\rm odd} \tag{1}$$

ATLAS:	$tth, h \to \gamma\gamma$	$\theta < 43^\circ$	2004.04545
CMS:	$h \to \tau\tau$	$\theta < 36^\circ$	HIG-20-006

- Γ_{125}= 4.2 MeV
- coupling accuracy is of $\sim 8\%$ ($W^\pm, Z$), 10%–20% (f).

In any SM extension, Higgs boson properties (within the precision of experiment) must be

$$y^{\rm THDM}/y^{\rm SM} \simeq 1 \qquad \text{Higgs alignment limit} \tag{2}$$

However, there is still a room of SM deviations in
– self-interations
– interactions with light quarks and leptons
– CP-violating interactions (unambiguous evidence of non-standard Higgs sector)

Among BSMs the most popular and investigated theory is the MSSM (THDM-II) – 5 Higgs bosons

- no direct evidence has been found but
- many free parameters → benchmark scenarios or BPs for the LHC searches
- model-dependence contraints → the standard MSSM benchmark scenarios are excluded
- non-stardard benchmark scenarios are still viable

THDM: Higgs sector

$SU(2)$ doublets with $Y_i = 1$

$$\Phi_1 = \left(\begin{array}{c} -i\omega_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \eta_1 + i\chi_1) \end{array} \right), \qquad \Phi_2 = e^{i\xi} \left(\begin{array}{c} -i\omega_2^+ \\ \frac{1}{\sqrt{2}}(v_2 e^{i\zeta} + \eta_2 + i\chi_2) \end{array} \right), \tag{3}$$

Achmetzjanova E., Dolgopolov M., Dubinin M., Phys.Rev.D 71, 2005, 075008

$$\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \left(\begin{array}{c} 0 \\ v_1 \end{array} \right), \qquad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \left(\begin{array}{c} 0 \\ v_2 e^{i\theta} \end{array} \right), \qquad \theta = \xi + \zeta \tag{4}$$

$$v^2 = v_1^2 + v_2^2 = (246 \text{ GeV})^2, \qquad \tan\beta = \frac{v_2}{v_1} \tag{5}$$

$$\mathcal{L}_H = (\mathcal{D}_\nu \Phi_1)^\dagger \mathcal{D}^\nu \Phi_1 + (\mathcal{D}_\nu \Phi_2)^\dagger \mathcal{D}^\nu \Phi_2 - U(\Phi_1, \Phi_2) \tag{6}$$

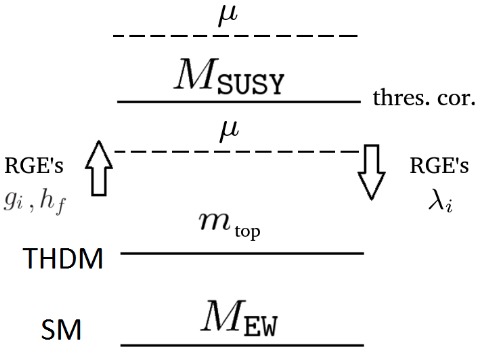
$SU(2) \times U(1)$ renormalizable potential

$$\begin{aligned} U = & -\mu_1^2(\Phi_1^\dagger \Phi_1) - \mu_2^2(\Phi_2^\dagger \Phi_2) - [\mu_{12}{}^2(\Phi_1^\dagger \Phi_2) + h.c.] \\ & + \lambda_1(\Phi_1^\dagger \Phi_1)^2 + \lambda_2(\Phi_2^\dagger \Phi_2)^2 + \lambda_3(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \\ & + [\lambda_5/2(\Phi_1^\dagger \Phi_2)(\Phi_1^\dagger \Phi_2) + \lambda_6(\Phi_1^\dagger \Phi_1)(\Phi_1^\dagger \Phi_2) + \lambda_7(\Phi_2^\dagger \Phi_2)(\Phi_1^\dagger \Phi_2) + h.c.] \end{aligned} \tag{7}$$

$$\mathcal{D}_\nu \Phi = \left(\partial_\nu - i\frac{g_2}{2}\sigma^a A_\nu^a - i\frac{g_1}{2} B_\nu \right) \Phi, \tag{8}$$

Loop level

Different methods, codes, assumptions



M_{SUSY} scale

$$\lambda_i = \lambda_i^{\text{tree}} + \Delta\lambda_i^{\text{thr}}$$

λ_i^{tree}

$$\lambda_{1,2}^{\text{tree}} = \frac{g_2^2 + g_1^2}{4}, \quad \lambda_3^{\text{tree}} = \frac{g_2^2 - g_1^2}{4}$$

$$\lambda_4^{\text{tree}} = -\frac{g_2^2}{2}, \quad \lambda_{5,6,7}^{\text{tree}} = 0$$

$\Delta\lambda_i^{\text{thr}}$

Coleman, Weinberg, Phys. Rev. D7, 6 (1973)

Haber, Hempfling, Phys. Rev. D48 (1993) 4280

$$U_{\text{CW}} = U^0 + \frac{3}{32\pi^2}\text{tr}\mathcal{M}^4\left(\ln\frac{\mathcal{M}^2}{\sigma^2} - \frac{3}{2}\right)$$

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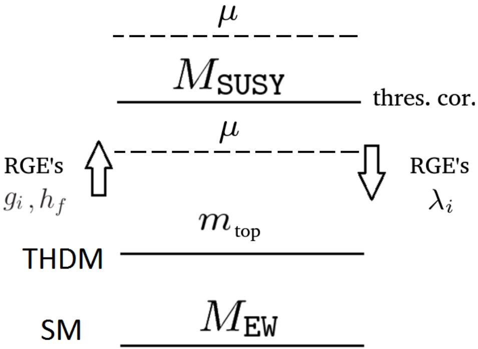
Loop level

Different methods, codes, assumptions



$M_{\rm SUSY}$ scale

$$\lambda_i = \lambda_i^{\rm tree} + \Delta\lambda_i^{\rm thr}$$

$\lambda_i^{\rm tree}$

$$\lambda_{1,2}^{\rm tree} = \frac{g_2^2 + g_1^2}{4}, \quad \lambda_3^{\rm tree} = \frac{g_2^2 - g_1^2}{4}$$

$$\lambda_4^{\rm tree} = -\frac{g_2^2}{2}, \quad \lambda_{5,6,7}^{\rm tree} = 0$$

$\Delta\lambda_i^{\rm thr}$

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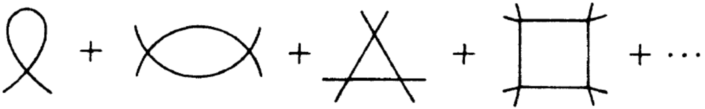
$$U_{\rm CW} = U^0 + \frac{3}{32\pi^2}{\rm tr}\mathcal{M}^4\left(\ln\frac{\mathcal{M}^2}{\sigma^2} - \frac{3}{2}\right)$$



$\mathcal{O}(\Phi^4)$: $\quad 2|m_{\rm top}\mu| < M_S^2, \quad 2|m_{\rm top}A| < M_S^2,$ where $A_t = A_b = A$

Carena *et al.*, Phys. Lett. B 355, 1995

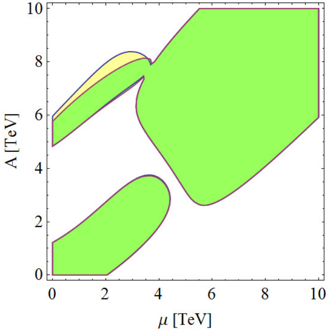
$$
\begin{aligned}
U^{(6)} &= \kappa_1(\Phi_1^\dagger\Phi_1)^3 + \kappa_2(\Phi_2^\dagger\Phi_2)^3 + \kappa_3(\Phi_1^\dagger\Phi_1)^2(\Phi_2^\dagger\Phi_2) + \kappa_4(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)^2 + \\
&+ \kappa_5(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \kappa_6(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \\
&+ [\kappa_7(\Phi_1^\dagger\Phi_2)^3 + \kappa_8(\Phi_1^\dagger\Phi_1)^2(\Phi_1^\dagger\Phi_2) + \kappa_9(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2)^2 + \\
&+ \kappa_{10}(\Phi_1^\dagger\Phi_2)^2(\Phi_2^\dagger\Phi_2) + \kappa_{11}(\Phi_1^\dagger\Phi_2)^2(\Phi_2^\dagger\Phi_1) + \kappa_{12}(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_2)^2 + \\
&+ \kappa_{13}(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_2) + h.c.].
\end{aligned}
$$

Threshold corrections to κ_i in Dubinin M., Petrova E., Phys.Rev.D 95, 2017, 055021

$|\kappa_i| \sim |\lambda_j|$ at

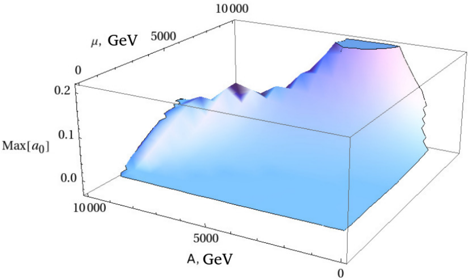
$$
\begin{aligned}
&|\mu| m_t \cot\beta, \quad |\mu| m_b \tan\beta, \quad |A_t| m_t, \quad |A_b| m_b \quad \geq M_S^2 \\
&|\mu A_t| m_t^2 \cot\beta, \quad |\mu A_b| m_b^2 \tan\beta \quad \geq M_S^4
\end{aligned}
$$

– large A, μ regime



M_S=1.5 TeV, tan β=1

Dubinin, Fedotova, EPJ WoC 158, 2017, 02005



$|a_0| \leq 1/2$

M_S=2 TeV, tan β=3,
m_A=28 GeV, $\sqrt{s}$=8 TeV

Dubinin, Fedotova, JETP, Vol. 131, 6, 2020

Perturbative unitarity analysis is based on the approach of Krauss, F. Staub, Phys.Rev. D98, no.1, 015041 (2018)

RG-effects to the dim-6 operators

Let's consider $\kappa_1(\Phi_1^\dagger\Phi_1)^3$

$$\begin{aligned}
\kappa_1 &= \frac{h_D^6}{32M_S^2\pi^2}\left(2-\frac{3|A_D|^2}{M_S^2}+\frac{|A_D|^4}{M_S^4}-\frac{|A_D|^6}{10M_S^6}\right)\\
&- h_D^4\frac{g_1^2+g_2^2}{128M_S^2\pi^2}\left(3-3\frac{|A_D|^2}{M_S^2}+\frac{|A_D|^4}{2M_S^4}\right)\\
&+ \frac{h_D^2}{512M_S^2\pi^2}\left(\frac{5}{3}g_1^4+2g_1^2g_2^2+3g_2^4\right)\left(1-\frac{|A_D|^2}{2M_S^2}\right)\\
&- h_U^6\frac{|\mu|^6}{320M_S^8\pi^2}+h_U^4\frac{(g_1^2+g_2^2)|\mu|^4}{256M_S^6\pi^2}\\
&- h_U^2\frac{(17g_1^4-6g_1^2g_2^2+9g_2^4)|\mu|^2}{3072M_S^4\pi^2}+\frac{g_1^2}{1024M_S^2\pi^2}(g_1^4-g_2^4)
\end{aligned}$$

RG-improved effective potentials in non-renormalizable theories

Kazakov D.I., Iakhibbaev R.M., Tolkachev D.M., 2209.08019v2 [hep-th]

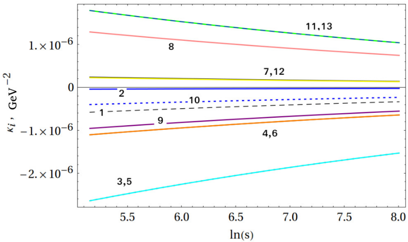
$$V(\phi)=g\phi^6/6!$$

– divergences are subtracted some way,

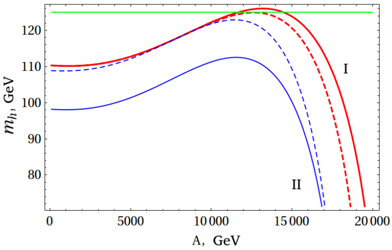
– no analytic expressions for this type of potential, only numeric estimations

RG-corrections to $g_{1,2}$, $h_{U,D}$

Haber H. E., Hempfling R., Phys. Rev. Lett. 1991. 66. P. 1815; Phys. Rev. D. 1993. 48. P. 4280.



$m_t \leq \sqrt{s} \leq M_{\text{SUSY}}$
$m_A \sim M_{\text{EW}}$, $A_{t,b}$=11 TeV



I: $m_A = M_{\text{SUSY}}$
II: m_A=200 GeV

κ_i $(i = 1, \ldots, 13)$ as functions of $\ln s$ and the Higgs mass as a function of $A_{t,b} = A$ with $\kappa_i(M_{\text{SUSY}})$ (solid lines) or $\kappa_i(M_{top})$ (dashed lines). Here M_{SUSY}=3 TeV, $\tan\beta$=5, μ=15 TeV

Mass states $h_1, h_2, h_3, H^{\pm}$

$$h_{\rm SM} = h_1, \qquad g = y^{\rm THDM}/y^{\rm SM} \simeq 1 \tag{9}$$

g	CP-conservation	CP-violation (θ=0)
h_1uu	c_α/s_β	$(s_\alpha a_{21} + c_\alpha a_{11} - i c_\beta a_{31}\gamma_5)/s_\beta$
h_1dd	$-s_\alpha/c_\beta$	$(c_\alpha a_{21} - s_\alpha a_{11} - i s_\beta a_{31}\gamma_5)/c_\beta$
h_1VV	$s_{\beta-\alpha}$	$c_{\alpha-\beta}a_{21} - s_{\alpha-\beta}a_{11}$

$$a_{ij} = a^{'}_{ij}/n_j, \qquad n_j = k_j\sqrt{a^{\prime 2}_{1j} + a^{\prime 2}_{2j} + a^{\prime 2}_{3j}}, \qquad k_j = \pm 1 \tag{10}$$

$$\begin{aligned}
a^{'}_{11} &= [(m^2_H - m^2_{h_1})(m^2_A - m^2_{h_1}) - c^2_2], a^{'}_{12} = -c_1c_2, a^{'}_{13} = -c_1(m^2_H - m^2_{h_3}),\\
a^{'}_{21} &= c_1c_2, a^{'}_{22} = -[(m^2_h - m^2_{h_2})(m^2_A - m^2_{h_2}) - c^2_1], a^{'}_{23} = -c_2(m^2_h - m^2_{h_3}),\\
a^{'}_{31} &= -c_1(m^2_H - m^2_{h_1}), a^{'}_{32} = c_2(m^2_h - m^2_{h_2}), a^{'}_{33} = (m^2_h - m^2_{h_3})(m^2_H - m^2_{h_3})
\end{aligned}$$

$h, H, A, H^{\pm}$ – states after rotations by angles α, β

Dubinin M.N., Semenov A.V., Eur. Phys. J. C 28, 2003, 223

$$g_{h_1 uu} = \frac{s_\alpha a_{21} + c_\alpha a_{11} - i c_\beta a_{31} \gamma_5}{s_\beta} \simeq 1 \qquad (11)$$

One can see that $a_{31} \simeq 0 \quad \rightarrow \quad a_{11} \simeq \sin(\beta - \alpha), \qquad a_{21} \simeq \cos(\beta - \alpha)$

Alignment	The general case	Only $\mathcal{O}(\Phi^4)$
I	$c_1 \simeq 0$	$\mathrm{Im}\mu_{12}^2 \simeq 0$
	$\beta - \alpha \simeq \pi/2$	
	$k_1 \xi_1^{\mathrm{I}} = 1$	
II	$m_H^2 \simeq m_{h_1}^2$	
	$c_2 \simeq -c_1 \tan(\beta - \alpha)$	$\mathrm{Im}\mu_{12}^2 \simeq 0$
	$k_1 \xi_1^{\mathrm{II}} \xi_2^{\mathrm{II}} \xi_3^{\mathrm{II}} = 1$	

Numerical estimations:

- The Higgs alignment limit I is very similar to the one in the CP-conserving limit. It is realized at $m_{H^\pm} \geq 500$ GeV
- The Higgs alignment limit II is realized at $m_{H^\pm} \sim \mathcal{O}(100)$ GeV, $M_{\mathrm{SUSY}} \geq 2$ TeV and large $\tan\beta$. Nevertheless, it is hardly believed that the case II is viable.

CP-vioalating interactions

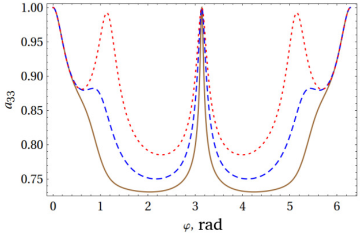
$$a^{\mathrm{I}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & a^{\mathrm{I}}_{22} & a^{\mathrm{I}}_{23} \\ 0 & a^{\mathrm{I}}_{32} & a^{\mathrm{I}}_{33} \end{pmatrix}, \qquad a^{\mathrm{II}} = \begin{pmatrix} \sin(\beta-\alpha) & a^{\mathrm{II}}_{12} & a^{\mathrm{II}}_{13} \\ \cos(\beta-\alpha) & a^{\mathrm{II}}_{22} & a^{\mathrm{II}}_{23} \\ 0 & a^{\mathrm{II}}_{32} & a^{\mathrm{II}}_{33} \end{pmatrix}, \quad (12)$$

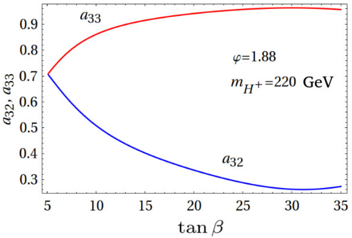
Interactions	Alignment I	Alignment II
h_2uu	$-a^{\mathrm{I}}_{22}\cot\beta$	$(s_\alpha a^{\mathrm{II}}_{22} + c_\alpha a^{\mathrm{II}}_{12} - ic_\beta a^{\mathrm{II}}_{32}\gamma_5)/s_\beta$
h_3uu	$-ia^{\mathrm{I}}_{33}\cot\beta\gamma_5$	$(s_\alpha a^{\mathrm{II}}_{23} + c_\alpha a^{\mathrm{II}}_{13} - ic_\beta a^{\mathrm{II}}_{33}\gamma_5)/s_\beta$
h_2dd	$a^{\mathrm{I}}_{22}\tan\beta$	$(c_\alpha a^{\mathrm{II}}_{22} - s_\alpha a^{\mathrm{II}}_{12} - is_\beta a^{\mathrm{II}}_{32}\gamma_5)/c_\beta$
h_3dd	$-ia^{\mathrm{I}}_{33}\tan\beta\gamma_5$	$(c_\alpha a^{\mathrm{II}}_{23} - s_\alpha a^{\mathrm{II}}_{13} - is_\beta a^{\mathrm{II}}_{33}\gamma_5)/c_\beta$
h_2VV	0	$c_{\beta-\alpha}a^{\mathrm{II}}_{22} + s_{\beta-\alpha}a^{\mathrm{II}}_{12}$
h_3VV	0	$c_{\beta-\alpha}a^{\mathrm{II}}_{23} + s_{\beta-\alpha}a^{\mathrm{II}}_{13}$

CP-violating effects can be observed in interactions of SM particles with

- h_3 in the case I,
- h_2, h_3 in the case II

and depend on the values of a_{32}, a_{33}





$m_{H^\pm}$=0.5 TeV (brown), 1 TeV (blue) or 3 TeV (red), $M_{\rm SUSY} = 2$ TeV, $\tan\beta = 5$

$M_{\rm SUSY}$	2 TeV	5 TeV		10 TeV
tgβ	5	5	10	10
φ_{al},°	66.1	97.5	94.4	105.8
$m_{H^\pm}^{\min}$, GeV	800	500	300	200
$\|A_{33}\|$	0.33	0.06	0.02	0.20

Higgs alignment limit I with $m_{h_1} = 125 \pm 3$ GeV, $|\mu| = 4M_{\rm SUSY}$, $|A_{t,b}| = 2M_{\rm SUSY}$

Summary

- Higgs sector with CP-violation and new model regime of large A, μ. For such a regime, radiative corrections to the dimension-six operators $\mathcal{O}(\Phi^6)$ that inevitably arise at the loop level of effective Higgs potential decomposition become considerable

- It is found that the renormalization group effects to additional radiative corrections improve predictions for the mass of the SM-like Higgs boson by about 2% for the Higgs alignment limit I and by about 9% for the Higgs alignment limit II

- If h_1 is SM-like, Higgs alignment limit is analyzed, two cases are obtained (I and II)

- Higgs alignment limit I is realized at
  – intermidiate $\tan\beta$ and M_{SUSY} in the decoupling regime,
  – intermidiate $\tan\beta$ and $M_{\mathrm{SUSY}} \sim 10$ TeV at $m_{H^\pm} \sim M_{\mathrm{EW}}$

- CP-violating effects can be found in processes with $h_{2,3}$ interactions with SM particles and model regimes are proposed

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Thank you for your attention!

$$
\begin{aligned}
a'^{\rm I}_{22} &\simeq m^2_{h_2} - m^2_A, \qquad a'^{\rm I}_{23} \simeq -c^{\rm I}_2, \qquad a'^{\rm I}_{32} \simeq c^{\rm I}_2, \qquad a'^{\rm I}_{33} \simeq m^2_H - m^2_{h_3}, \\
n^{\rm I}_2 &= k_2 \xi^{\rm I}_2 \sqrt{(m^2_A - m^2_{h_2})^2 + (c^{\rm I}_2)^2}, \qquad n^{\rm I}_3 = k_3 \xi^{\rm I}_3 \sqrt{(m^2_H - m^2_{h_3})^2 + (c^{\rm I}_2)^2}, \qquad (13)
\end{aligned}
$$

$$
\begin{aligned}
a'^{\rm II}_{12} &\simeq c^2_1 \tan(\beta - \alpha), \qquad a'^{\rm II}_{22} = a'_{22}, \qquad a'^{\rm II}_{32} \simeq -c_1 \tan(\beta - \alpha)(m^2_h - m^2_{h_2}), \\
a'^{\rm II}_{13} &= a_{13}, \qquad a'^{\rm II}_{23} \simeq c_1 \tan(\beta - \alpha)(m^2_h - m^2_{h_3}), \qquad a'^{\rm II}_{33} = a'_{33}, \qquad (14)
\end{aligned}
$$

$$
c^{\rm I}_2 \simeq v^2({\rm Im}\lambda_7 \tan\beta - {\rm Im}\lambda_6 \cot\beta)/2, \qquad c^{\rm II}_2 \simeq -c^{\rm II}_1 \tan(\beta - \alpha), \quad (15)
$$

$$
\xi^{\rm I}_1 = \begin{cases} +1, \text{ если } a'_{11} > 0, \\ -1, \text{ если } a'_{11} < 0, \end{cases} \qquad \xi^{\rm I}_{2,3} = \begin{cases} +1, \text{ если } m^2_h > m^2_{h_{2,3}}, \\ -1, \text{ если } m^2_h < m^2_{h_{2,3}}, \end{cases} \qquad (16)
$$

$$
\xi^{\rm II}_{1,2} = \begin{cases} +1, \text{ если } c_{1,2} > 0, \\ -1, \text{ если } c_{1,2} < 0, \end{cases} \qquad \xi^{\rm II}_3 = \begin{cases} +1, \text{ если } (\beta - \alpha) \in (0, \pi/2), \\ -1, \text{ если } (\beta - \alpha) \in (\pi/2, \pi). \end{cases} \qquad (17)
$$

$$\begin{aligned}
\Delta\mathcal{M}^2_{11} &= -v^2(\Delta\lambda_1 c^2_\beta + \mathtt{Re}\Delta\lambda_5 s^2_\beta + \mathtt{Re}\Delta\lambda_6 s_{2\beta}) + \\
&+ v^4[3\kappa_1 c^4_\beta + 4\mathrm{Re}\kappa_8 c^3_\beta s_\beta + (\kappa_3 + \kappa_5 + 3\mathrm{Re}\kappa_9) c^2_\beta s^2_\beta + \\
&+ (3\mathrm{Re}\kappa_7 + \mathrm{Re}\kappa_{11} + \mathrm{Re}\kappa_{13}) c_\beta s^3_\beta + \mathrm{Re}\kappa_{10} s^4_\beta],
\end{aligned} \tag{18}$$

$$\begin{aligned}
\Delta\mathcal{M}^2_{22} &= -v^2(\Delta\lambda_2 s^2_\beta + \mathtt{Re}\Delta\lambda_5 c^2_\beta + \mathtt{Re}\Delta\lambda_7 s_{2\beta}) + \\
&+ v^4[\mathrm{Re}\kappa_9 c^4_\beta + (3\mathrm{Re}\kappa_7 + \mathrm{Re}\kappa_{11} + \mathrm{Re}\kappa_{13}) c^3_\beta s_\beta + \\
&+ (\kappa_4 + \kappa_6 + 3\mathrm{Re}\kappa_{10}) c^2_\beta s^2_\beta + 4\mathrm{Re}\kappa_{12} c_\beta s^3_\beta + 3\kappa_2 s^4_\beta],
\end{aligned} \tag{19}$$

$$\begin{aligned}
\Delta\mathcal{M}^2_{12} &= -v^2(\Delta\lambda_{34} s_\beta c_\beta + \mathtt{Re}\Delta\lambda_6 c^2_\beta + \mathtt{Re}\Delta\lambda_7 s^2_\beta) + \\
&+ v^4[\mathrm{Re}\kappa_8 c^4_\beta + (\kappa_3 + \kappa_5 + \mathrm{Re}\kappa_9) c^3_\beta s_\beta + \\
&+ 2(\mathrm{Re}\kappa_{11} + \mathrm{Re}\kappa_{13}) c^2_\beta s^2_\beta + (\kappa_4 + \kappa_6 + \mathrm{Re}\kappa_{10}) c_\beta s^3_\beta + \mathrm{Re}\kappa_{12} s^4_\beta].
\end{aligned} \tag{20}$$

Appendix C: mass basis

$$\left(\begin{array}{c}\eta_1\\ \eta_2\end{array}\right)=\mathcal{O}_\alpha\left(\begin{array}{c}H\\ h\end{array}\right),\qquad \left(\begin{array}{c}\chi_1\\ \chi_2\end{array}\right)=\mathcal{O}_\beta\left(\begin{array}{c}G^0\\ A\end{array}\right),\qquad \left(\begin{array}{c}\omega_1^\pm\\ \omega_2^\pm\end{array}\right)=\mathcal{O}_\beta\left(\begin{array}{c}G^\pm\\ H^\pm\end{array}\right)$$

Local minimum conditions

$$\begin{aligned}
\mu_1^2 &= -\mathrm{Re}\mu_{12}^2 t_\beta + \frac{v^2}{4}(4\lambda_1 c_\beta^2 + 3\mathrm{Re}\lambda_6 s_{2\beta} + 2s_\beta^2(\lambda_{345} + \mathrm{Re}\lambda_7 t_\beta)) + \\
&+ \frac{v^4}{4}(3\kappa_1 c_\beta^4 + 5\mathrm{Re}\kappa_8 c_\beta^3 s_\beta + 3(\mathrm{Re}\kappa_7 + \mathrm{Re}\kappa_{11} + \mathrm{Re}\kappa_{13}) c_\beta s_\beta^3 + \\
&+ (\mathrm{Re}\kappa_9 + (\kappa_3 + \kappa_5)/2) s_{2\beta}^2 + (\kappa_4 + \kappa_6 + 2\mathrm{Re}\kappa_{10} + \mathrm{Re}\kappa_{12} t_\beta) s_\beta^4), \quad (21) \\
\mu_2^2 &= -\mathrm{Re}\mu_{12}^2 \cot\beta + \frac{v^2}{4}(4\lambda_2 s_\beta^2 + 3\mathrm{Re}\lambda_7 s_{2\beta} + 2c_\beta^2(\lambda_{345} + \mathrm{Re}\lambda_6 \cot_\beta)) + \\
&+ \frac{v^4}{4}(3\kappa_2 s_\beta^4 + 5\mathrm{Re}\kappa_{12} s_\beta^3 c_\beta + 3(\mathrm{Re}\kappa_7 + \mathrm{Re}\kappa_{11} + \mathrm{Re}\kappa_{13}) s_\beta c_\beta^3 + \\
&+ (\mathrm{Re}\kappa_{10} + (\kappa_4 + \kappa_6)/2) s_{2\beta}^2 + (\kappa_3 + \kappa_5 + 2\mathrm{Re}\kappa_9 + \mathrm{Re}\kappa_8 \cot_\beta) c_\beta^4). \quad (22)
\end{aligned}$$

$$U(h,H,A,H^{\pm},G^0,G^{\pm}) = c_0 A + c_1 hA + c_2 HA + \frac{m_h^2}{2}h^2 + \frac{m_H^2}{2}H^2 + \frac{m_A^2}{2}A^2 + m_{H^\pm}^2 H^+H^- + I_k$$

$c_0 = 0$:

$$\mathrm{Im}\mu_{12}^2 = \frac{v^2}{2}(s_\beta c_\beta \mathrm{Im}\lambda_5 + c_\beta^2 \mathrm{Im}\lambda_6 + s_\beta^2 \mathrm{Im}\lambda_7) + \frac{v^4}{4}\{\mathrm{Im}\kappa_8 c_\beta^4 + 2\mathrm{Im}\kappa_9 c_\beta^3 s_\beta + (3\mathrm{Im}\kappa_7 + \mathrm{Im}\kappa_{11} + \mathrm{Im}\kappa_{13})c_\beta^2 s_\beta^2 + 2\mathrm{Im}\kappa_{10} c_\beta s_\beta^3 + \mathrm{Im}\kappa_{12} s_\beta^4\} \quad (23)$$

$$c_1 = v^2(-1/2 \cdot \mathrm{Im}\lambda_5 c_{\alpha+\beta} + \mathrm{Im}\lambda_6 s_\alpha c_\beta - \mathrm{Im}\lambda_7 c_\alpha s_\beta) + \frac{v^4}{4}(-c_{\alpha+\beta} s_{2\beta}(3\mathrm{Im}\kappa_7 + \mathrm{Im}\kappa_{11} + \mathrm{Im}\kappa_{13}) + 4(s_\alpha c_\beta^3 \mathrm{Im}\kappa_8 - c_\alpha s_\beta^3 \mathrm{Im}\kappa_{12}) + 2(s_\beta^2(-3c_\alpha c_\beta + s_\alpha s_\beta)\mathrm{Im}\kappa_{10} - c_\beta^2(c_\alpha c_\beta - 3 s_\alpha s_\beta)\mathrm{Im}\kappa_9), \quad (24)$$

$$c_2 = -\frac{v^2}{2}\{\mathrm{Im}\lambda_5 s_{\alpha+\beta} + 2(\mathrm{Im}\lambda_6 c_\beta c_\alpha + \mathrm{Im}\lambda_7 s_\beta s_\alpha) + v^2[2\mathrm{Im}\kappa_8 c_\beta^3 c_\alpha + \mathrm{Im}\kappa_9 c_\beta^2(s_{\alpha+\beta} + 2c_\alpha s_\beta) + \mathrm{Im}\kappa_{10} s_\beta^2 (s_{\alpha+\beta} + 2 c_\beta s_\alpha) + 2\mathrm{Im}\kappa_{12} s_\beta^3 s_\alpha + \frac{1}{2}(3\mathrm{Im}\kappa_7 + \mathrm{Im}\kappa_{11} + \mathrm{Im}\kappa_{13}) s_{2\beta} s_{\alpha+\beta}]\} \quad (25)$$

$$m_{H,h}^2 = \frac{1}{2}(m_A^2 + m_Z^2 + \Delta\mathcal{M}_{11}^2 + \Delta\mathcal{M}_{22}^2 \pm \sqrt{m_A^4 + m_Z^4 - 2m_A^2 m_Z^2 c_{4\beta} + C}),$$

$$m_A^2 = m_{H\pm}^2 - m_W^2 + \frac{v^2}{2}(\mathrm{Re}\Delta\lambda_5 - \Delta\lambda_4) +$$

$$- \frac{v^4}{4}[c_\beta^2(2\mathrm{Re}\kappa_9 - \kappa_5) + s_\beta^2(2\mathrm{Re}\kappa_{10} - \kappa_6) - s_{2\beta}(\mathrm{Re}\kappa_{11} - 3\mathrm{Re}\kappa_7)] \quad (26)$$

$$\tan 2\alpha = \frac{2\Delta\mathcal{M}_{12}^2 - (m_Z^2 + m_A^2)s_{2\beta}}{(m_Z^2 - m_A^2)c_{2\beta} + \Delta\mathcal{M}_{11}^2 - \Delta\mathcal{M}_{22}^2}, \quad (27)$$

where

$$C = 4\Delta\mathcal{M}_{12}^4 + (\Delta\mathcal{M}_{11}^2 - \Delta\mathcal{M}_{22}^2)^2 - 2(m_A^2 - m_Z^2)(\Delta\mathcal{M}_{11}^2 - \Delta\mathcal{M}_{22}^2)c_{2\beta} - 4(m_A^2 + m_Z^2)\Delta\mathcal{M}_{12}^2 s_{2\beta},$$

$$m_{h_{1,2}}^2 = 2\sqrt{(-q)}\cos\left(\frac{\Theta \pm 2\pi}{3}\right) - \frac{a_2}{3}, \qquad m_{h_3}^2 = 2\sqrt{(-q)}\cos\left(\frac{\Theta}{3}\right) - \frac{a_2}{3},$$

$$q = \frac{1}{9}(3a_1^2 - a_2^2), \qquad \Theta = \arccos\left(\frac{r}{\sqrt{(-q)^3}}\right), \qquad r = \frac{1}{54}(9a_1 a_2 - 27a_0 - 2a_2^3), \quad (28)$$

$$a_0 = c_1^2 m_H^2 + c_2^2 m_h^2 - m_h^2 m_H^2 m_A^2, \quad a_1 = m_h^2 m_H^2 + m_h^2 m_A^2 + m_H^2 m_A^2 - c_1^2 - c_2^2, \quad (29)$$

$$a_2 = -m_h^2 - m_H^2 - m_A^2$$

$$\begin{pmatrix} h \\ H \\ A \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} \qquad (30)$$

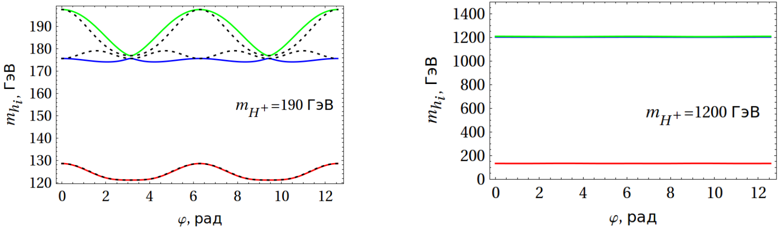

Рис.: Higgs masses: m_{h_1} – red, m_{h_2} – blue, m_{h_3} – green. $M_{SUSY} = 2$ TeV, $A_{t,b} = 2M_{SUSY}$, $\mu = 4M_{SUSY}$, $\tan\beta = 5$. Dashed lines correspond to m_h, m_H, m_A.