The 7th International Conference on Particle Physics and Astrophysics

# Self-similarity and cumulative hadron production in heavy ion collisions at high energies

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# Contents

# Motivation & Goals



## Search for new symmetries in Nature

Systematic analysis of inclusive cross sections of particle production in p+p, p+A and A+A collisions to search for general features of constituent structure, interaction and fragmentation over a wide scale range.

## z-Scaling as a tool in high energy physics

Development of z-scaling approach for description of cumulative hadron production in inclusive p+A and A+A collisions and verification of self-similarity principle.

## The approach can be used to study

- Symmetry of constituent interactions at small scales
- Origin of flavor, spin,…
- Similarity and difference of u,d,s,c,b,t quark fragmentation
- Strangeness as a probe to search for new physics
- Phase transitions in p+p, p+A and A+A systems
- …………..

# Fundamental principles & symmetries



"Fundamental symmetry principles dictate the basic laws of physics, control the structure of matter and define the fundamental forces in nature."

Leon M. Lederman

"…for every conservation law there must exist a continuous symmetry...."

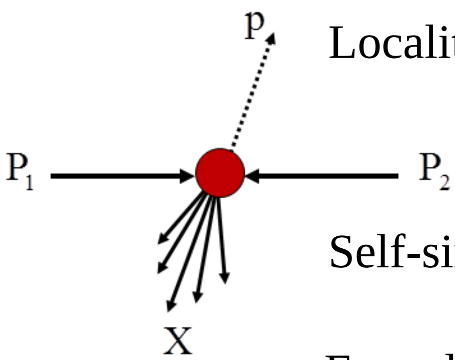
Emmy Nöether

**Self-similarity is a property of physical phenomena and principle to construct theories**

- Self-similarity is the symmetry of repeatability of structures and processes with change in a scale.
- The self-similarity principle states that structures and processes repeats with change of a scale.
- Self-similarity is the unifying concept for theories of fractals and chaos.
- Phenomenon that is self-similar looks the same or behaves the same when viewed at different magnifications.

# Self-similarity in inclusive reactions & z-scaling

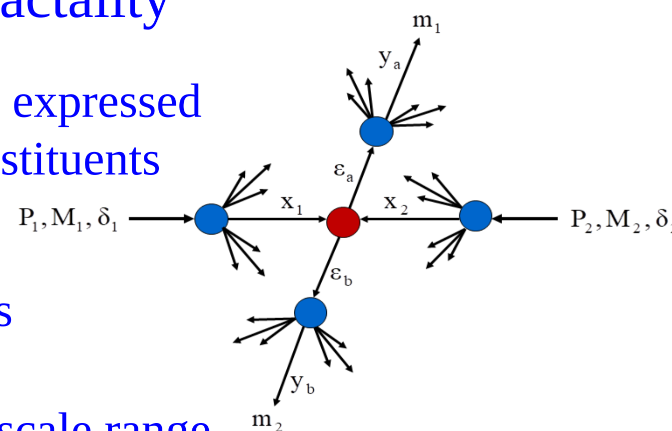
## Principles: locality, self-similarity, fractality




Locality: collisions of hadrons and nuclei are expressed via binary interactions of their constituents (partons, quarks and gluons,...).

Self-similarity: interactions of the constituents are mutually similar.

Fractality: self-similarity is valid over a wide scale range.




- The principles are reflected as regularities in measurable observables and can be usually expressed as scalings in a suitable representation of data.
- z-Scaling of differential cross sections of inclusive particle production in p+p, p+A and A+A is used as a tool to search for and study of principles and symmetries that reflect properties of interactions at constituent level.
- z-Scaling is based on the principles of *locality*, *self-similarity*, *fractality*.

Int. J. Mod. Phys. A 32 (2017) 1750029

# Self-similarity in inclusive reactions & z-scaling

The assumption of self-similarity of hadron interactions at a constituent level transforms to the requirement of universal description of inclusive spectra by a scaling function $\Psi(z)$ that depends on a self-similarity parameter z.

## Hypothesis of z-scaling :

$s^{1/2}$, $p_T$, $\theta_{cms}$ — Inclusive particle distributions can be described in terms of constituent sub-processes and parameters characterizing general properties of the system. — $x_1, x_2, y_a, y_b$ $\delta_1, \delta_2, \varepsilon_a, \varepsilon_b, c$

$Ed^3\sigma/dp^3$ — Scaled cross section $\Psi(z)$ of inclusive particle production depends in a self-similar way on a single scaling variable z. — $\Psi(z)$

The self-similarity parameter z is a dimensionless quantity, expressed through the dimensional values $P_1$, $P_2$, $p$, $M_1$, $M_2$, $m_1$, $m_2$, characterizing the process of inclusive particle production.

Procedure to construct function $\Psi(z)$ based on maximum fractal entropy was suggested.

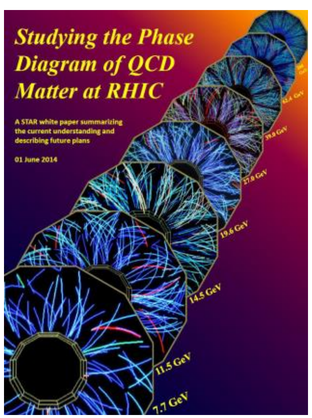
Int. J. Mod. Phys. A 32 (2017) 1750029.
Physics 5(2) (2023) 537.

# Scaling, Universality vs. basic principles & symmetries

"Scaling" and "Universality" are concepts developed to understanding critical phenomena. Scaling means that systems near the critical points exhibiting self-similar properties are invariant under transformation of a scale. According to universality, quite different systems behave in a remarkably similar fashion near the respective critical points. Critical exponents are defined only by symmetry of interactions and dimension of the space.

Harry E. Stanley, Grigory I. Barenblatt,…

## Beam Energy Scan program at RHIC to search for and study phase transition and critical phenomena in nuclear matter

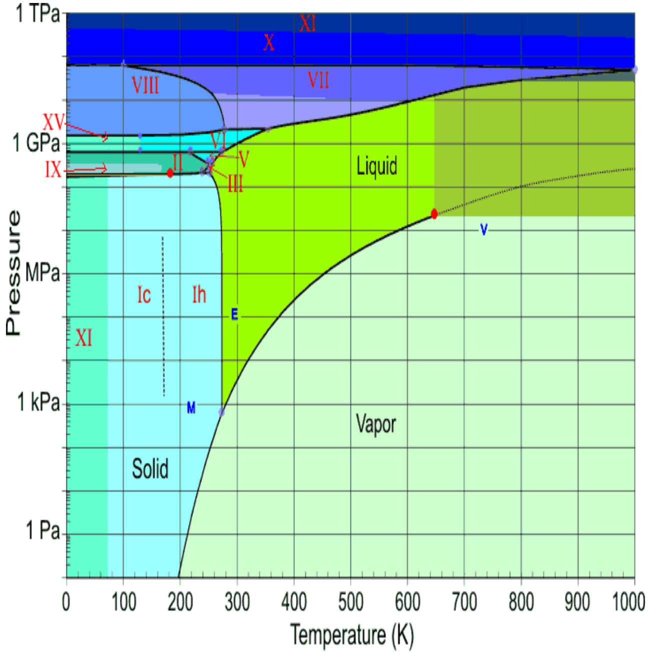



- The idea is to vary the collision energy and look for the signatures of QCD phase boundary and QCD critical point i.e. to span the phase diagram from the top RHIC energy (lower $\mu_B$ ) to the lowest possible energy (higher $\mu_B$).
- To look for the phase boundary, we would study the established signatures of QGP at 200 GeV as a function of beam energy. Turn-off of these signatures at particular energy would suggest the crossing of phase boundary.
- Near a critical point, there would be enhanced fluctuations in multiplicity distributions of conserved quantities (net-charge, net-baryon).

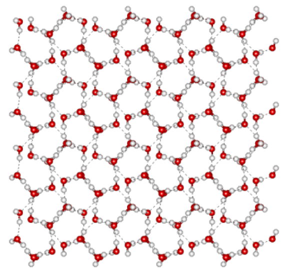
STAR collaboration

# Phase diagram of $H_2O$ and Nuclear Matter

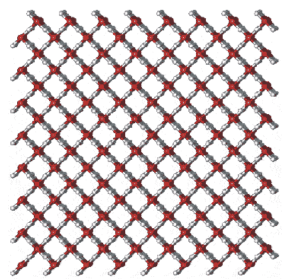
The phase diagram of water
is established



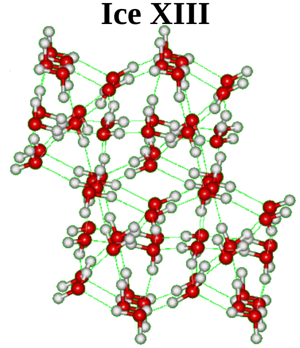

Ice III



Ice X

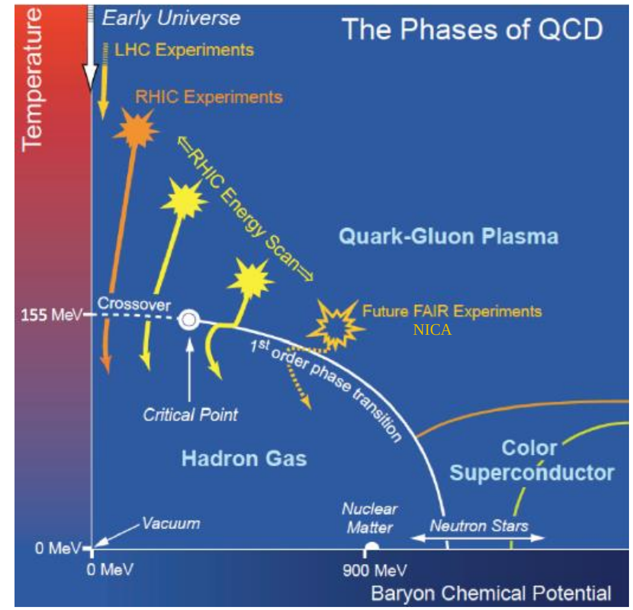


Ice XIII



- Phases (ice I-XV, liquid, vapor)
- Phase boundaries
- Phase transitions
- Triple Point (16)
- Critical Point (1)

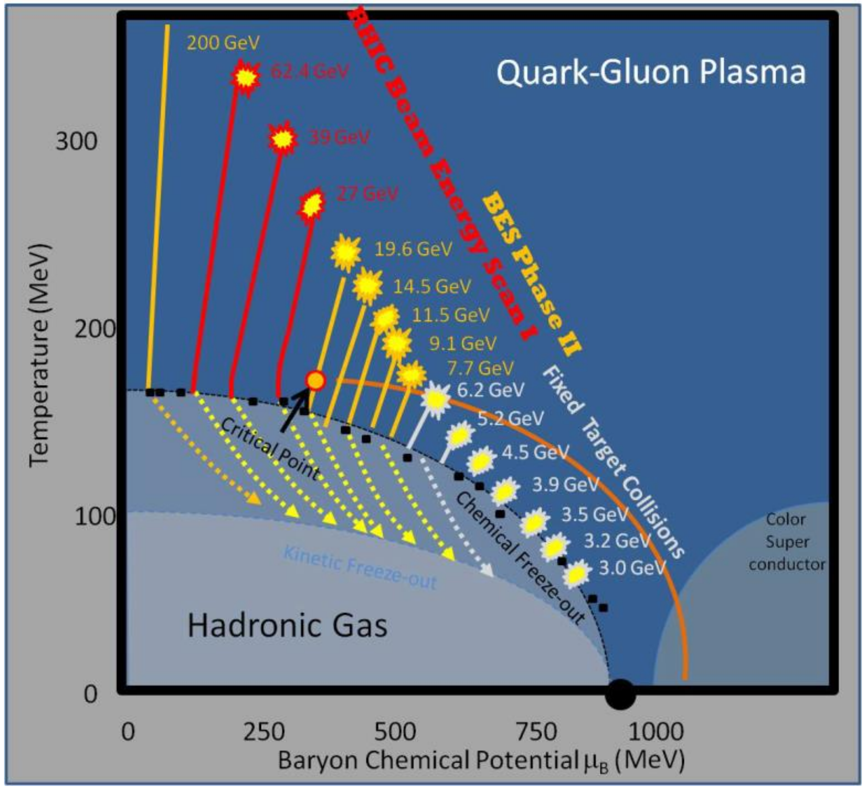
The phase diagram of strongly interacting
nuclear matter is under study




- Phases - ?
- Phase boundaries -?
- Phase transitions - ?
- Triple Point - ?
- Critical Point - ?

# STAR Beam Energy Scan Program

## STAR & BES-I, BES-II, FXT




**Collider mode**

$7.7 < \sqrt{s_{NN}} < 200$ GeV

$420 > \mu_B > 25$ MeV

**Fixed target mode**

$3 < \sqrt{s_{NN}} < 7.7$ GeV

$750 > \mu_B > 420$ MeV

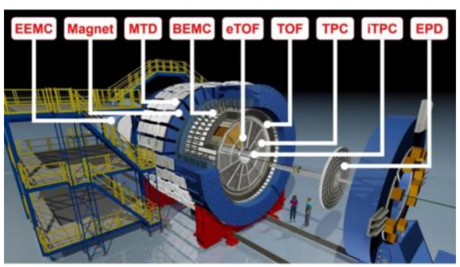
# High-Energy Nuclear Collisions at RHIC

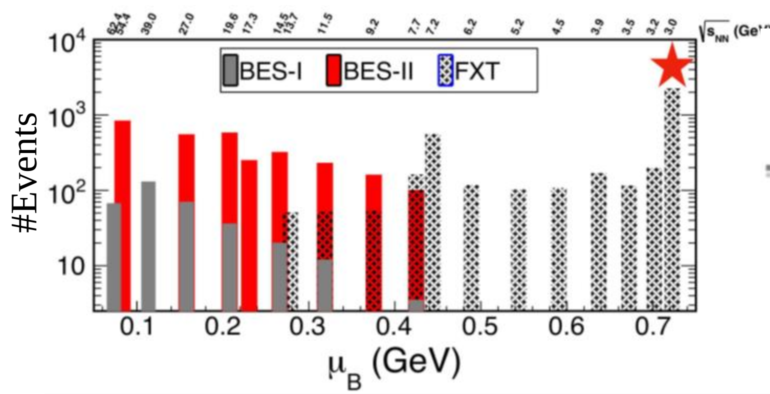


**Au+Au Collisions at RHIC**

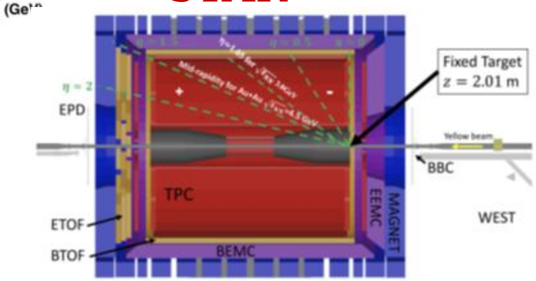
| Collider Runs | | | | | | Fixed-Target Runs | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\sqrt{s_{NN}}$ (GeV) | #Events | $\mu_B$ | $y_{beam}$ | run | | $\sqrt{s_{NN}}$ (GeV) | #Events | $\mu_B$ | $y_{beam}$ | run |
| 1 | 200 | 380 M | 25 MeV | 5.3 | Run-10, 19 | 1 | 13.7 (100) | 50 M | 280 MeV | -2.69 | Run-21 |
| 2 | 62.4 | 46 M | 75 MeV | | Run-10 | 2 | 11.5 (70) | 50 M | 320 MeV | -2.51 | Run-21 |
| 3 | 54.4 | 1200 M | 85 MeV | | Run-17 | 3 | 9.2 (44.5) | 50 M | 370 MeV | -2.28 | Run-21 |
| 4 | 39 | 86 M | 112 MeV | | Run-10 | 4 | 7.7 (31.2) | 260 M | 420 MeV | -2.1 | Run-18, 19, 20 |
| 5 | 27 | 585 M | 156 MeV | 3.36 | Run-11, 18 | 5 | 7.2 (26.5) | 470 M | 440 MeV | -2.02 | Run-18, 20 |
| 6 | 19.6 | 595 M | 206 MeV | 3.1 | Run-11, 19 | 6 | 6.2 (19.5) | 120 M | 490 MeV | 1.87 | Run-20 |
| 7 | 17.3 | 256 M | 230 MeV | | Run-21 | 7 | 5.2 (13.5) | 100 M | 540 MeV | -1.68 | Run-20 |
| 8 | 14.6 | 340 M | 262 MeV | | Run-14, 19 | 8 | 4.5 (9.8) | 110 M | 590 MeV | -1.52 | Run-20 |
| 9 | 11.5 | 157 M | 316 MeV | | Run-10, 20 | 9 | 3.9 (7.3) | 120 M | 633 MeV | -1.37 | Run-20 |
| 10 | 9.2 | 160 M | 372 MeV | | Run-10, 20 | 10 | 3.5 (5.75) | 120 M | 670 MeV | -1.2 | Run-20 |
| 11 | 7.7 | 104 M | 420 MeV | | Run-21 | 11 | 3.2 (4.59) | 200 M | 699 MeV | -1.13 | Run-19 |
| | | | | | | 12 | 3.0 (3.85) | 2000 M | 750 MeV | -1.05 | Run-18, 21 |

Solenoidal Tracker At RHIC




STAR



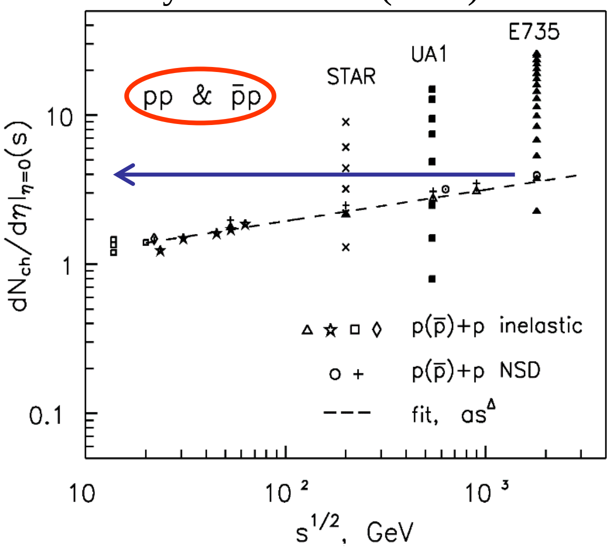

Unprecedented wide range of collision energies and centrality.

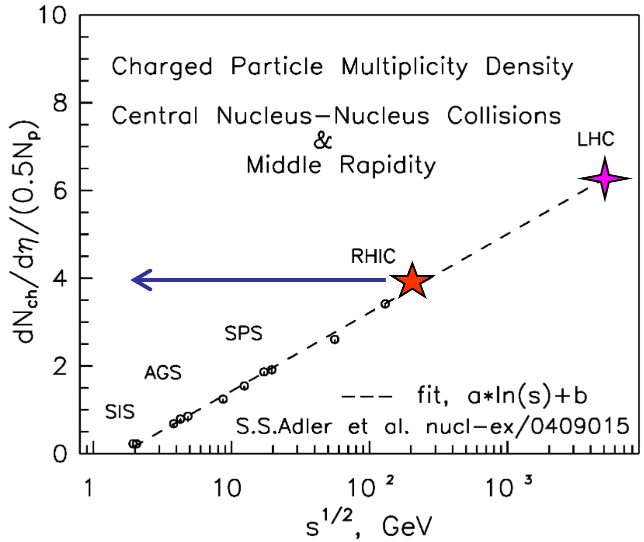
# Characteristics of produced medium in pp/p̄p & AA



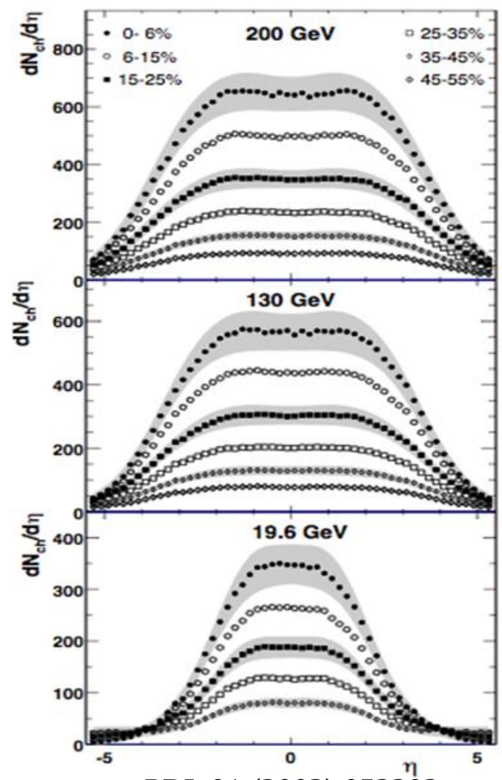
- Particle multiplicity $N_{ch}$
- Multiplicity density $dN_{ch}/d\eta$
- Mean transverse momentum $<p_T>$
- Energy density $\varepsilon_{Bj} = 1/(\pi R^2 \tau)\, dE_T/dy$

Phys.At.Nucl.70 (2007)1294




RHIC & PHOBOS




PRL 91 (2003) 052303

- Multiplicity density $dN_{ch}/d\eta$ in pp & p̄p collisions is much larger than $dN_{ch}/d\eta/(0.5N_p)$ in central AA collisions at AGS, Sp̄pS and RHIC.
- Is medium produced in pp at high $dN_{ch}/d\eta$ similar one then in AA ?
- Are there common properties of hadron production in pp & AA ?

# Self-similarity of negative hadron production in p+p collisions

p+p is of interest by itself:

- verification and search for new features
- search for a phase transition with different probes

p+p interaction is a reference for p+A and A+A physics

V.V. Abramov et al., Sov. J. Nucl. Phys. 31 (1980) 484.
V.V. Abramov et al., JETP Lett. 33 (1981) 289.
J.W. Cronin et. al., Phys. Rev. D11 (1975) 3105.
D. Antreasyan, J.W Cronin et al., Phys. Rev. D 19 (1979) 764.
D.E. Jaffe et al., Phys. Rev. D 40 (1989) 2777.

I.Zborovský & MT, Int. J. Mod. Phys. (2015) 1560103.

# Self-similarity of $h^-$ production in p+p

## Self-similarity parameter

$$z = z_0 \Omega^{-1}$$

$$z_0 = \frac{s_\perp^{1/2}}{(dN_{ch}/d\eta|_0)^c m_N}$$

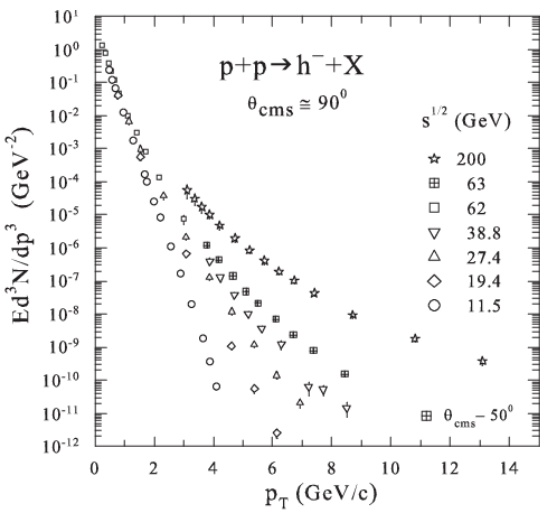
$$\Omega = (1-x_1)^\delta (1-x_2)^\delta (1-y_a)^{\varepsilon_F} (1-y_b)^{\varepsilon_F}$$

- $dN_{ch}/d\eta|_0$ - multiplicity density
- c - "specific heat" of bulk matter
- $\delta$ - proton fractal dimension
- $\varepsilon_F$ - fragmentation fractal dimension

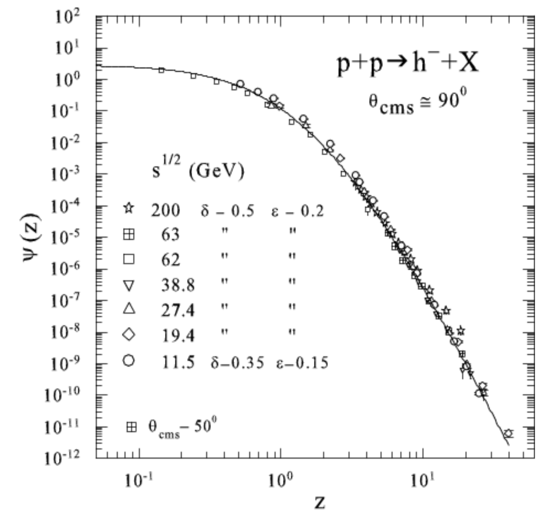
## Scaling function

$$\Psi(z) = \frac{\pi}{(dN/d\eta)\cdot\sigma_{inel}} \cdot J^{-1} \cdot E\frac{d^3\sigma}{dp^3}$$

Strong sensitivity of data point to √s at high $p_T$




"Collapse" of data point onto a single curve



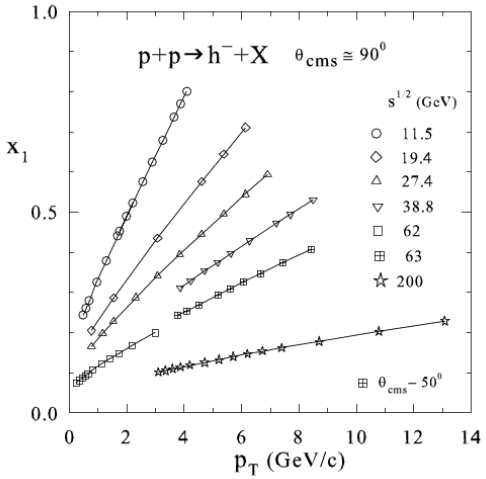

- Energy independence of $\Psi(z)$
- Centrality independence of $\Psi(z)$
- Power law at high z
- Saturation at low z

- Universality: the same shape of $\Psi(z)$ vs. $\sqrt{s}$, $p_T$
- Asymptotic behavior of $\Psi(z)$ at high z – power law.

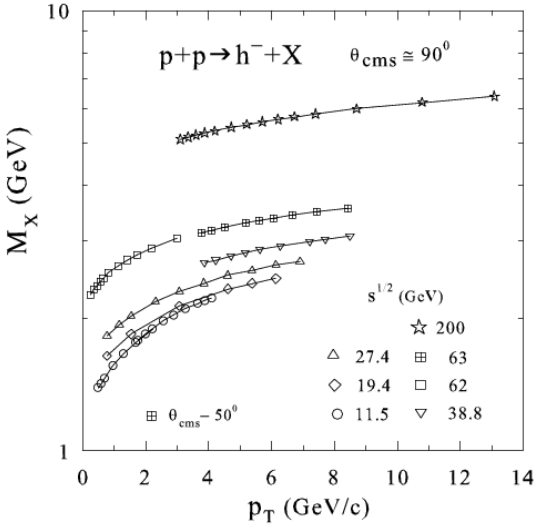
# Constituent sub-process of h⁻ production in p+p

## Constituent level of particle production in terms of

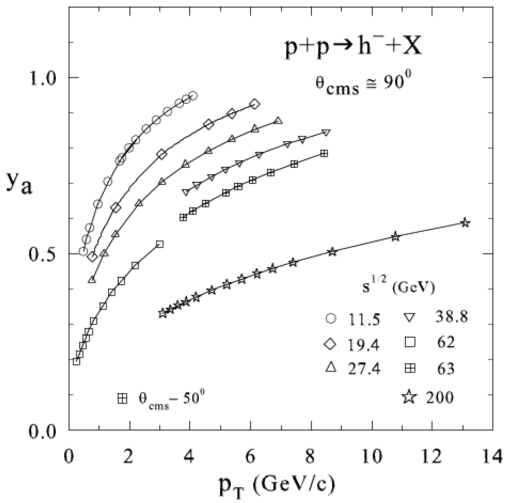
Momentum fraction $x_1$




Recoil mass $M_X$




Energy loss $\Delta E/E=(1-y_a)$




$x_1$

- increases with $p_T$
- decreases with $\sqrt{s}$

$M_X$

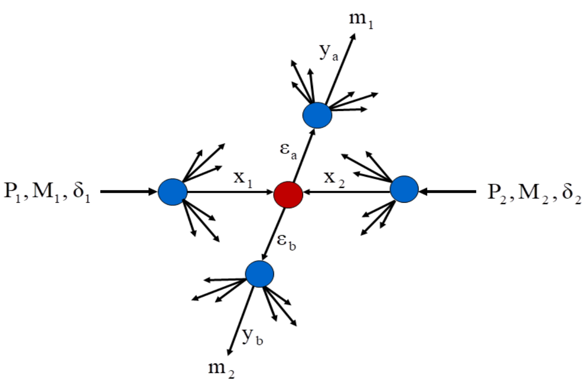
- increases with $p_T$
- increases with $\sqrt{s}$

$\Delta E/E$

- decreases with $p_T$
- increases with $\sqrt{s}$

- p+p is a reference for p+A and A+A
- high $x_1$ and $p_T$ – physics nearby kinematic boundary

# Energy loss in p+p



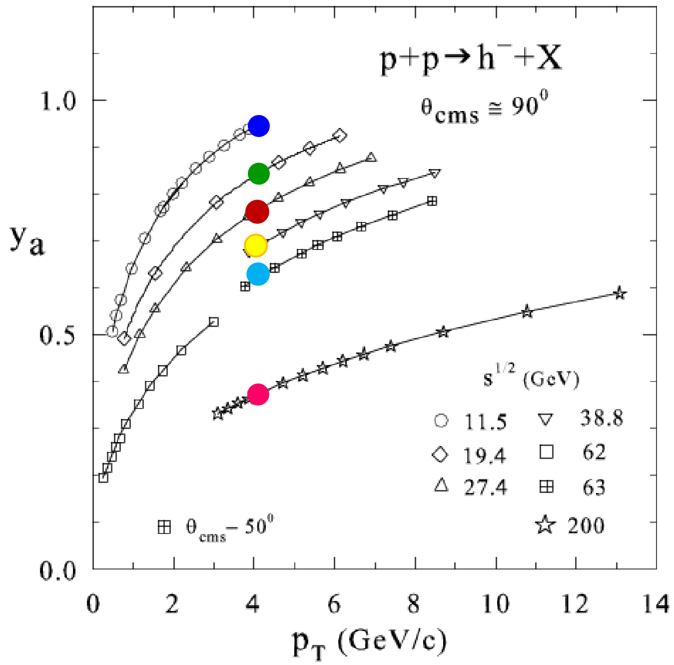

$(x_1 P_1 + x_2 P_2 - q)^2 = M_X^2$

$q = p/y_a$

p – momentum of produced hadron

q – momentum of scattered constituent

Energy loss $\Delta E/E = (1 - y_a)$




$\Delta E/E$

- decreases with $p_T$
- increases with $\sqrt{s}$

$p_T = 4$ GeV/c

**5 %**
**energy loss**
$q \approx 4.2$ GeV/c

**17 %**
**energy loss**
$q \approx 4.8$ GeV/c

**25 %**
**energy loss**
$q \approx 5.3$ GeV/c

**32 %**
**energy loss**
$q \approx 5.9$ GeV/c

**38 %**
**energy loss**
$q \approx 6.5$ GeV/c

**68 %**
**energy loss**
$q \approx 12.5$ GeV/c

# Conclusions I

- Self-similarity of particle structure, constituent interactions and fragmentation process of hadron production in p+p collisions was found.
- Properties of the scaling function were reviewed.
- Model parameters – structural and fragmentation dimensions and specific heat, were determined from data analysis.

# Self-similarity of cumulative hadron production in p+A collisions

p+A is of interest by itself:

- verification of scaling and search for new phenomena
- search for a phase transition with different probes

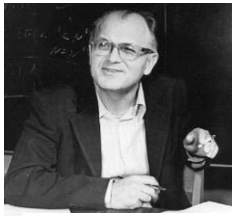
p+A interaction is a reference for A+A physics

I. Zborovský, MT, Phys. Rev. D75 (2007) 094008.
I. Zborovský, MT et. al., Int . J. Mod. Phys. A16 (2001) 1281.
A. Aparin, MT, Nucl. Phys. B 245 (2013) 149-152.
A. Aparin, MT, Phys. Part. Nucl. Lett., 11 (2014) 91-100;
11 (2014) 381-390; 11 (2014) 391-403.

# Cumulative particle, process, region,…

A.M.Baldin & V.S.Stavinsky (1971,1973)

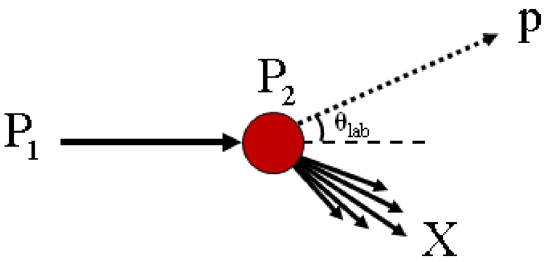

A.M.Baldin


V.S.Stavinsky

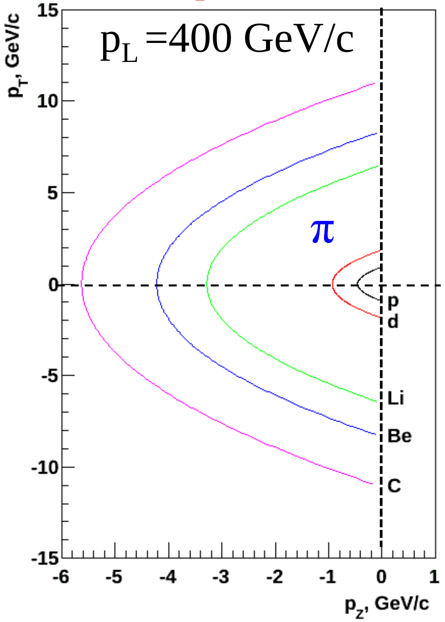
The cumulative particle is a particle produced in the region forbidden for free nucleon kinematics:

$$P_1 + P_2 \to p + X$$

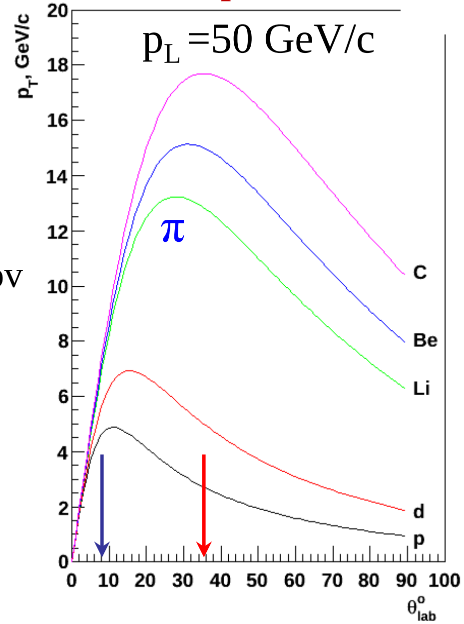
$$(P_1 + P_2 - p)^2 = M_X^2 \qquad \min M_X \Rightarrow p^A_{max} > p^p_{max}$$




G.A.Leksin (1980)



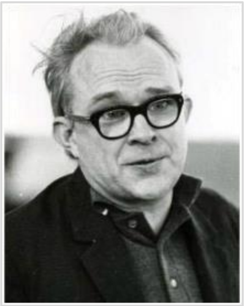

V.V.Ammosov (2013)




Conservation laws:
- 4-momentum
- electric charge
- baryon number
- flavor

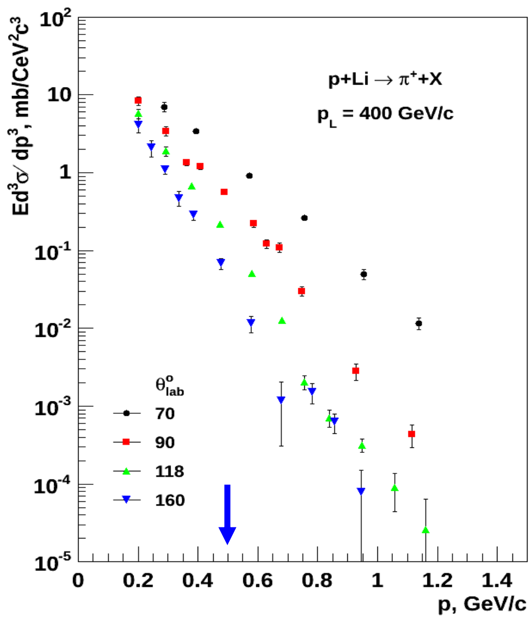
# Cumulative pion spectra in p+A at FNAL

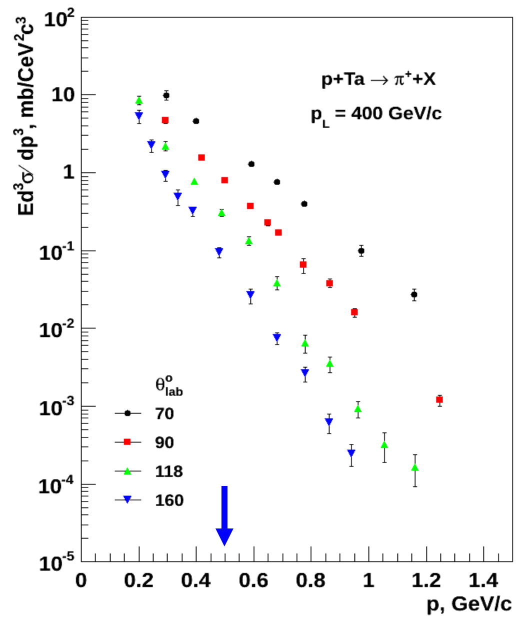




G. Leksin

$p_L$ = 400 GeV/c, A=Li,Be,C,Al,Cu,Ta $\theta_{lab}$=70,90,118,160 deg.

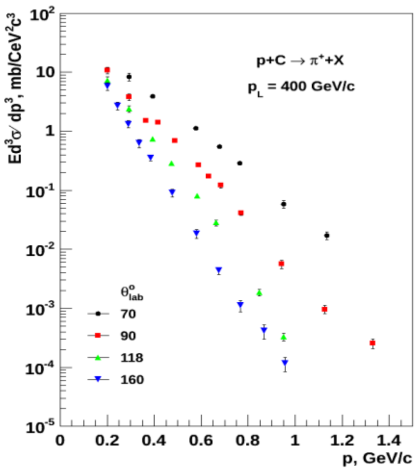


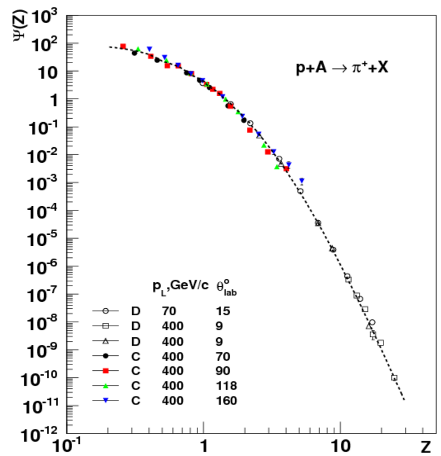



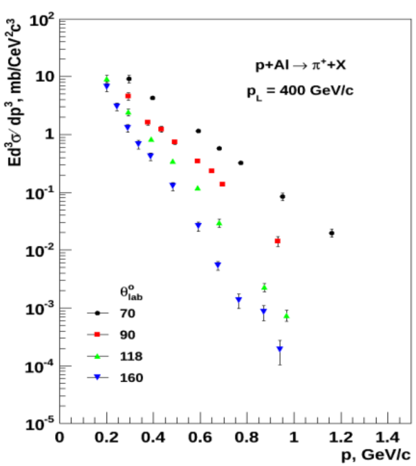

- Spectra in cumulative region: p > 0.5 GeV/c.
- Smooth behavior of spectra vs. p.
- Strong angular dependence with p.
- A-dependence of spectra (A=7-181 ).

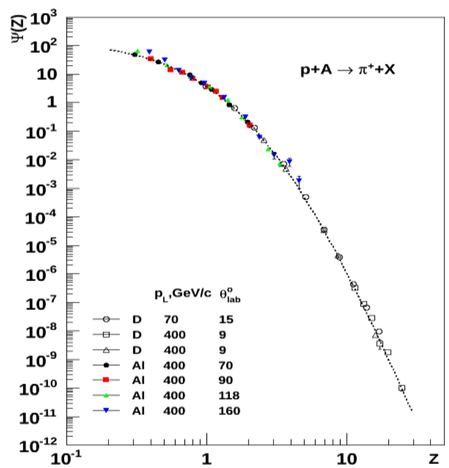
N.A. Nikiforov et al., Phys. Rev. C22 (1980)700.

# High-$p_T$ and low-$p_T$ pion production in p+A




## C, Al & D

- Collapse of data points
- Universal shape of Ψ(z)
- Self-similarity over a wide kinematic range

$$\theta^{\pi}_{lab} = 180^0$$

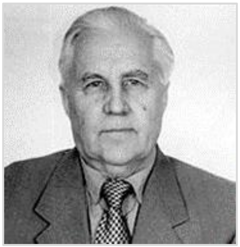
| $p^{\pi}_{max}$ (GeV/c) | p | D | C | Al | $p_L$ (GeV/c) |
|---|---|---|---|---|---|
| | 0.447 | 0.905 | 5.13 | 10.6 | 70 |
| | 0.456 | 0.928 | 5.53 | 12.2 | 400 |

**Cumulation under nucleus compression**


N.A. Nikiforov et al., Phys. Rev. C22 (1980) 700.
&
J.W. Cronin et.al., Phys. Rev. D11 (1975) 3105.
D. Antreasyan et al., Phys. Rev. D19 (1979) 764.
V.V. Abramov et al., Sov. J. Nucl. Phys. 41 (1985) 357.
D.E. Jaffe et al., Phys. Rev. D40 (1989) 2777.

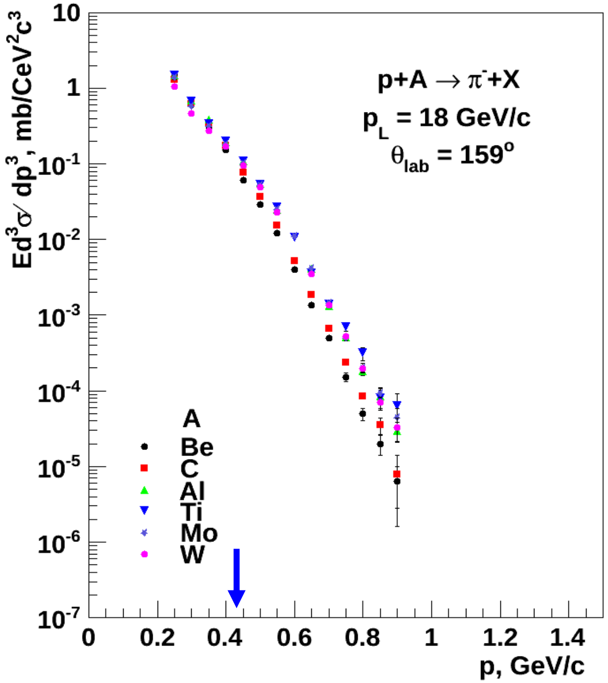

D.Toivonen, M.T. (2003)
A. Aparin, M.T. (2014)

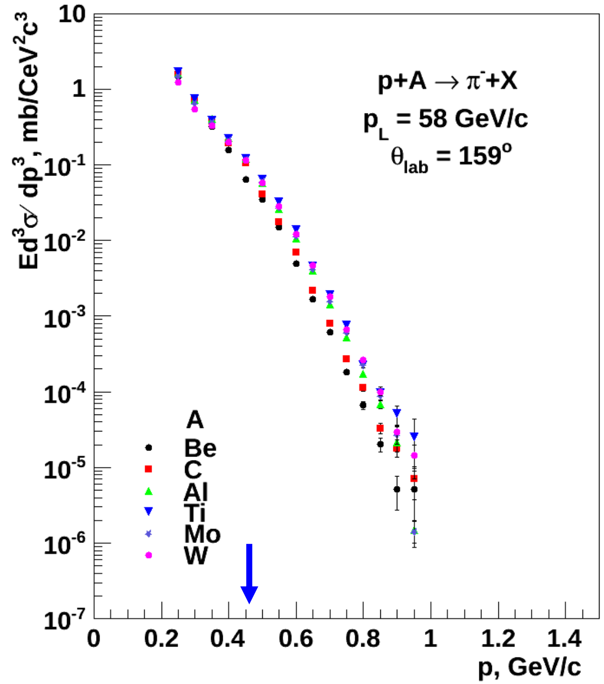
# Low-$p_T$ cumulative pion spectra in p+A at U70



L. Zolin

$p_L$ =17, 58 GeV/c, A= Be,C,Al,Ti,Mo,W, $\theta_{lab}$=159 deg.




- Spectra in cumulative region: p > 0.5 GeV/c.
- Smooth behavior of spectra vs. p .
- A-dependence of spectra (A= 9-184).

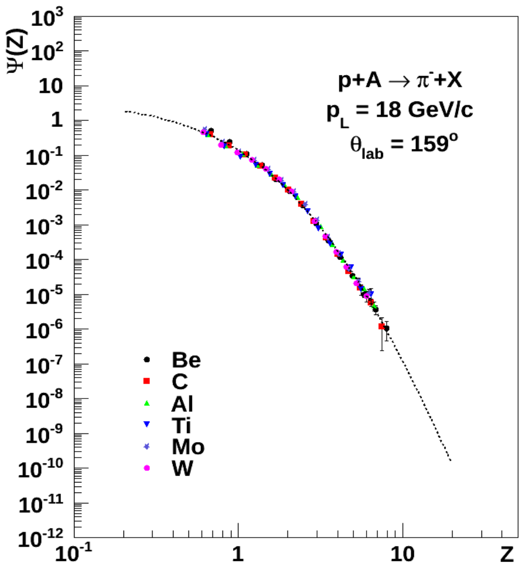
O.P.Gavrishchuk et al., Nucl. Phys. A523 (1991) 589.

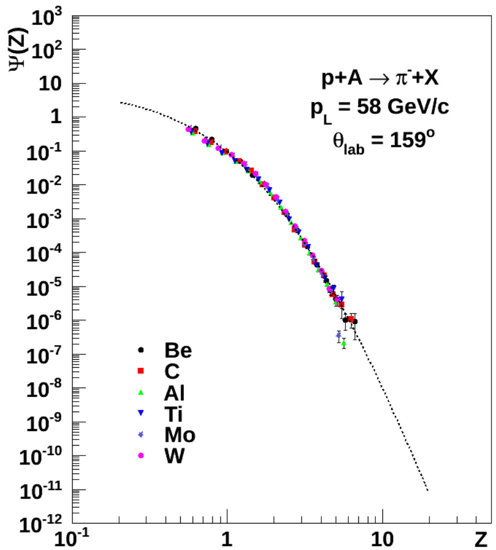
# Low-$p_T$ cumulative pion spectra in p+A at U70



## z-presentation of spectra

U70 (L. Zolin)




Be,C, Al,Ti,Mo,W & D

- Collapse of data points
- Universal shape of $\Psi(z)$
- Self-similarity over a wide kinematic range

$\theta_{lab} = 159^0$

| | p | C | Ti | W | $p_L$ (GeV/c) |
|---|---|---|---|---|---|
| $p^{\pi}_{max}$ (GeV/c) | 0.43 | 4.1 | 9.56 | 14.1 | 18 |
| | 0.46 | 5.2 | 16.3 | 34.5 | 58 |

Cumulation under nucleus compression

- "Universal shape " of $\Psi(z)$
- Power law for $z > 4$
- No discontinuity of $\delta_A$

Self-similar properties of nuclear matter

A=9-184

A.Aparin, MT, Phys. Part. Nucl. Lett., 11 (2014) 391

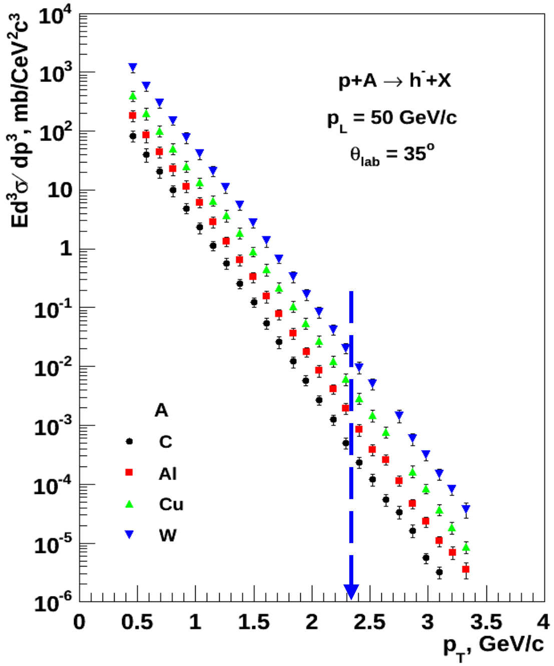
# High-$p_T$ cumulative hadron spectra in p+A at U70

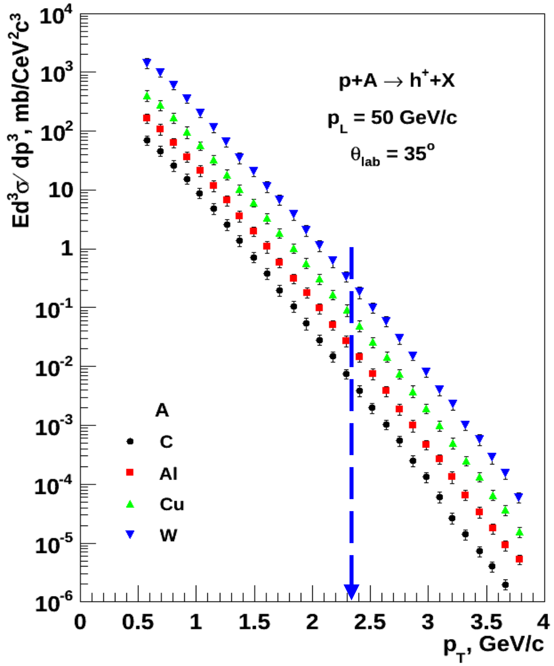


V. Gapienko

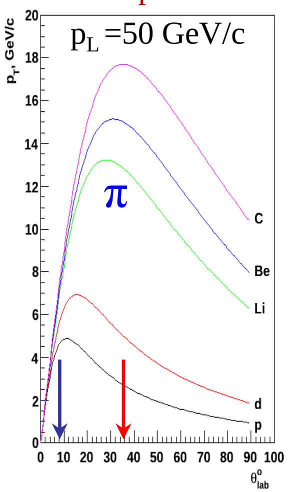
$p_L$ =50 GeV/c, A=C,Al,Cu,W, $\theta_{lab}$=35 deg.

V. Ammosov




pA



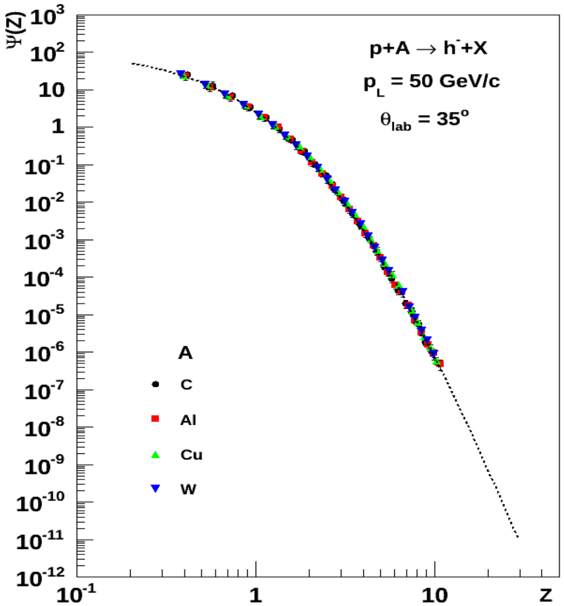

- Spectra in cumulative region: $p_T > 2.5$ GeV/c.
- Smooth behavior of spectra vs. $p_T$.
- A-dependence of spectra (A=12-184 ).

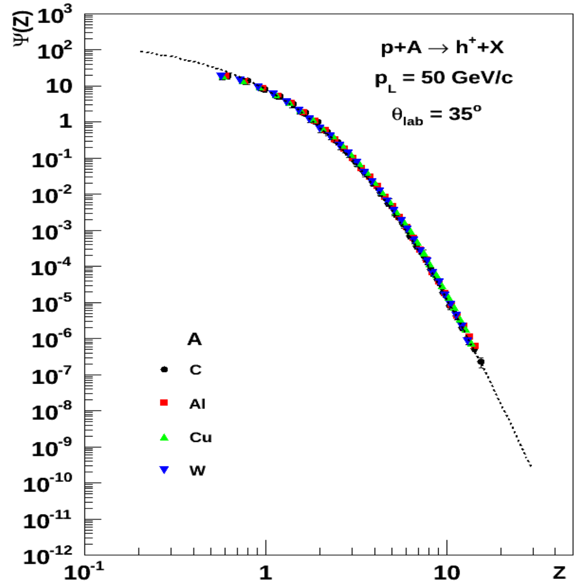
N.N.Antonov et al. (IHEP, Protvino) "Physics of Fundamental Interactions",
Russian Academy of Science, ITEP,Moscow, Russia, 21-25 November, 2011.
V.V.Ammosov et al., Yad.Fiz. 76 (2013) 1276.

# High-$p_T$ cumulative hadron spectra in p+A at U70

## z-presentation of spectra

FNAL (J.Cronin, D.Jaffe) & U70 (R.Sulyaev, V.Gapienko)




C, Al, Cu, W & D

- Collapse of data points
- Universal shape of $\Psi(z)$
- Self-similarity over a wide kinematic range

$p_L = 50$ (GeV/c) $\theta_{lab} = 35^0$

| $p^h_{T\,max}$ (GeV/c) | p | C | Al | Cu | W |
|---|---|---|---|---|---|
| | 2.62 | 15.6 | 20.7 | 24.4 | 26.7 |

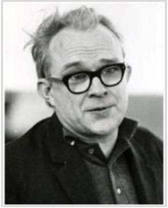
- Universal shape of $\Psi(z)$
- Power law for $z > 4$
- No discontinuity of $\delta_A$

Cumulation under nucleus compression

Self-similar properties of nuclear matter $A=12\text{-}184$

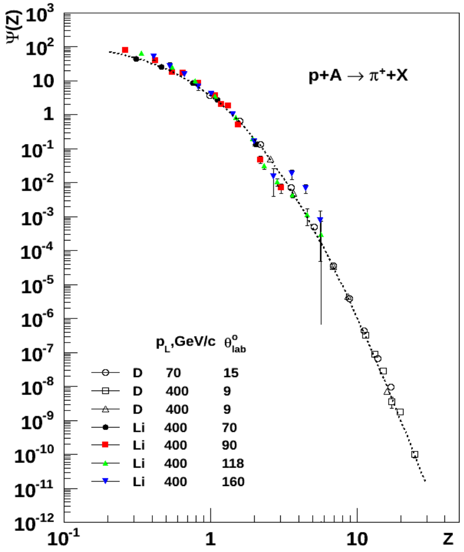
A.Aparin, MT
Phys. Part. Nucl. Lett., 11 (2014) 381.

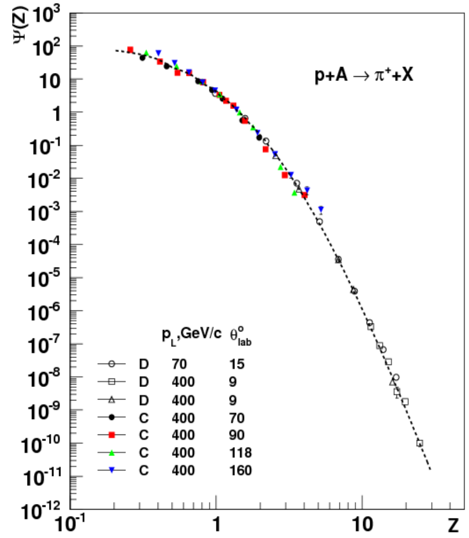
# Low-$p_T$ cumulative pion production in p+A at FNAL

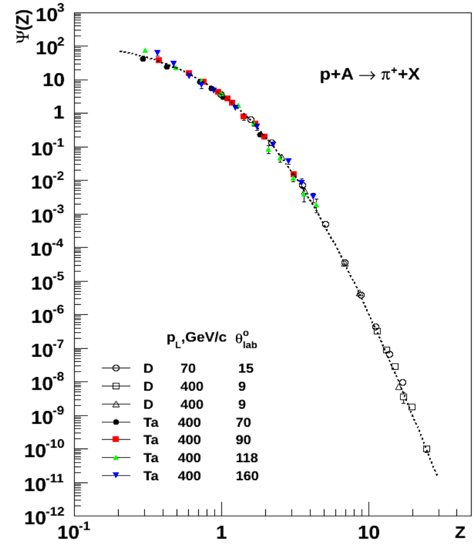


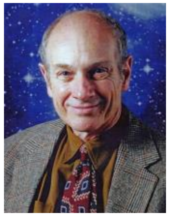
G. Leksin

FNAL (J.Cronin, G.Leksin, D.Jaffe) & U70 (R.Sulyaev)



J. Cronin

R. Sulyaev

D. Jaffe

- Universal shape of $\Psi(z)$
- Power law for $z > 4$
- No discontinuity of $\delta_A = A_2\delta$

Scale invariance

Independence of the shape of the curve on $\{z,\Psi\}$ plane on scale quantities $\sqrt{s}$, $p_T$, $\theta$

N.A. Nikiforov et al., Phys. Rev. C22 (1980) 700.
A.Aparin, MT, Phys. Part. Nucl., Lett. 11 (2014) 91

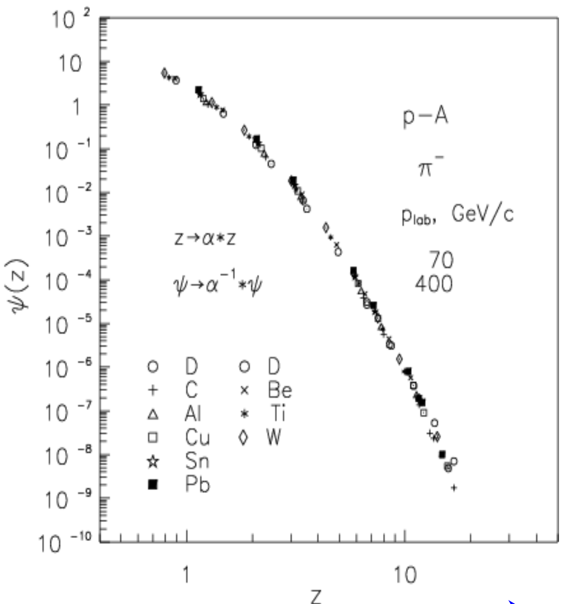
$z \to \alpha(A)z$ $\quad$ $\Psi \to \alpha^{-1}(A)\Psi$

# Self-similarity of hadron production in p+A

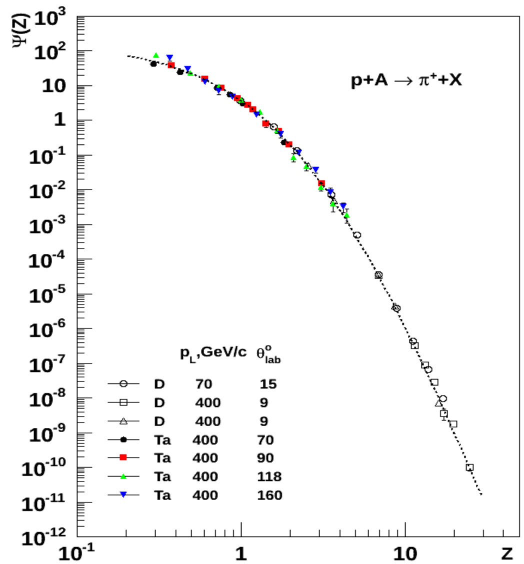


FNAL (J.Cronin, G.Leksin, D.Jaffe) & U70 (R.Sulyaev, V.Gapienko)

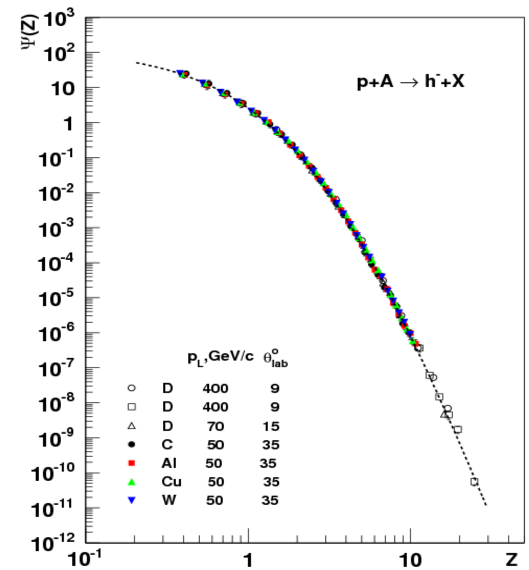
high-$p_T$ & non-cumulative




low-$p_T$ & cumulative




high-$p_T$ & cumulative




- Beam Energy Scan in p+A
- Spectra of cumulative identified particles
- Multiplicity density $dN_{ch}/d\eta$ vs. $\sqrt{s}$ and $\eta$
- Centrality dependence of spectra

- Verification of z-scaling in cumulative range
- Smooth transition from high-$p_T$ to cumulative range

# Conclusions II

- Self-similarity of particle structure, constituent interactions and fragmentation process of hadron production in non-cumulative and cumulative regions in p+A collisions were verified.
- Smooth behavior of $\Psi(z)$ in the overlapping range was found.

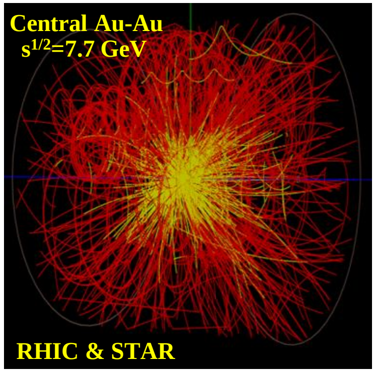
# Scaling features of hadron production in Au+Au at RHIC

Probing microscopic structure of the hot- and high-density nuclear matter at multiple length scales

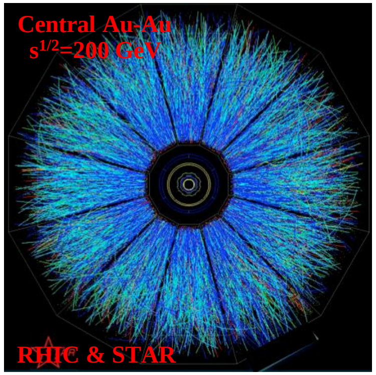
Self-similarity of hadron production

RHIC beam energy scan with Au+Au:

$\sqrt{s_{NN}}$ = 7.7, 11.5, 14.5, 19.6, 27, 39, 62, 130, 200 GeV








Int. J. Mod. Phys. (2015) 1560103
Nucl. Phys. A993 (2020) 121646
Nucl. Phys. A1025 (2022) 122492

# Self-similarity in Au+Au collisions

## Self-similarity parameter

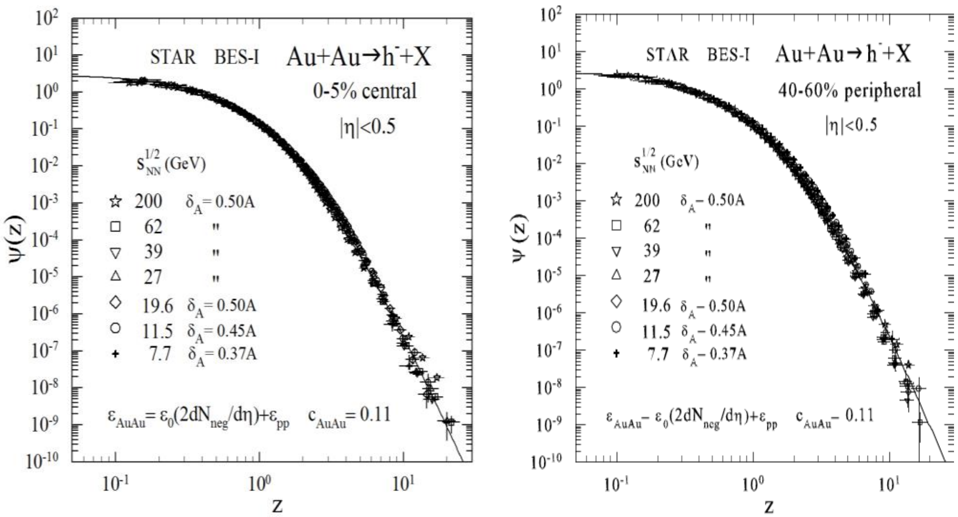
$$z = z_0 \Omega^{-1}$$

$$z_0 = \frac{s_\perp^{1/2}}{(dN_{ch}/d\eta|_0)^c m_N}$$

$$\Omega = (1-x_1)^{\delta_{A_1}}(1-x_2)^{\delta_{A_2}}(1-y_a)^{\varepsilon_{AA}}(1-y_b)^{\varepsilon_{AA}}$$

- $dN_{ch}/d\eta|_0$ - multiplicity density
- $c_{AA}$ - "specific heat" of bulk matter
- $\delta_A$ - nucleus fractal dimension
- $\varepsilon_{AA}$ - fragmentation dimension

"Collapse" of data points onto a single curve




## A+A collisions:

$$\delta_A = A\delta$$

$$\varepsilon_{AA} = \varepsilon_0 (dN_{AA}/d\eta) + \varepsilon_{pp}$$

$$\Psi(z) = \frac{\pi}{(dN/d\eta)\,\sigma_{inel}} J^{-1} E \frac{d^3\sigma}{dp^3}$$

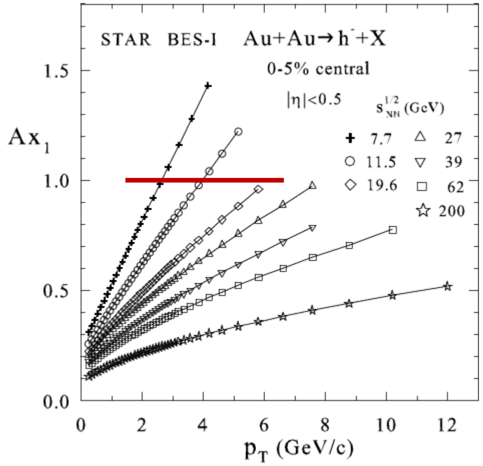
- Energy independence of $\Psi(z)$
- Centrality independence of $\Psi(z)$
- Dependence of $\varepsilon_{AA}$ on multiplicity
- Power law at low- and high-z regions

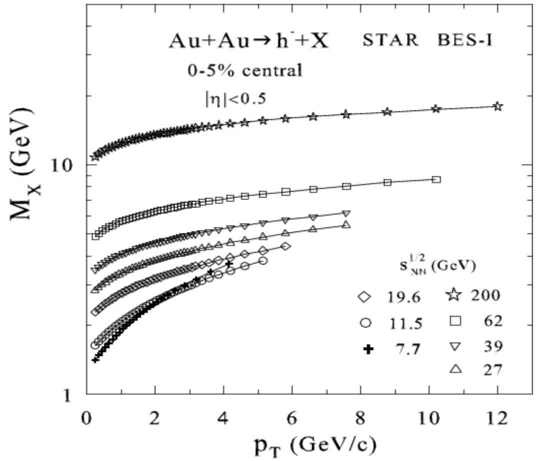
Indication of a decrease of $\delta$ for $\sqrt{s_{NN}} < 19.6$ GeV

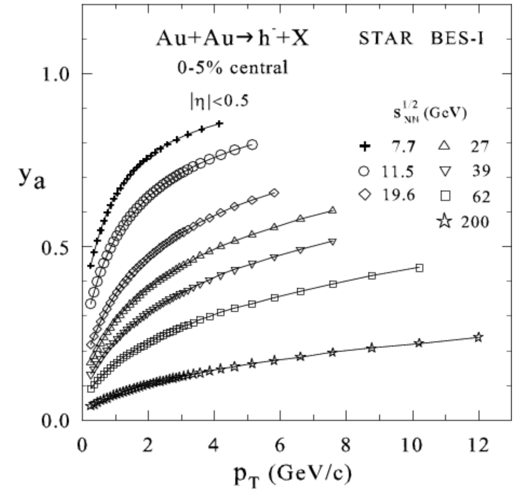
# Self-similarity of h⁻ production in central Au+Au

## Constituent sub-process in terms of

Momentum fraction $Ax_1$ | Recoil mass $M_X$ | Energy loss $\Delta E/E=(1-y_a)$




$Ax_1$
- increases with $p_T$
- decreases with $\sqrt{s_{NN}}$

$M_X$
- increases with $p_T$
- increases with $\sqrt{s_{NN}}$

$\Delta E/E$
- decreases with $p_T$
- increases with $\sqrt{s_{NN}}$

**Smooth behavior of $x_1$, $y_a$, $M_X$ vs. $p_T$, centrality, collision energy**

- High $x_1$ and $p_T$ → compressed matter
- Large $M_X$ → high density recoil system
- High $y_a$ → small energy loss

# Energy loss in Au+Au

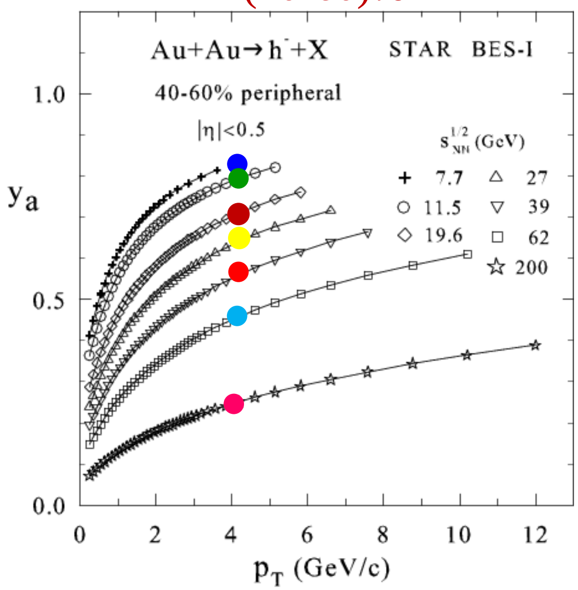
Energy loss $\Delta E/E = (1-y_a)$

$p_T$ – transverse momentum of produced hadron
$q$ – momentum of scattered constituent

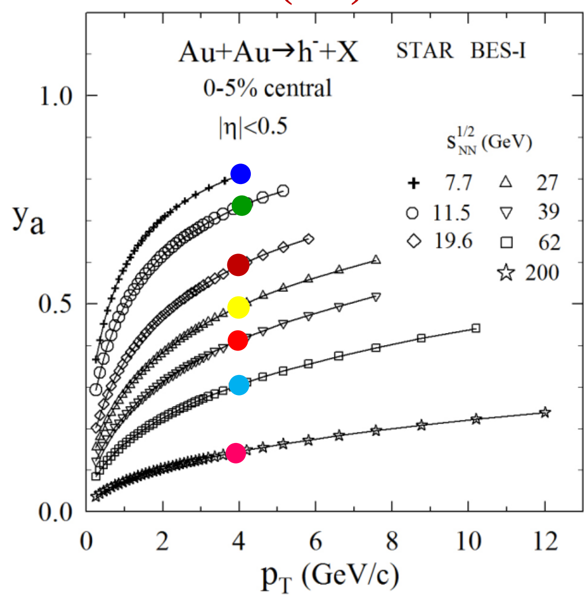
$p_T = 4$ GeV/c , $q = p_T / y_a$

| (40-60)% | | (0-5)% | |
|---|---|---|---|
| **20 %** **energy loss** $q \approx 5$ GeV/c | | **20 %** **energy loss** $q \approx 5$ GeV/c | |
| **22 %** **energy loss** $q \approx 5.1$ GeV/c | | **25 %** **energy loss** $q \approx 5.3$ GeV/c | |
| **30 %** **energy loss** $q \approx 5.7$ GeV/c | | **40 %** **energy loss** $q \approx 6.7$ GeV/c | |
| **35 %** **energy loss** $q \approx 6.2$ GeV/c | | **50 %** **energy loss** $q \approx 8$ GeV/c | |
| **45 %** **energy loss** $q \approx 7.3$ GeV/c | | **60 %** **energy loss** $q \approx 10$ GeV/c | |
| **55 %** **energy loss** $q \approx 8.9$ GeV/c | | **70 %** **energy loss** $q \approx 13.3$ GeV/c | |
| **75 %** **energy loss** $q \approx 16$ GeV/c | | **85 %** **energy loss** $q \approx 26.6$ GeV/c | |

(40-60)%




(0-5)%




$\Delta E/E$

- decreases with $p_T$
- increases with $\sqrt{s_{NN}}$
- increases with centrality

# Model parameters: $\delta_A$, $\varepsilon_{AA}$, $c_{AA}$

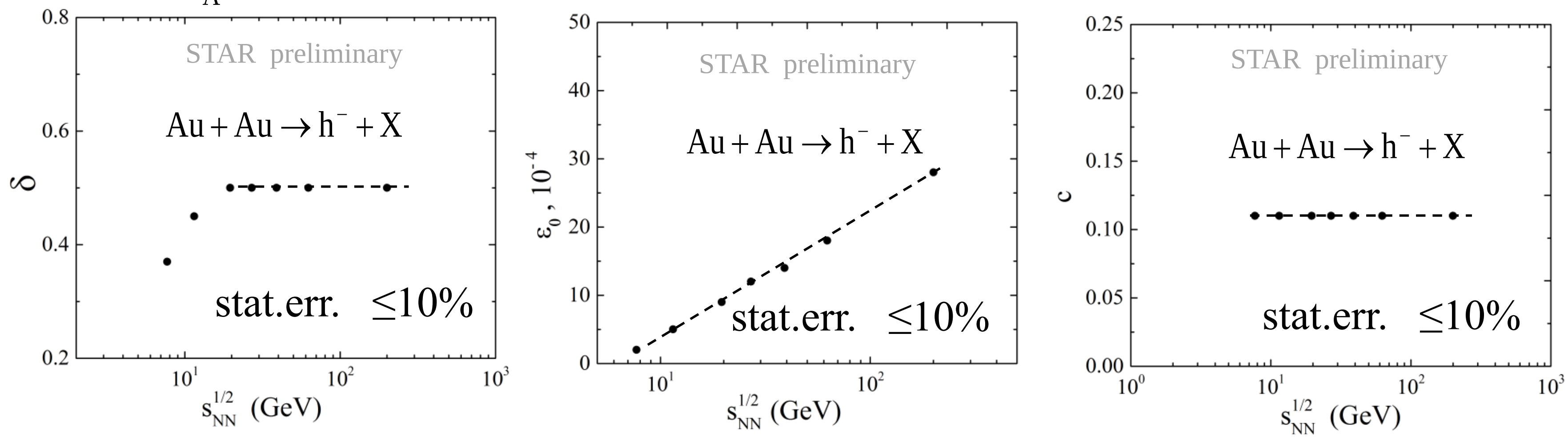


Parameters $\delta_A$, $\varepsilon_{AA}$, $c_{AA}$ are determined from the requirement of scaling behavior of $\Psi$ as a function of self-similarity parameter $z$

**Nucleus fractal dimension** $\delta_A = A \cdot \delta$

**Fragmentation dimension** $\varepsilon_{AA} = \varepsilon_0 (dN_{AA}/d\eta) + \varepsilon_{pp}$

**"Specific heat"** $c_{AA}$



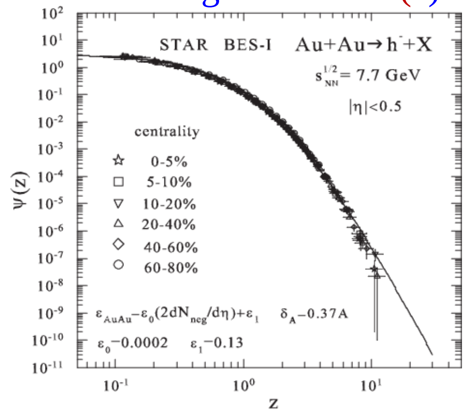

- $\delta_A$ decreases with energy for $\sqrt{s_{NN}} \leq 20$ GeV
- $\delta_A$ is independent of energy for $\sqrt{s_{NN}} \geq 20$ GeV
- $\varepsilon_{AA}$ increases with energy
- $c_{AA}$ is independent of energy

Search for discontinuity and correlations of the model parameters as signatures of Phase Transition and Critical Point.

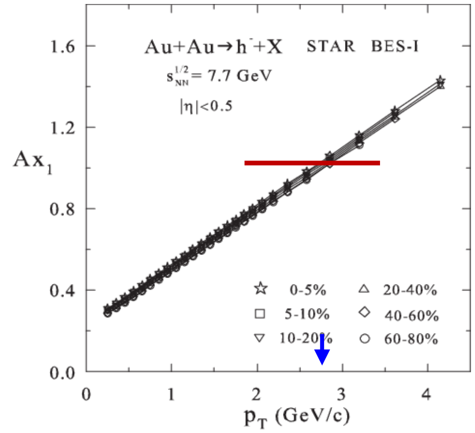
# Cumulative hadron production at RHIC in Au+Au at STAR & $\sqrt{s_{NN}}$ =7.7 GeV



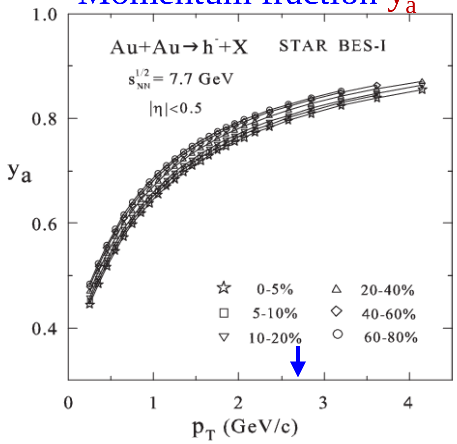
### Scaling function $\Psi(z)$



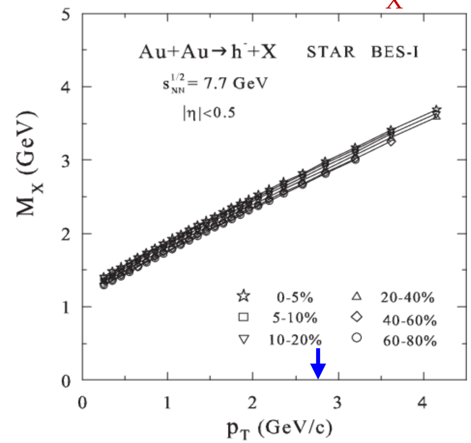

### Momentum fraction $Ax_1$




### Momentum fraction $y_a$



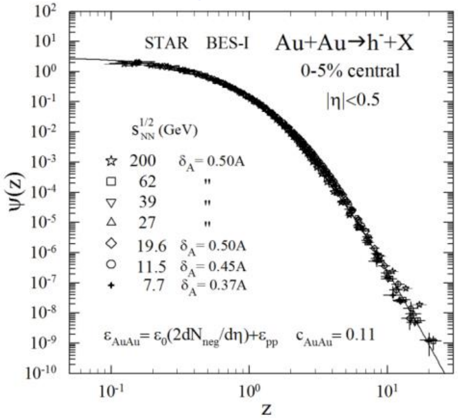

### Recoil mass $M_X$



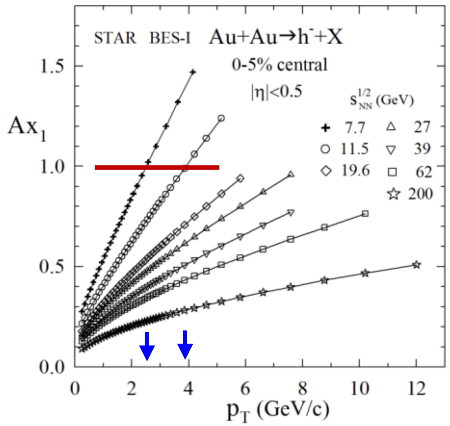

Smooth behavior of $\Psi(z)$, $Ax$, $y_a$, $M_x$ vs. $p_T$

# Central Au+Au at STAR & $\sqrt{s_{NN}}$ = 7.7 – 200 GeV

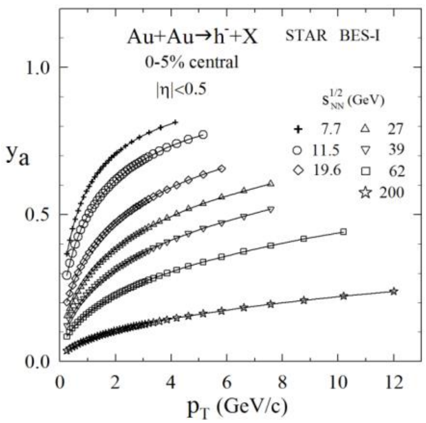
Scaling function Ψ(z)



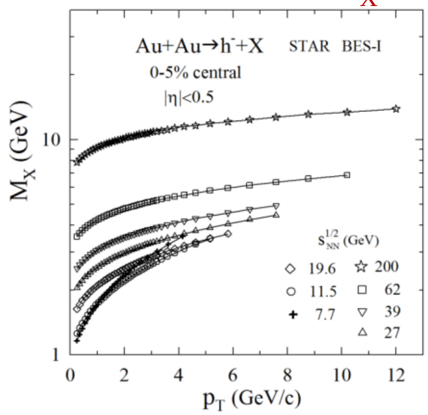

Momentum fraction $Ax_1$




Momentum fraction $y_a$




Recoil mass $M_X$




Cumulative region was only reached at $\sqrt{s_{NN}}$ < 11.5 GeV

# Conclusions III



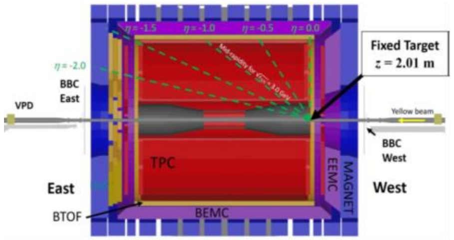
- The STAR BES-I data on negative hadrons produced in Au+Au collisions cover cumulative region at $\sqrt{s_{NN}}$ =7.7, 11.5 GeV.
- Results of analysis demonstrate smooth behavior in $z$-presentation vs. collision energy, centrality over a wide range of $p_T$.
- $z$-Scaling of particle production manifests self-similarity, locality and fractality of hadron interactions at a constituent level.

# Self-similarity of cumulative production in collider and fixed target mode at RHIC & NICA ?

RHIC & STAR

CM: two beams, $\sqrt{s_{NN}}$ = 7.7-200 GeV

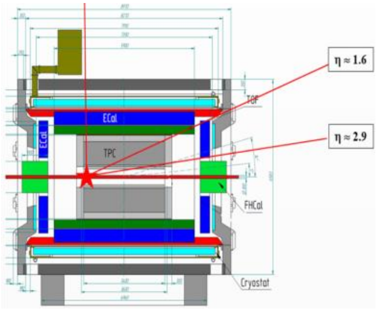
FXT: one beam, $\sqrt{s_{NN}}$ = 3.0-7.7 GeV




NICA & MPD

CM: two beams, $\sqrt{s_{NN}}$ = 4-11 GeV

FXT: one beam, $\sqrt{s_{NN}}$ = 2.4-3.5 GeV




We consider that:

- Smaller energy losses is better for localization of a Critical Point.
- High-$p_T$ region is most preferable region to search for a Critical Point.
- Colliding ions should be not very heavy.
- Collision energy $\sqrt{s_{NN}}$ should be not very high.

G.A.Leksin
Phys. At. Nucl.
65 (2002) 1985.

I.Zborovský, MT
Phys. Part. Nucl., Lett.
7 (2010) 271.

I.Zborovský, MT, A.Aparin
Phys. Part. Nucl., Lett.
12 (2015) 221.

# Self-similarity of $\pi^-$ production in Au+Au

## Self-similarity parameter

$$z = z_0 \Omega^{-1}$$

$$z_0 = \frac{s_\perp^{1/2}}{(dN_{ch}/d\eta|_0)^c m_N}$$

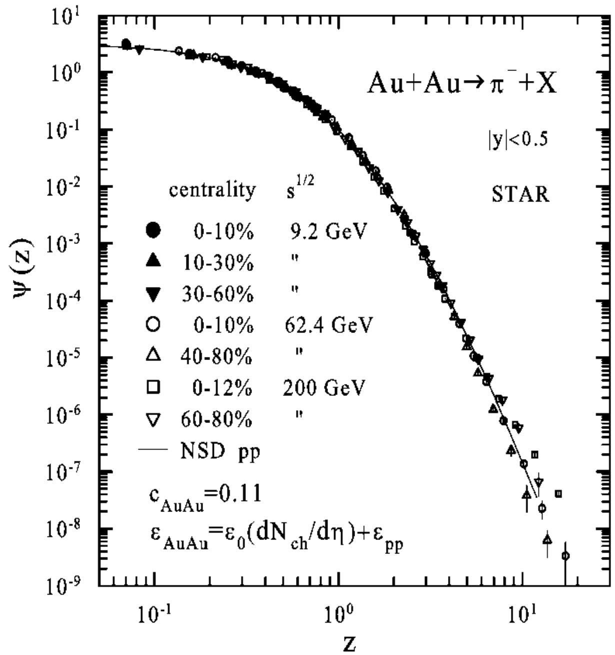
$$\Omega = (1-x_1)^\delta (1-x_2)^\delta (1-y_a)^{\varepsilon_F} (1-y_b)^{\varepsilon_F}$$

- $dN_{ch}/d\eta|_0$ - multiplicity density
- c - "specific heat" of bulk matter
- $\delta$ - nucleus fractal dimension
- $\varepsilon_F$ - fragmentation fractal dimension

## Scaling function

$$\Psi(z) = \frac{\pi}{(dN/d\eta) \cdot \sigma_{inel}} \cdot J^{-1} \cdot E \frac{d^3\sigma}{dp^3}$$

"Collapse" of data points onto a single curve



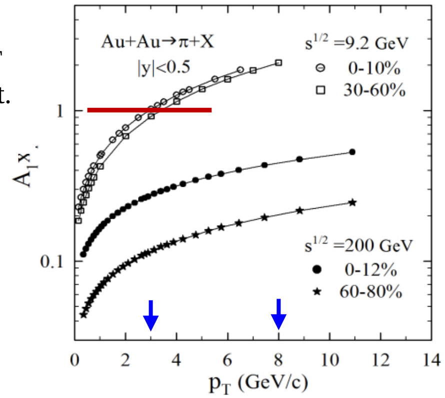

- Energy independence of $\Psi(z)$
- Centrality independence of $\Psi(z)$
- Power law at high z
- Saturation at low z

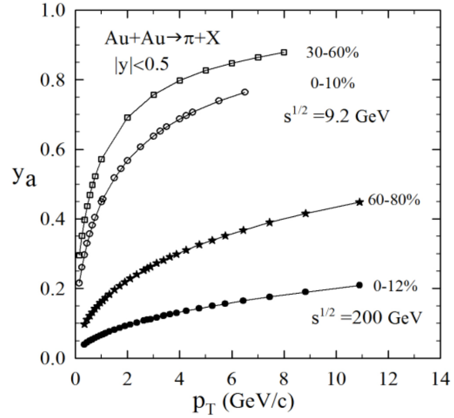
I.Zborovský, MT
Phys. Part. Nucl., Lett. 7 (2010) 271

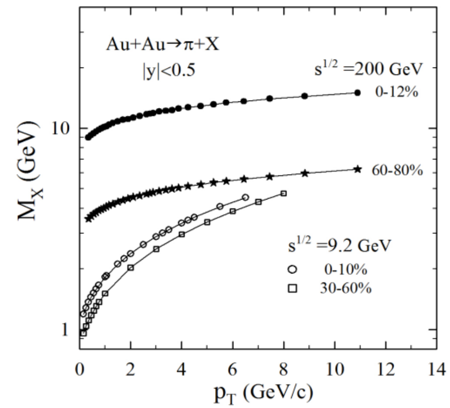
# Collider mode of $\pi^-$ meson production in Au+Au

## Momentum fractions $x_1$, $x_2$, $y_a$ & recoil mass $M_X$

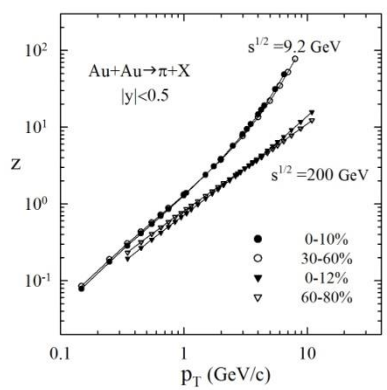
I.Zborovský & MT
Phys.Part.Nucl.,Lett.
7 (2010) 271



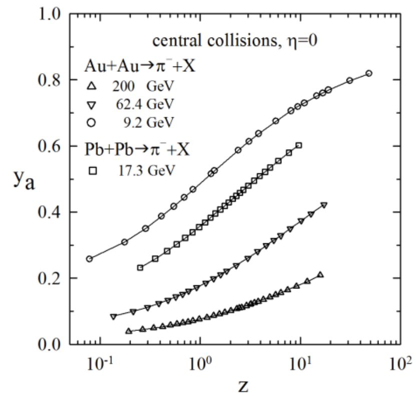



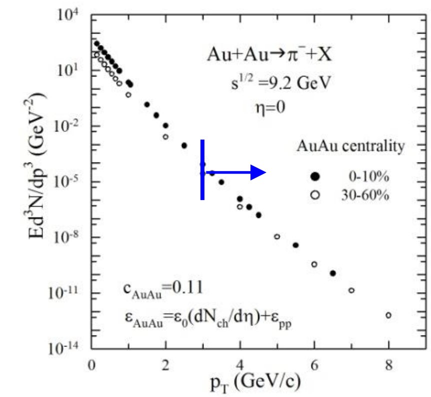

## z-$p_T$ plot




## $y_a$-z plot




## $p_T$ -spectra



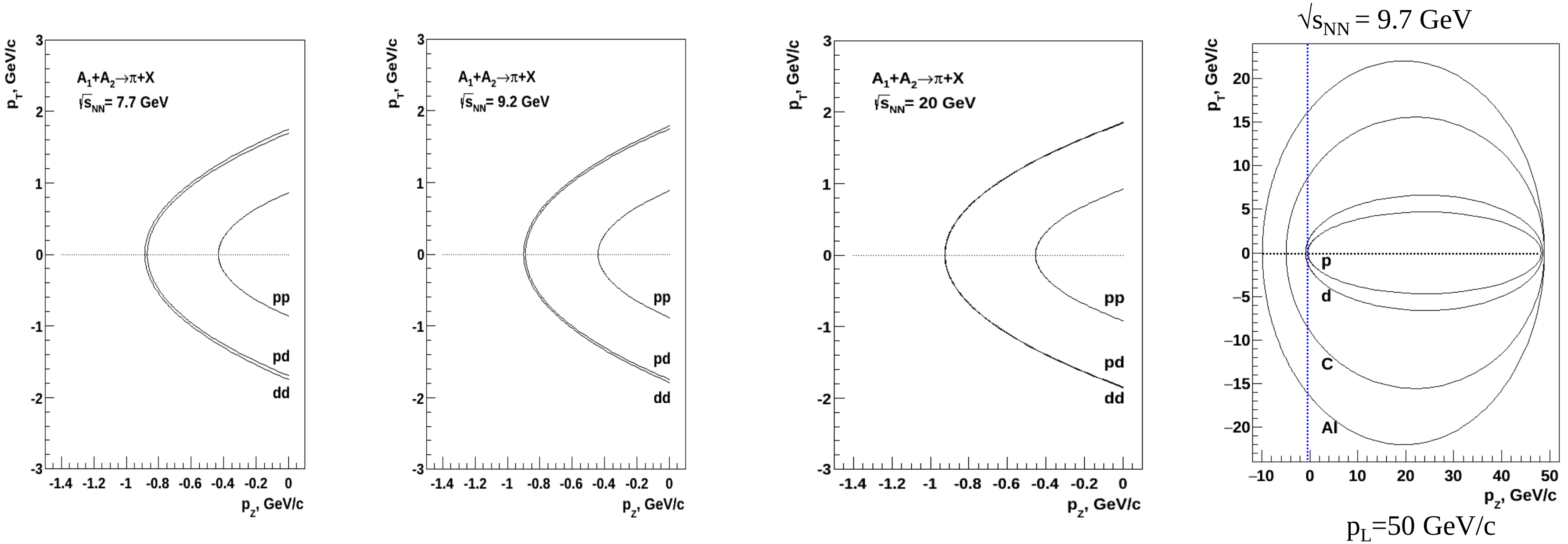

Cumulative region: $A_1x_1 > 1$, $p_T > 3$ GeV/c $\rightarrow$ $A_1x_1 = 2$, $p_T = 8$ GeV/c

Energy loss: $\Delta E/E < 40\%$ $\rightarrow$ 20%

Recoil mass: $M_X > 2$ GeV $\rightarrow$ 4 GeV

# Fixed target mode of $\pi^-$ meson production in Au+Au

## Kinematics of backward pion production in p+p & p+A




- Cumulative region is achievable
- Wide kinem. range - $\sqrt{s_{NN}}$ , $\theta_{lab}$ , $p_T$
- Target fragmentation – p+A, A+A

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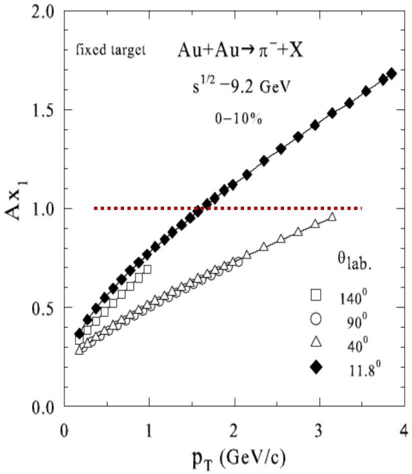
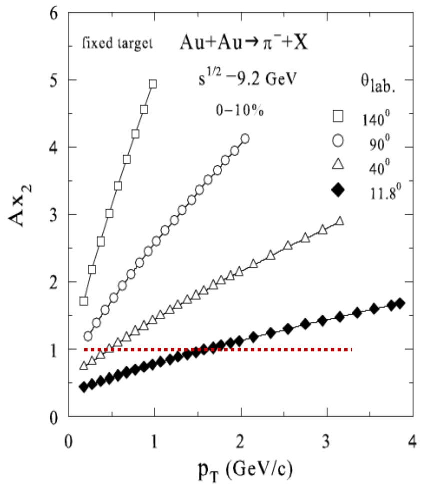
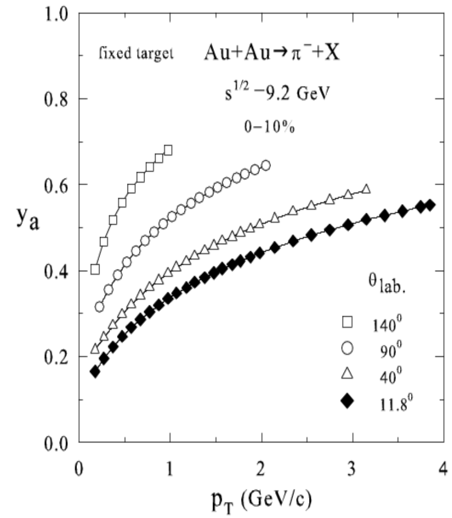
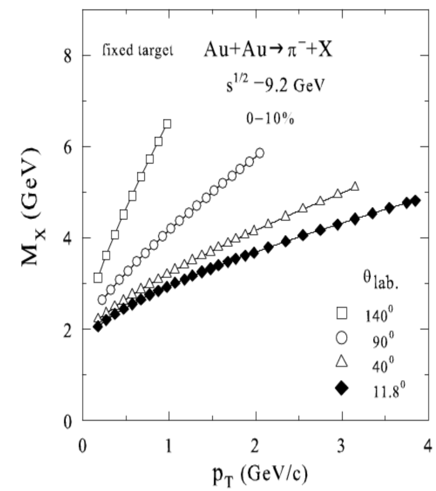
# Microscopic scenario of pion production in Au+Au

## Momentum fractions $x_1$, $x_2$, $y_a$ & recoil mass $M_X$




### Kinematics and event selection

- Cumulative range: $Ax_2 > 1$
- Small energy loss : $\Delta E/E = (1 - y_a)$
- Probe with high $p_T$
- Events with high multiplicity

### In cumulative region

- Nuclear matter compressed
- Phase transition not smeared

↓

**Clear signatures of phase transition and critical point**

# Conclusions IV



- Pion production in collider and fixed target modes in Au+Au collisions in the framework of z-scaling approach were analyzed.
- Dependence of momentum fractions $x_1$, $x_2$, $y_a$ and recoil mass $M_X$ on transverse momentum and angle of inclusive particle was studied.
- Verification of self-similarity of cumulative high-$p_T$ pion production in Au+Au collisions was suggested.
- Discontinuity of fractal dimensions of nuclei and fragmentation process and "heat capacity " as a signature of phase transition was discussed.

# Conclusions



- Data on cumulative hadron spectra obtained by G.Leksin, L.Zolin and V.Gapienko groups in p+A collisions at $\sqrt{s_{NN}}$ = 11.5 – 27.4 GeV were reviewed in the framework of z-scaling approach.
- Results of this analysis were compared with previous data obtained by J.Cronin, R.Sulyaev and D.Jaffe groups.
- Indication on self-similarity of hadron production in p+A collisions at high energies in the cumulative region were found.
- Universality of the shape of $\Psi(z)$ was used to predict pion spectra in Au+Au collisions at $\sqrt{s_{NN}}$ = 9.2 GeV in cumulative range.
- Collider and fixed target mode for cumulative production were discussed.

The results can be used to develop programs
to search for new physics phenomena in p+A and A+A collisions
at U70, RHIC, LHC & NICA, FAIR.


**Thank You for Your Attention !**