Holographic model for color superconductivity in d-dimension

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Outline

introduction

- Holographic model for the color superconductivity (CSC) phase in QCD is one of the interesting problem in high energy physics
- The previous holographic model for the CSC phase always use AdS_6 to study this
- The main problem in holographic model for CSC phase is the number of color $N_c > 1$. If we use the Einstein-Maxwell gravity we only have CSC phase with $N_c = 1$ (Kazuo 2019).
- In this project, we extend the notion of holo CSC phase for the d-dimension instead of 6 to study generally and we also use the arbitrary $SU(N_c)$. And we will try to use Einstein-Maxwell gravity for d-dimension

holographic dictionary

- d-dimension AdS $\leftrightarrow$ (d-1)-dimension boundary
- The Cooper pair (diquark in QCD) $\leftrightarrow$ the scalar field ψ and q is its U(1) charge correspond to the charge (in analogy to QCD CSC the baryon number) of the Cooper pair operator and this relate the color number as $q = \frac{2}{N_c}$
- The U(1) gauge field A_μ in the bulk $\leftrightarrow$ the current in analogy to current baryon number in QCD CSC
- The temperature $\leftrightarrow$ the Hawking temperature of RN planar black hole
- In this talk, we assume that the $SU(N_c)$ theory have not the confinement phase

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Einstein-Maxwell action

The action of the d-dimension Einstein-Maxwell gravity for the CSC phase transition is given by

$$S = \int d^d x \sqrt{-g} \left(\mathcal{R} + \frac{(d-1)(d-2)}{L^2} - \frac{1}{4} F^2 - |(\partial_\mu - iqA_\mu)\psi|^2 - m^2 |\psi|^2 \right) \tag{1}$$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, the cosmological constant Λ is determined by $\Lambda = -\frac{(d-1)(d-2)}{2L^2}$. And we set $L^2 = 1$

the ansatz

The ansatz for the vector field and scalar fields read

$$A_\mu dx^\mu = \phi(r)dt, \psi = \psi(r) \tag{2}$$

The RN planar black hole solution whose the metric is given by the following ansatz:

$$ds^2 = r^2(-f(r)dt^2 + h_{ij}dx^i dx^j) + \frac{dr^2}{r^2 f(r)} \tag{3}$$

where $h_{ij}dx^i dx^j = dx_1^2 + ... + dx_{d-2}^2$ is the line element of the $(d-2)$-dimension hypersurface

equation of motion

$$
\begin{aligned}
\phi''(r) + \frac{d-2}{r}\phi'(r) - \frac{2q^2\psi^2(r)}{r^2 f(r)}\phi(r) = 0 \\
\psi''(r) + \left[\frac{f'(r)}{f(r)} + \frac{d}{r}\right]\psi'(r) + \frac{1}{r^2 f(r)}\left[\frac{q^2\phi^2(r)}{r^2 f(r)} - m^2\right]\psi(x) = 0
\end{aligned}
\tag{4}
$$

The blackening function of this AdS black hole is

$$
f(r) = 1 - \left(1 + \frac{3\mu^2}{8r_+^2}\right)\left(\frac{r_+}{r}\right)^{d-1} + \frac{3\mu^2 r_+^d}{8r^{d+2}} \tag{5}
$$

The Hawking temperature

The temperature of physics system is dual to the Hawking temperature and

$$T = \frac{r_+^2 f'(r_+)}{4\pi} = \frac{1}{4\pi}\left((d-1)r_+ - \frac{9\mu^2}{8r_+}\right), \tag{6}$$

the condition $T \geq 0$ we have the constraint of μ

$$\frac{\mu^2}{r_+^2} \leq \frac{8(d-1)}{9} \tag{7}$$

Near boundary conditon

Near the boundary $(r \to \infty)$ we have the form of the matter fields:

$$\begin{aligned} \phi(r) &= \mu - \frac{\bar{d}}{r^{d-3}} \\ \psi(r) &= \frac{J_C}{r^{\Delta_-}} + \frac{C}{r^{\Delta_+}} \end{aligned} \tag{8}$$

where $\mu, \bar{d}, J_C$ and C are regarded as the chemical potential, charge density, source, and the condensates value (VEV) of the Cooper pair operator dual to ψ, like the diquark Cooper pair in QCD, respectively, and we can see that one constraint $d > 3$.

The conformal dimension $\Delta_\pm$ in this case read

$$\Delta_\pm = \frac{1}{2}\left((d-1) \pm \sqrt{(d-1)^2 + 4m^2}\right) \tag{9}$$

And the BF bound is

$$m^2 \geq -\frac{(d-1)^2}{4} \tag{10}$$

We choose $m^2 = 2 - d$ to have $\Delta_- = 1$ and we also obtain $\Delta_+ = d - 2$. Hence

$$\psi(r) = \frac{J_C}{r} + \frac{C}{r^{d-2}} \tag{11}$$

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Near the critical chemical potential

At the critical point, $\psi = 0$ hence in near critical chemical potential (it means $\mu > \mu_c$ but near μ_c) the back reaction of the scalar field is negligible. We obtain the bulk configuration is determined by:

$$S = \int d^d x \sqrt{-g} \left(\mathcal{R} - 2\Lambda - \frac{1}{4} F^2 \right) \tag{12}$$

Hence, the solution of the gauge field in this case is given by

$$\phi(r) = \mu \left(1 - \left(\frac{r_+}{r} \right)^{d-3} \right) \tag{13}$$

the condition

From the equation of motion, we introduce the effective mass

$$m_{eff}^2 = m^2 - \Delta m^2 = m^2 - \frac{q^2\phi^2(r)}{r^2 f(r)} \tag{14}$$

To the Cooper pair condensed state appear, we must to have the instability of the bulk scalar field (with the effective mass). And this correspond to the BF bound is broken, we have a condition

$$m^2 - \frac{q^2\phi^2(r)}{r^2 f(r)} < \frac{-(d-1)^2}{4} \tag{15}$$

We obtain

$$\frac{q^2\phi^2(r)}{r^2 f(r)} > \frac{(d-3)^2}{4} \tag{16}$$

Plug the value of $\phi(r)$ and $f(r)$ we obtain

$$\frac{q^2\hat{\mu}^2 z^2\left(1-z^{d-3}\right)^2}{1-\left(1+\frac{3\hat{\mu}^2}{8}\right)z^{d-1}+\frac{3\hat{\mu}^2 z^{d+2}}{8}} = q^2 F(\hat{\mu}, z, d) > \frac{(d-3)^2}{4} \tag{17}$$

with $\hat{\mu} = \frac{\mu}{r_+}$, and $z = \frac{r_+}{r}$
And we obtain

$$N_c < \frac{4\sqrt{F_{max}(\hat{\mu}, z)}}{d-3} \tag{18}$$

We define the following two functions:

$$F_{\max}(d,\tilde{\mu}) = \max_{z\in[0,1]} F(d,\tilde{\mu},z) \quad , \quad G(d,\tilde{\mu}) \equiv \frac{4\left[F_{\max}(d,\tilde{\mu})\right]^{1/2}}{d-3} \ , \tag{19}$$

Result

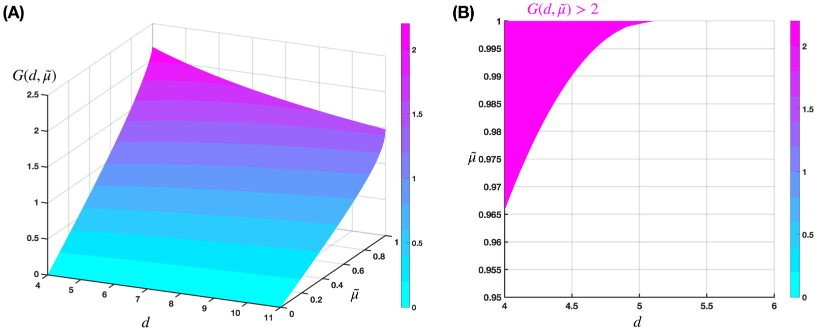

Figure: **Our numerical investigation for F.** This calculation was done using MatLab R2023a. **(A)** The surface function $G(d, \tilde{\mu})$ inside the region of interests i.e. $(d, \tilde{\mu}) \in [4, 11] \times [0, 1]$. **(B)** We zoom into the small corner where $G(d, \tilde{\mu}) > 2$ can be realized.

- We see that with onlly with $d = 4$ the Einstein-Maxwell will give $N_c = 2$ CSC phase
- If $N_c = 3$ the normal Einstein-Maxwell gravity can't give CSC phase hence if we want to study this phase with $N_c \geq 3$ we must to use modify gravity for example Einstein-Gauss-Bonnet with the negative α (Nam 2021) or add some non linear term in Maxwell law (Nam 2022)

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Conclusion

- We built the holographic model for CSC phase by the Einstein-Maxwell gravity in d-dimension
- Only with $d = 4$ we can obtain the color superconductivity with $N_c = 2$. BUT in holo QCD CSC phase $d = 6$ (because the QCD scale) hence we only obtain CSC phase with $N_c = 1$ by Einstein-Maxwell gravity
- If we want to study CSC phase with $N_c \geq 3$ we must to change the gravity or the Maxwell law for all bulk dimension (Nam 2021) (Nam 2022)

Thank you for attention!