Four-vector deformations and holographic principle

work based on works
[2302.08749, 2011.11424]

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Holographic interpretation

Bivector transformation - hidden symmetry of solutions space of 10d supergravity

$$(G+B)^{-1} = (g+b)^{-1} + \beta, \qquad \beta^{mn} = k_a{}^m k_b{}^n \boxed{r^{ab}} \tag{1}$$

Backgrounds looks like $AdS_D \ltimes M_{10-D}$: duals to CFT deformations

$AdS_5 \times \mathbb{S}^5 \Longleftrightarrow \mathcal{N} = 4$ SYM

$\downarrow$ Deformed background $\Longleftrightarrow$

- adding of operators $(\mathrm{Tr}[\Phi^3])$;
- changing of RG behaviour (AdS cut-off);
- non-commutativity ;

Holographic interpretation

- There are three possibilities:
  1. All isometrics were taken through M: **marginal deformations**;
  2. All isometrics were taken through AdS: non-commutativity;
  3. Mixed case: dipole deformations.
- In case of using of basic hidden symmetry of space of solutions of supergravity, isometrics of M_{10-D} must to be commutative, thus acceptable only abelian deformations

$$[k_a, k_b] = 0$$

- U-duality (in following advanced) hidden symmetry of supegravity, allow us expand acceptable view of deformation
- Advanced hidden symmetry allow non-abelian isometrics of compact space M_{10-D}

$$[k_a, k_b] = f_{ab}{}^c k_c$$

Non-abelian deformations

- YB bi-vector transformation of vary solution with b-field
  [Bakhmatov, Colgain, Sheikh-Jabbari, Yavatanoo (2018)]

$$(G+B)^{-1} = (g+b)^{-1} + \beta \tag{2}$$

- Necessary to define

$$\begin{aligned} [k_a, k_b] &= f_{ab}{}^{c} k_c && \text{(algebra of symmetries)} \\ \beta^{mn} &= k_a{}^{m} k_b{}^{n} r^{ab} && \text{(bi-vector anzatz);} \\ r^{b_1[a_1} r^{|b_2|a_2} f_{b_1 b_2}{}^{a_3]} &= 0 && \text{(classical YB equation);} \\ r^{b_1 b_2} f_{b_1 b_2}{}^{a} k_a{}^{m} &= I^m = 0 && \text{(unimodularity);} \end{aligned} \tag{3}$$

In case of compact isometrics:

- Abelian $\mathfrak{u}(1)^n$: $f_{ab}{}^{c} = 0 \implies \forall r_{ab}$
- Non-abelian (SU(N), SO(N), ...): $r_{ab} \equiv 0$
  [Lichnerowicz, Medina (1988), Pop, Stolin (2007)]

Deformation of $M_5 \times N_5$ solutions of IIB

Consider solution in view $M_5 \times N_5$ with

$$\begin{aligned} ds^2 &= \underbrace{g_{\mu\nu}dx^\mu dx^\nu}_{AdS_5} + \underbrace{g_{mn}dy^m dy^n}_{\mathbb{S}^5}, \\ C_{\mathbb{S}_5} &= \frac{1}{4!}C_{mnkl}(y)dy^m \wedge dy^n \wedge dy^k \wedge dy^l, \\ C_{AdS_5} &= \frac{1}{4!}c_{\mu\nu\kappa\lambda}(x)dx^\mu \wedge dx^\nu \wedge dx^\kappa \wedge dx^\lambda \end{aligned} \tag{4}$$

- Solution $AdS_5 \times \mathbb{S}^5$ corresponds to anzats and dual to $\mathcal{N} = 4, D = 4$ SYM

Four-vector deformation

Four-vector transformation of IIB supergravity fields in clear view:

$$\begin{aligned}
K &= (1+W)^2 \pm W^2 \\
m'^{\alpha\beta} &= m^{\alpha\beta} \\
G'_{\mu\nu} &= K^{\frac{1}{2}} G_{\mu\nu} \\
'^m &= K^{-\frac{1}{4}} {}^m (1+W) \pm K^{-\frac{1}{4}} W^m \\
g'_{mn} &= K^{-\frac{1}{2}} \left(g_{mn} + 2\, W_{(mn)} \pm \left(1 \pm {}^2\right) W_m W_n \right), \\
W_m &= \frac{1}{4!} \epsilon_{mp_1 \dots p_4}^{p_1 \dots p_4}, \\
g' &= K^{-\frac{3}{2}} g,
\end{aligned} \tag{5}$$

where were in bi case:

$$(G+B)^{-1} = (g+b)^{-1} + \beta \tag{6}$$

Generalization of the Yang–Baxter equation in IIB case

- As in case of bi–vector deformation, we can span Ω^{mnkl} on Killing vectors:

$$\Omega^{mnkl} = \rho^{i_1 i_2 i_3 i_4} k^m_{i_1} k^n_{i_2} k^k_{i_3} k^l_{i_4} \tag{7}$$

- Enough conditions on coordinates of four-vector for generation of IIB solution from IIB solution

Linear conditions: IIB analogue of unimodularity condition

$$\rho^{[a_1 a_2 | a_3 a_4 |} f_{a_3 a_4}{}^{a_5]} = 0. \tag{8}$$

Quadratic condition:Generalization of the classical Yang–Baxter equation in case of four-vector

$$\rho^{[a_1 a_2 | a_3 a_4 |} \rho^{a_5 a_6 a_7] a_8} f_{a_3 a_8}{}^{a_9} - 3\rho^{[a_1 a_2 | a_3 a_4 |} \rho^{a_5 a_6 | a_9 a_8 |} f_{a_3 a_8}{}^{a_7]} = 0. \tag{9}$$

Results

Four-vector deformation of $AdS_5 \times \mathbb{S}^5$ on AdS isometries:

$$K = 1 - \left(\frac{R}{z}\right)^3 \left(\left(\frac{x^a}{z}\right)\rho_a\right) \tag{10}$$

$$ds^2 = K^{-\frac{1}{2}}\left(\frac{R}{z}\right)^2\left[-(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + K\,dz^2 + \frac{1}{2}\left(\frac{R}{z}\right)^3 \rho_a dx^a dz\right]$$
$$+ R^2 K^{\frac{1}{2}} d\Omega^2_{(5)},$$
$$F = -\frac{1}{R}\left(-K^{-2} d\mathrm{Vol}(AdS_5) + d\mathrm{Vol}(\mathbb{S}^5)\right). \tag{11}$$

where ρ^a must be such, that $\rho^2 = -(\rho^0)^2 + (\rho^1)^2 + (\rho^2)^2 + (\rho^3)^2 = 0$ for (11) to be IIB supergravity solution.

Summary and discussion

- For IIB supergravity were found new type of symmetry of space of solutions by four vector, and found conditions on it, built example of four–vector deformation
- In following works try to find full solution of conditions on four-vector on $\mathbb{S}^5$ isometrics
- Find precisely view of new deformations of $AdS_5 \times \mathbb{S}^5$ that will be corresponds to non-supersymmetric conform manifold

Thanks for your attention!