Gribov copy effects in lattice gluodynamics

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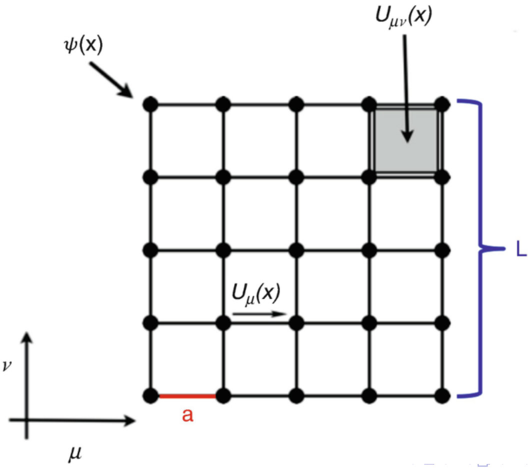
Plan

1. Nonperturbative gauge fixing and Gribov copy problem
2. Dual superconductor model for the confinement mechanism
3. Monopoles and maximal Abelian gauge (MAG)
4. New results for decomposition of a gauge field in MAG
5. Conclusions

Lattice regularization

$$U_\mu(x) = P\mathrm{exp}\left(i \int_{C_{x,x+\hat{\mu}}} A_\mu(s)ds\right) \approx 1 + iaA_\mu(x) + O(a^2)$$

$$S_G = \beta \sum_{x,\mu<\nu} (1 - \frac{1}{N}\mathrm{ReTr}U_{\mu\nu}) = \frac{\beta}{2N_c} \sum_{x,\mu<\nu} \mathrm{Tr}[F_{\mu\nu}^2] + \mathscr{O}(a^2)\,, \quad \beta = \frac{2N_c}{g^2}$$



Gribov copies

Faddeev-Popov formalism:

$$\langle O \rangle = \frac{1}{Z} \int \mathscr{D}A e^{-S(A)} \Delta(A) \delta(f(A)) O(A)$$

Gribov copies were discovered for the Coulomb gauge by Gribov 1978
Gribov's statement was generalized to other gauges by Singer, 1978
Non-perturbative gauge fixing (Zwanziger, 1994):

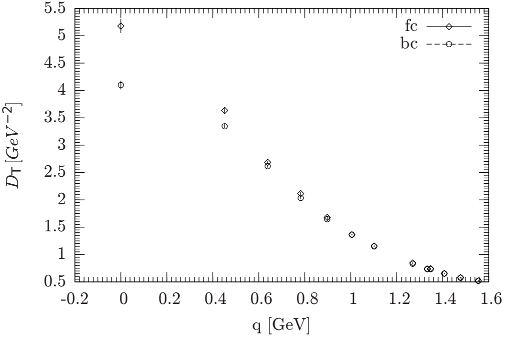
$$\langle O \rangle = \frac{1}{Z} \int \mathscr{D}A e^{-S(A)} I^{-1}(A) \int \mathscr{D}g e^{-\lambda F(A^g)} O(A^g)$$

where $F(A)$ - gauge fixing functional,

$$I(A) = \int \mathscr{D}g e^{-\lambda F(A^g)}$$

$\lambda \to \infty$ limit corresponds to restricting integration to fundamental modular region, i.e. a subset of global minima of $F(A)$

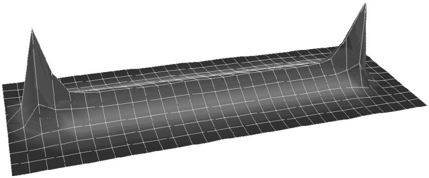
Example: Landau gauge gluon propagator



Transverse gluon propagator $D_T(p)$ in $SU(3)$ gluodynamics for two sets of Gribov copies

Bornyakov, Mitryushkin, Mueller-Preussker, 2011

Confinement problem



Quark confinement:
- is confirmed experimentally and in lattice calculations
- linear dependence of static quark interaction potential on a distance between them
- hasn't been proven analytically so far
- one of the approaches - to describe QCD vacuum as a dual superconductor, t'Hooft, 1976, Mandelstam, 1976

Maximal Abelian gauge in SU(3) gludynamics

Suggested by t'Hooft, 1981
to define color-magnetic monopoles
Gauge fixing functional (breaks $SU(N_c)$ to $U(1)^{N_c-1}$)

$$F_{SU(3)} = \frac{1}{12V} \int d^4x \sum_{\mu=1}^{4} \sum_{a\neq 3,8} (A_\mu^a(x))^2$$

$$f^a(A) = \sum_{b\neq 3,8} (\partial_\mu \delta^{ab} - g f^{ab3} A_\mu^3 - g f^{ab8} A_\mu^8) A_\mu^b = 0\,, \qquad a \neq 3,8$$

Gauge fixing functional in lattice regularization:

$$F_{SU(3)}^{latt} = 1 - \frac{1}{12V} \sum_{x,\mu,a=3,8} \mathrm{Tr}\{U_\mu(x)\lambda_a U_\mu^\dagger(x)\lambda_a\} \approx a^2 F_{SU(3)}$$

Color-magnetic monopoles

It is known that the t'Hooft-Polyakov monopoles in the Higgs model have a form of Dirac monopole in a unitary gauge

In $SU(N_c)$ theory without Higgs field we search for nonabelian color-magnetic monopoles making three steps

- fixing MA gauge
- making Abelian projection

$$A_\mu(x) = \sum_{a\neq 3,8} A^a_\mu(x)\lambda_a + A^3_\mu(x)\lambda_3 + A^8_\mu(x)\lambda_8 \equiv A^{offd}_\mu(x) + A^{abel}_\mu(x)$$

- using $A^{abel}_\mu(x)$ to locate Dirac monopoles via procedure introduced for compact $U(1)$ in DeGrand, Toussaint, 1980

Thus, MA gauge is used as a tool to locate color-magnetic monopoles

We are free to choose 'proper' Gribov copies

Abelian field is decomposed into 'monopole' and 'photon' components (names are borrowed from compact $U(1)$)

$$A_\mu^{abel}(x) = A_\mu^{mon}(x) + A_\mu^{phot}(x)$$

Then we introduce the decomposition (Bornyakov, Polikarpov, Schierholz, Suzuki, Syritsyn, 2006)

$$A_\mu(x) = A_\mu^{mod}(x) + A_\mu^{mon}(x)$$

where

$$A_\mu^{mod}(x) = A_\mu^{offd}(x) + A_\mu^{phot}(x)$$

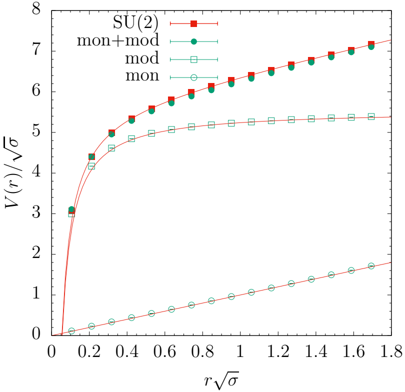
Static quark potential

$$\langle W(R,T)\rangle \propto e^{-TV(R)}(1+O(e^{-T\Delta E}))$$

$$V(R) = -\lim_{T\to\infty}\frac{1}{T}\log\langle W(R,T)\rangle$$

We measure three types of Wilson loops $\langle W(R,T)\rangle$:
- for nonabelian gauge field $A_\mu(x)$,
- for monopole component $A_\mu^{mon}(x)$,
- for modified nonabelian gauge field $A_\mu^{mod}(x)$,

Decomposition of static potential $V(r)$ in $SU(2)$ QCD

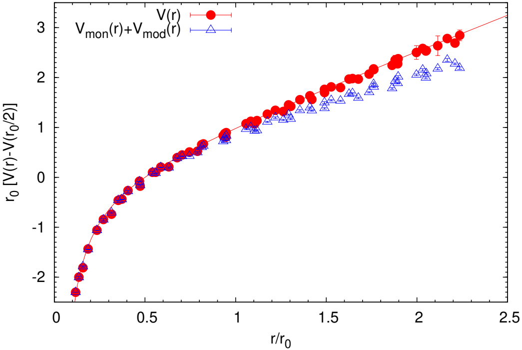


Static potentials $V_{mon}(r)$ and $V_{mod}(r)$ computed on 'global' minima of gauge fixing functional vs. physical static potential $V(r) = V_0 + \alpha/r + \sigma r$
Bornyakov, Kudrov, Rogalyov, 2022

$$V(r) = V_{mon}(r) + V_{mod}(r)$$

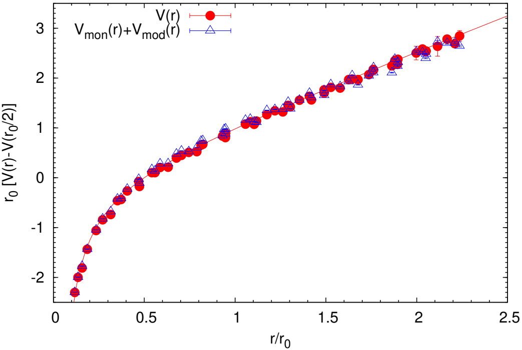
Interpretation: $A_\mu^{mon}(x)$ is responsible for the linear part of $V(r)$, $A_\mu^{mod}(x)$ - for perturbative part (at small r) and for hadron string fluctuations (at large r).

Decomposition of static potential in $SU(3)$ gluodynamics



$V_{mon}(r) + V_{mod}(r)$ is compared with $V(r)$,
both are computed for few values of lattice spacing $a \in [0.06, 0.09]$ fm
With 'global' minima (Gribov copies) we find agreement at small r and disagreement at large r
Disagreement comes from low string tension in $V_{mon}(r)$

Decomposition of static potential in $SU(3)$ gluodynamics



$V_{mon}(r) + V_{mod}(r)$ is compared with $V(r)$,
With 'proper' minima (Gribov copies) we find agreement at all distances r

Conclusions

We study the gauge field decomposition

$$A_\mu(x) = A_\mu^{mon}(x) + A_\mu^{mod}(x) \tag{1}$$

in MA gauge of $SU(3)$ gluodynamics and demonstrate that Gribov copies exist which produce nice decomposition for the static potential

$$V(r) = V_{mon}(r) + V_{mod}(r) \tag{2}$$

Future plans:

1. better understanding of differences between Gribov copies found in our study
2. to study properties of this gauge field decomposition in QCD
3. to study decomposition for other observables, in particular, for hadron spectrum