Probing the nuclear matter equation of state with light nuclei

Viktar Kireyeu for the PHQMD team

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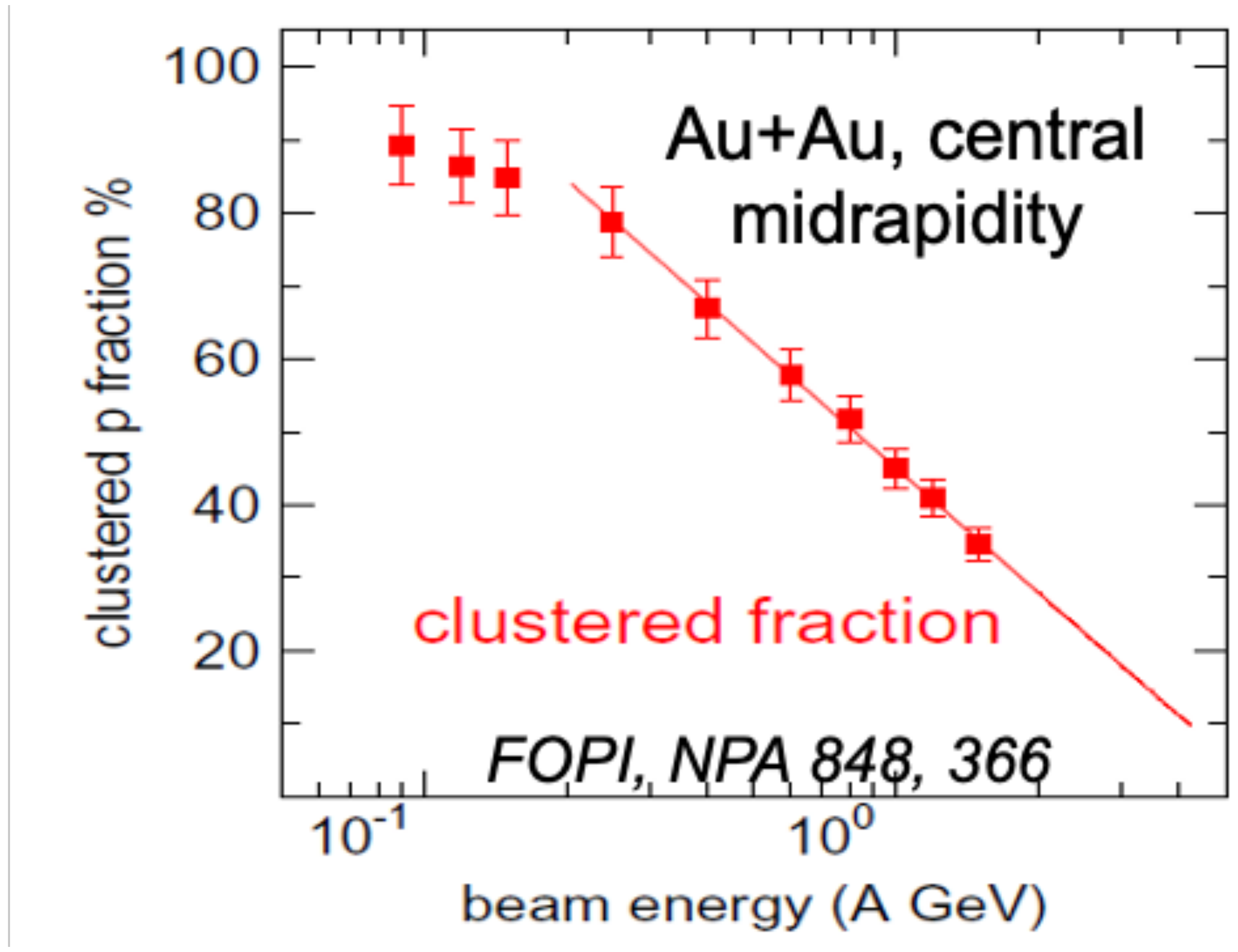
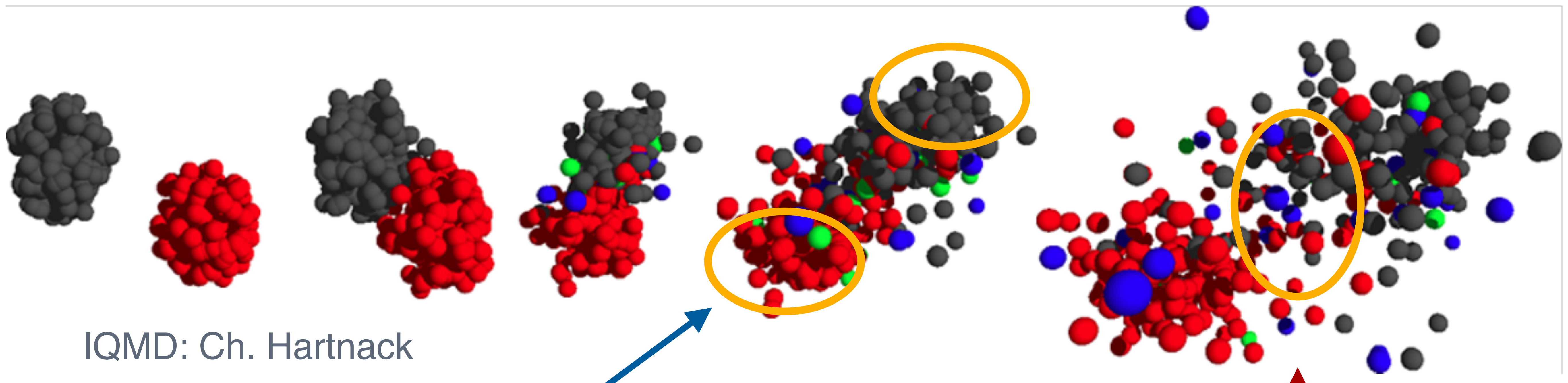
Cluster formation in heavy-ion collisions



FOPI, NPA 848, 366

Clusters and (anti-) hypernuclei are observed experimentally at all energies



IQMD: Ch. Hartnack

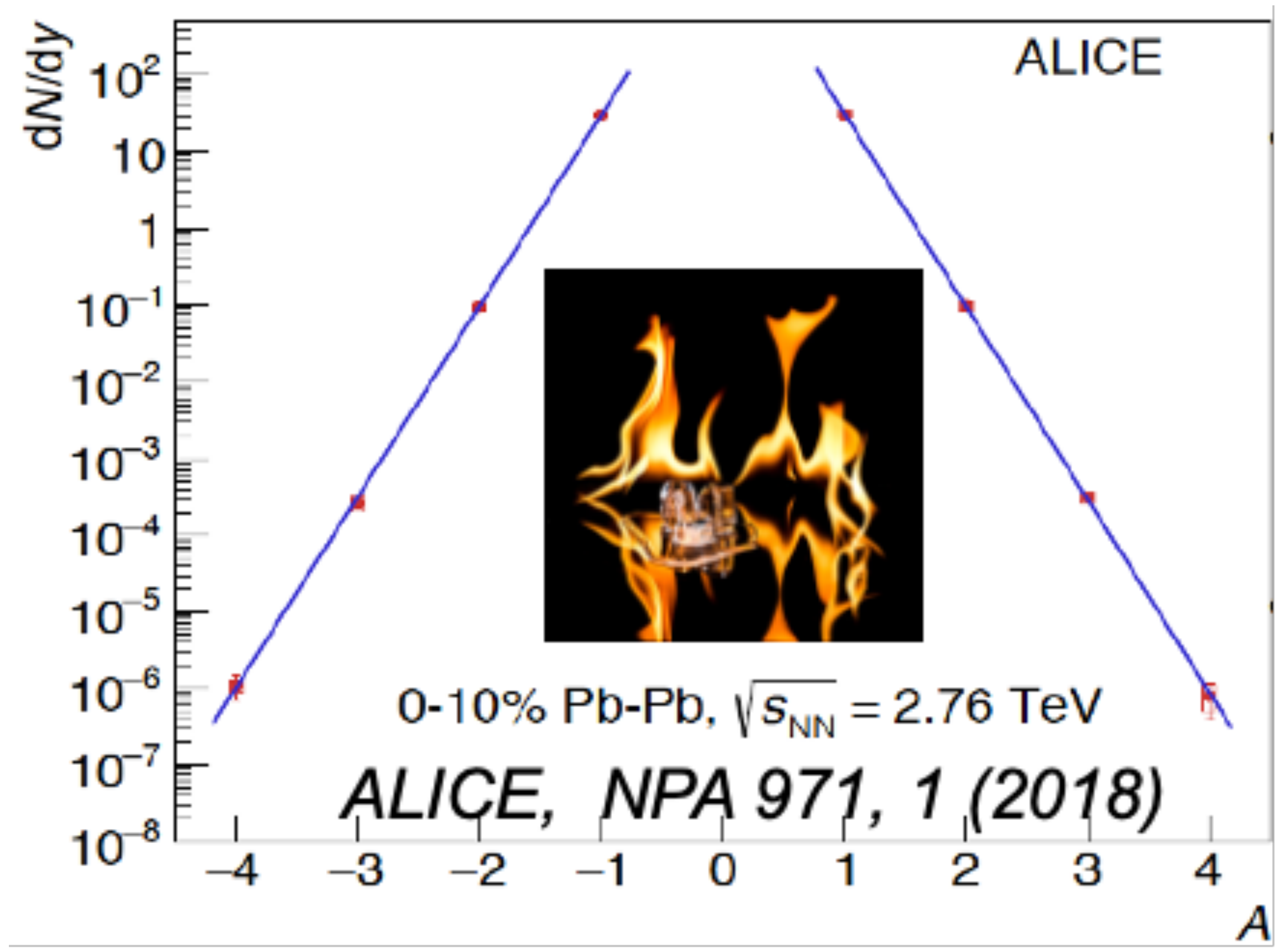
Projectile/target spectators: heavy cluster formation

Midrapidity: light clusters

(Anti)hypernuclei production:

at mid-rapidity by Λ coalescence during expansion

at projectile/target rapidity by re-scattering/absorption of Λ by spectators



ALICE, NPA 971, 1 (2018)

«Ice in a fire» puzzle: how the weakly bound objects can be formed and survive in a hot environment?

Modelling of cluster formation in HIC

Statistical models

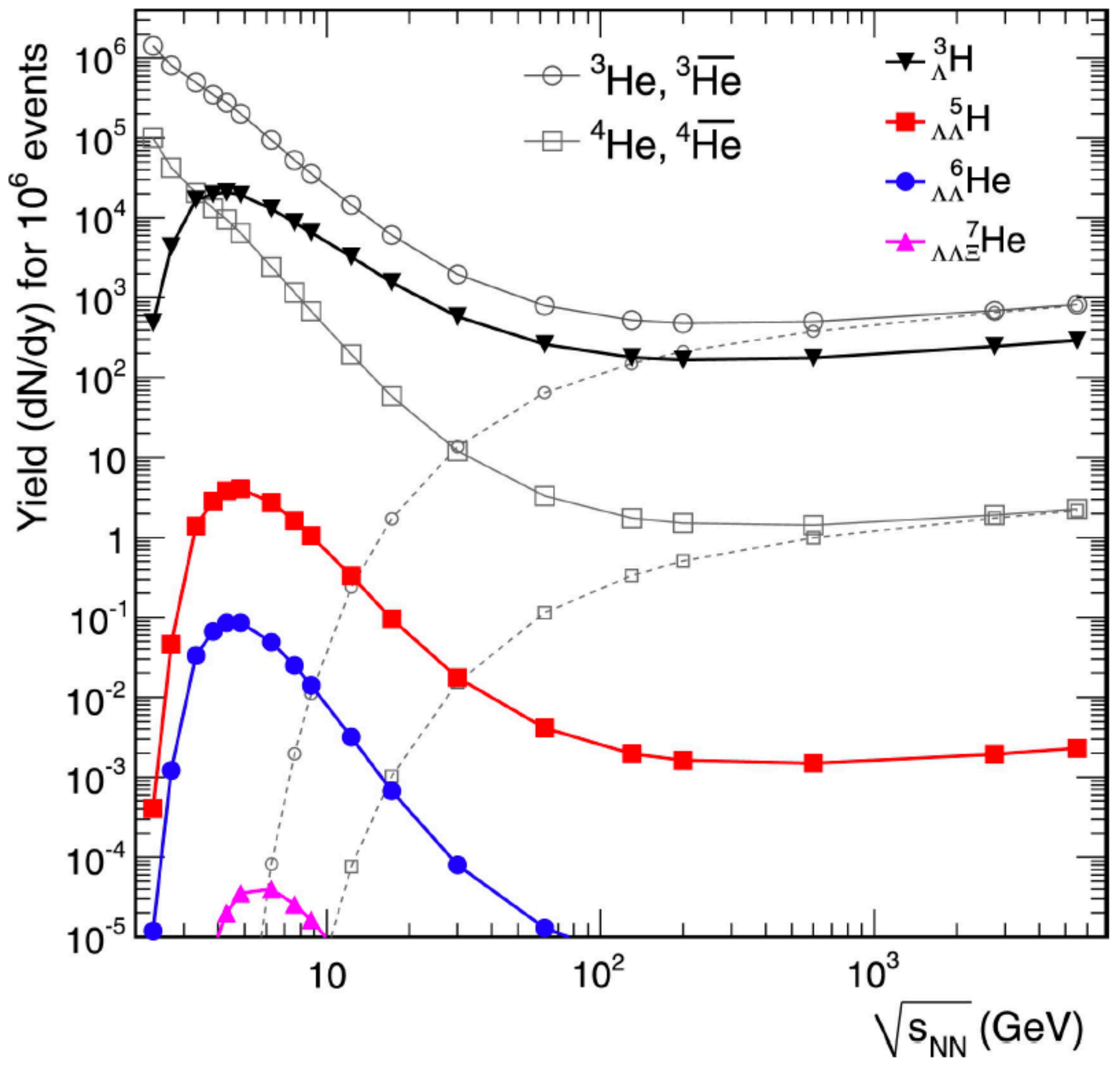
- Production of nuclei depending on T and μ_B at chemical freeze-out & particle mass

Coalescence models

- Formation of nuclei by nucleons & hyperons that are close in coordinate and momentum spaces at freeze-out time

=> no dynamical cluster formation during time evolution

=> no information on the dynamics of clusters formation & microscopic origin



A. Andronic et al., Phys. Lett. B697 (2011) 203-207.

In order to understand the microscopic **origin of cluster formation** one needs a realistic model for the **dynamical time evolution** of the HIC

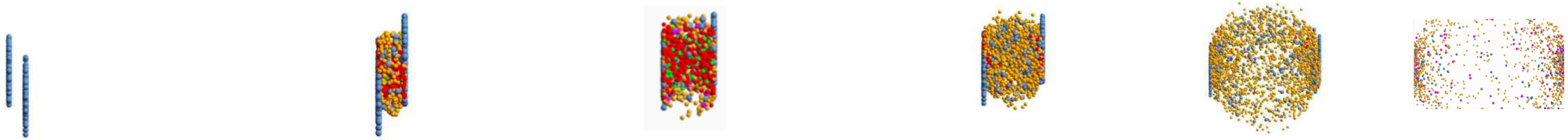
Transport models — dynamical modelling of cluster formation based on interactions:
via potential interaction – **'potential' mechanism**
by scattering – **'kinetic' mechanism**

Parton-Hadron-Quantum-Molecular Dynamics

= n-body microscopic transport approach for the description of heavy-ion dynamics with dynamical cluster formation from low to ultra-relativistic energies

Relativistic considerations + Correlations between nucleons + Cluster recognition

Initial A+A collisions — Formation of QGP — Partonic phase — Hadronization — Hadronic phase



QMD Initialization nuclei — propagation of baryons

local $\varepsilon > \varepsilon_c$ dissolution of pre-hadrons

propagation of partons

PHSD — Primary collisions pre-hadronic states

interactions of partons — collision integral

interactions of hadrons

propagation of mesons

Cluster recognition — MST or SACA

J. Aichelinet al., PRC 101 (2020) 044905

PHSD: W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919; NPA831 (2009) 215; W. Cassing, EPJ ST 168(2009)

Potentials in PHQMD

Nucleon-nucleon potential:

$$V_{i,j} = V(\mathbf{r}_i, \mathbf{r}_j, \mathbf{r}_{i0}, \mathbf{r}_{j0}, t) = V_{\text{Skyrme}} + V_{\text{Coul}} = \frac{1}{2} t_1 \delta(\mathbf{r}_i - \mathbf{r}_j) + \frac{1}{\gamma+1} t_2 \delta(\mathbf{r}_i - \mathbf{r}_j) \rho^{\gamma-1}(\mathbf{r}_i, \mathbf{r}_j, \mathbf{r}_{i0}, \mathbf{r}_{j0}, t) + \frac{1}{2} \frac{Z_i Z_j e^2}{|\mathbf{r}_i - \mathbf{r}_j|}$$

New!

$$V_{i,j} = V(\mathbf{r}_i, \mathbf{r}_j, \mathbf{r}_{i0}, \mathbf{r}_{j0}, \mathbf{p}_{i0}, \mathbf{p}_{j0}, t) = V_{\text{Skyrme loc.}} + \boxed{V_{\text{mom}}} + V_{\text{Coul}} = \frac{1}{2} t_1 \delta(\mathbf{r}_i - \mathbf{r}_j) + \frac{1}{\gamma+1} t_2 \delta(\mathbf{r}_i - \mathbf{r}_j) \rho^{\gamma-1}(\mathbf{r}_i, \mathbf{r}_j, \mathbf{r}_{i0}, \mathbf{r}_{j0}, t) + \boxed{V(\mathbf{r}_i, \mathbf{r}_j, \mathbf{p}_{i0}, \mathbf{p}_{j0})} + \frac{1}{2} \frac{Z_i Z_j e^2}{|\mathbf{r}_i - \mathbf{r}_j|}$$

Skyrme potential:

$$\langle V_{Skyrme}(\mathbf{r_{i0}}, t) \rangle = \alpha \left(\frac{\rho_{int}(\mathbf{r}_{i0}, t)}{\rho_0} \right) + \beta \left(\frac{\rho_{int}(\mathbf{r}_{i0}, t)}{\rho_0} \right)^{\gamma}$$

modified interaction density (with relativistic extension):

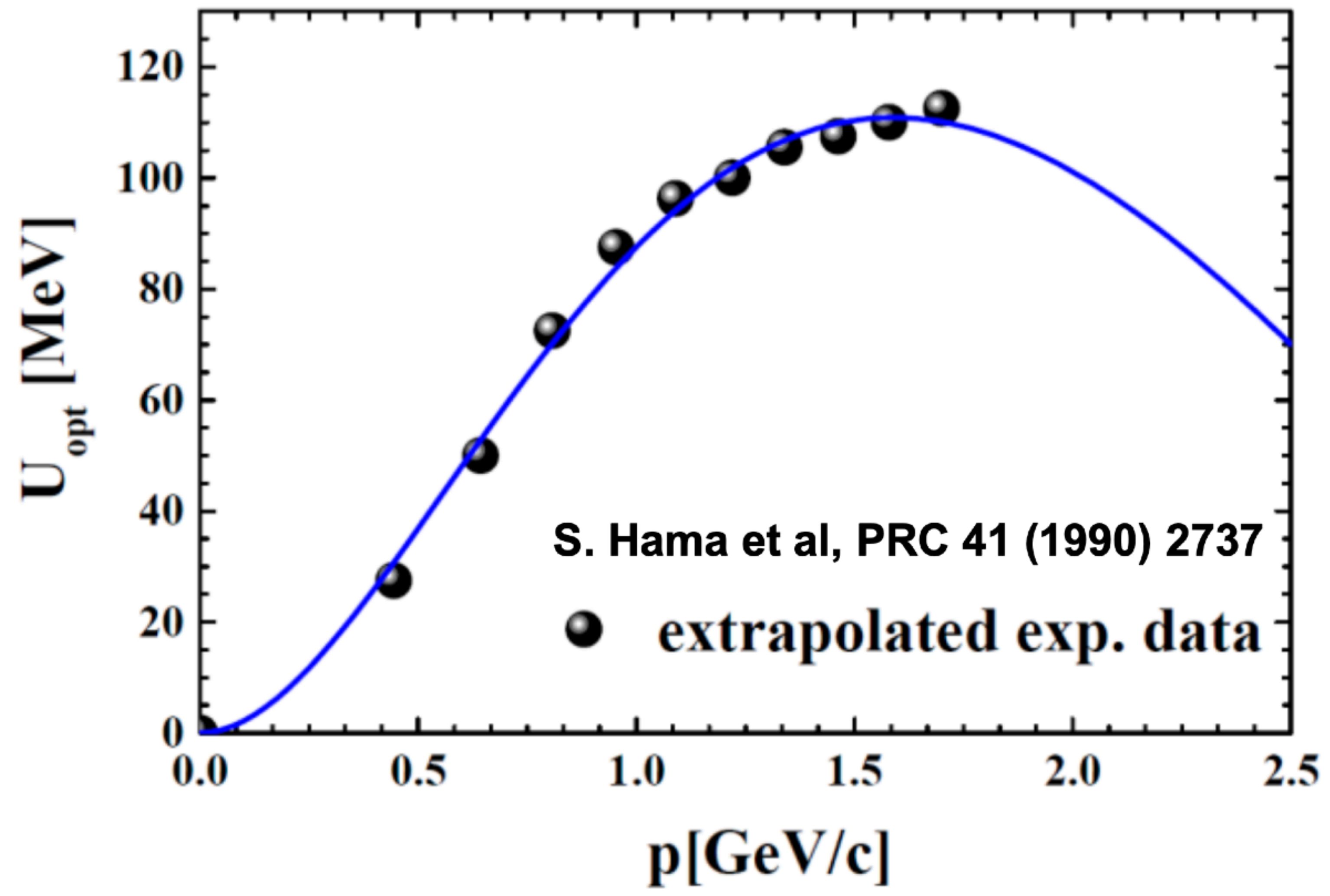
$$\tilde{\rho}_{int}(\mathbf{r}_{i0}, t) \to C \sum_j \left(\frac{4}{\pi L} \right)^{3/2} \mathrm{e}^{-\frac{4}{L}(\mathbf{r}_{i0}^T(t) - \mathbf{r}_{j0}^T(t))^2} \times \mathrm{e}^{-\frac{4\gamma_{cm}^2}{L}(\mathbf{r}_{i0}^L(t) - \mathbf{r}_{j0}^L(t))^2}$$

Momentum depended potential:

$$V(\mathbf{r}_1, \mathbf{r}_2, \mathbf{p}_{01}, \mathbf{p}_{02}) = (a\Delta p + b\Delta p^2)\ exp[-c\sqrt{\Delta p}]\ \delta(\mathbf{r}_1 - \mathbf{r}_2)$$

$$\Delta p = \sqrt{(\mathbf{p}_{01} - \mathbf{p}_{02})^2}$$

Parameters **a, b, c** are fitted to the 'optical' potential extracted from elastic scattering data in pA:



S. Hama et al, PRC 41 (1990) 2737

Potentials in PHQMD

In infinite matter a potential corresponds to the EoS:

$$E/A(\rho) = \frac{3}{5}E_F + V_{Skyrme\ stat}(\rho) + V_{mom}(\rho)$$

$$V_{mom} = (a\Delta p + b\Delta p^2)\ exp(-c\sqrt{\Delta p})\ \frac{\rho}{\rho_0}$$

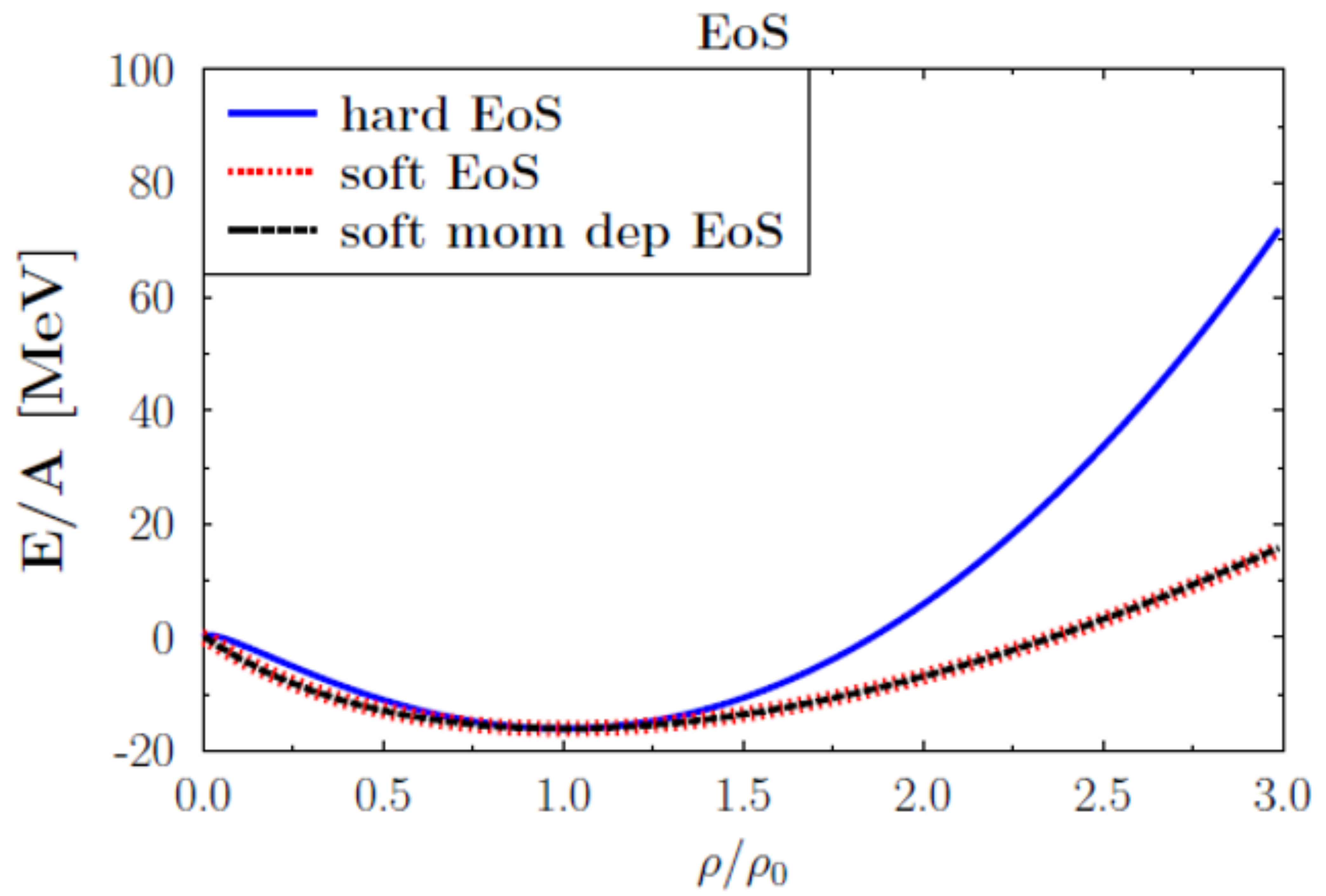
$$V_{Skyrme\ stat} = \alpha\frac{\rho}{\rho_0} + \beta\frac{\rho}{\rho_0}^{\gamma}$$

Compression modulus **K** of nuclear matter:

$$K = -V\frac{dP}{dV} = 9\rho^2\frac{\partial^2(E/A(\rho))}{(\partial\rho)^2}\Big|_{\rho=\rho_0}$$

E.o.S.	α [MeV]	β [MeV]	γ	K [MeV]
S	-383.5	329.5	1.15	200
H	-125.3	71.0	2.0	380
SM	-478.87	413.76	1.1	200
	a [MeV^{-1}]	b [MeV^{-2}]	c [MeV^{-1}]	
	236.326	-20.73	0.901	

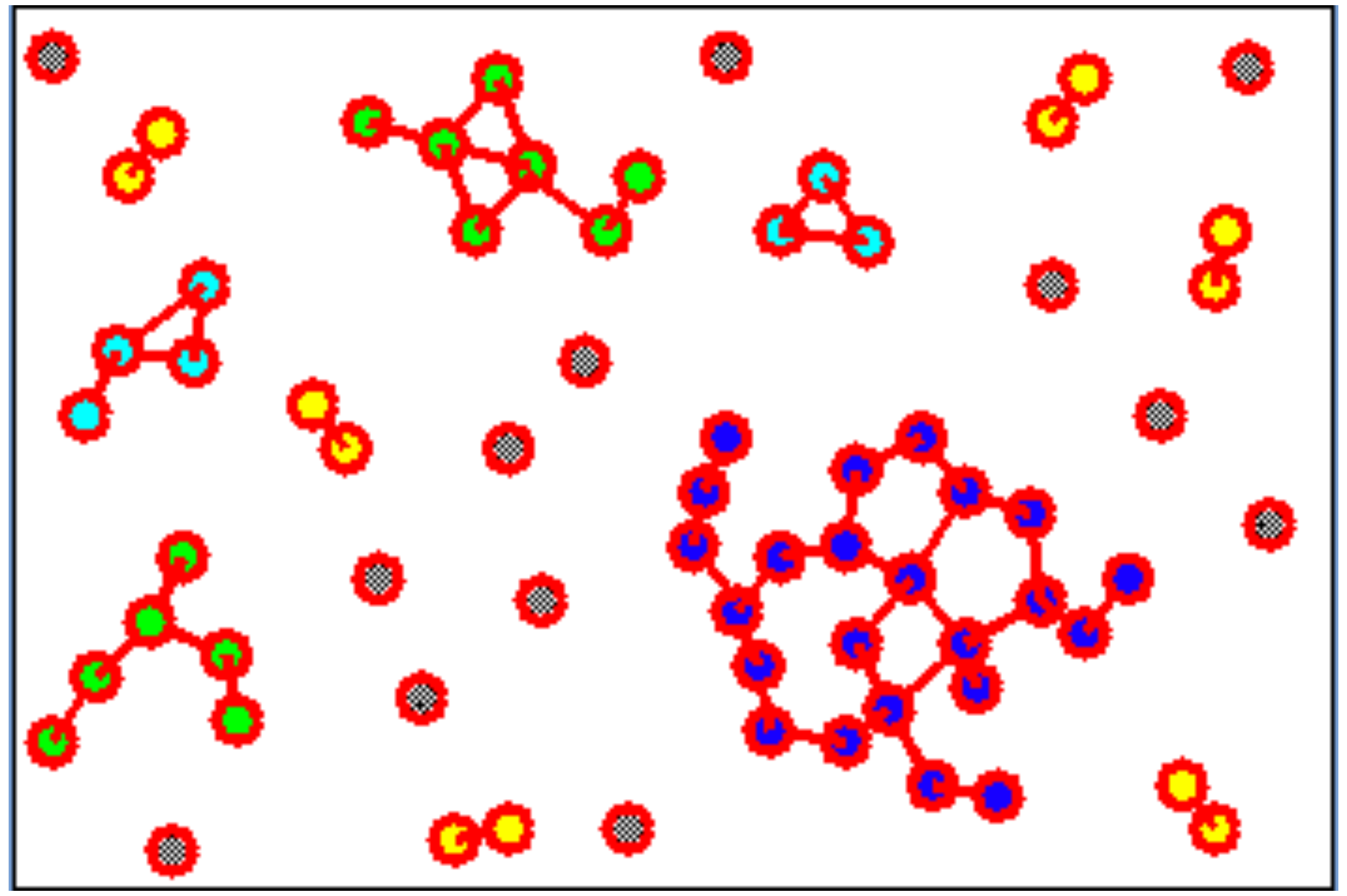
EoS for infinite cold nuclear matter



Minimum Spanning Tree (MST)

Cluster criterion: distance of nuclei

Algorithm: search for accumulations of particles in coordinate space



1. Two particles i & j are bound if:

   $|r_i - r_j| < 4.0$ fm

2. Particle is bound to cluster if bound with at least one particle of cluster

Remark: additional momentum cuts lead to a small changes: particles with large relative momentum are mostly not at the same position (**V. Kireyeu, Phys.Rev.C 103 (2021) 5, 054905**).

Kinetic mechanism for deuteron formation

G. Coci et al., PRC 108 (2023) 1, 014902

$$\pi^{\pm,0} + p + n \leftrightarrow \pi^{\pm,0} + d$$

$$\pi^{-} + p + p \leftrightarrow \pi^{0} + d$$

$$\pi^{+} + n + n \leftrightarrow \pi^{0} + d$$

$$\pi^{0} + p + p \leftrightarrow \pi^{+} + d$$

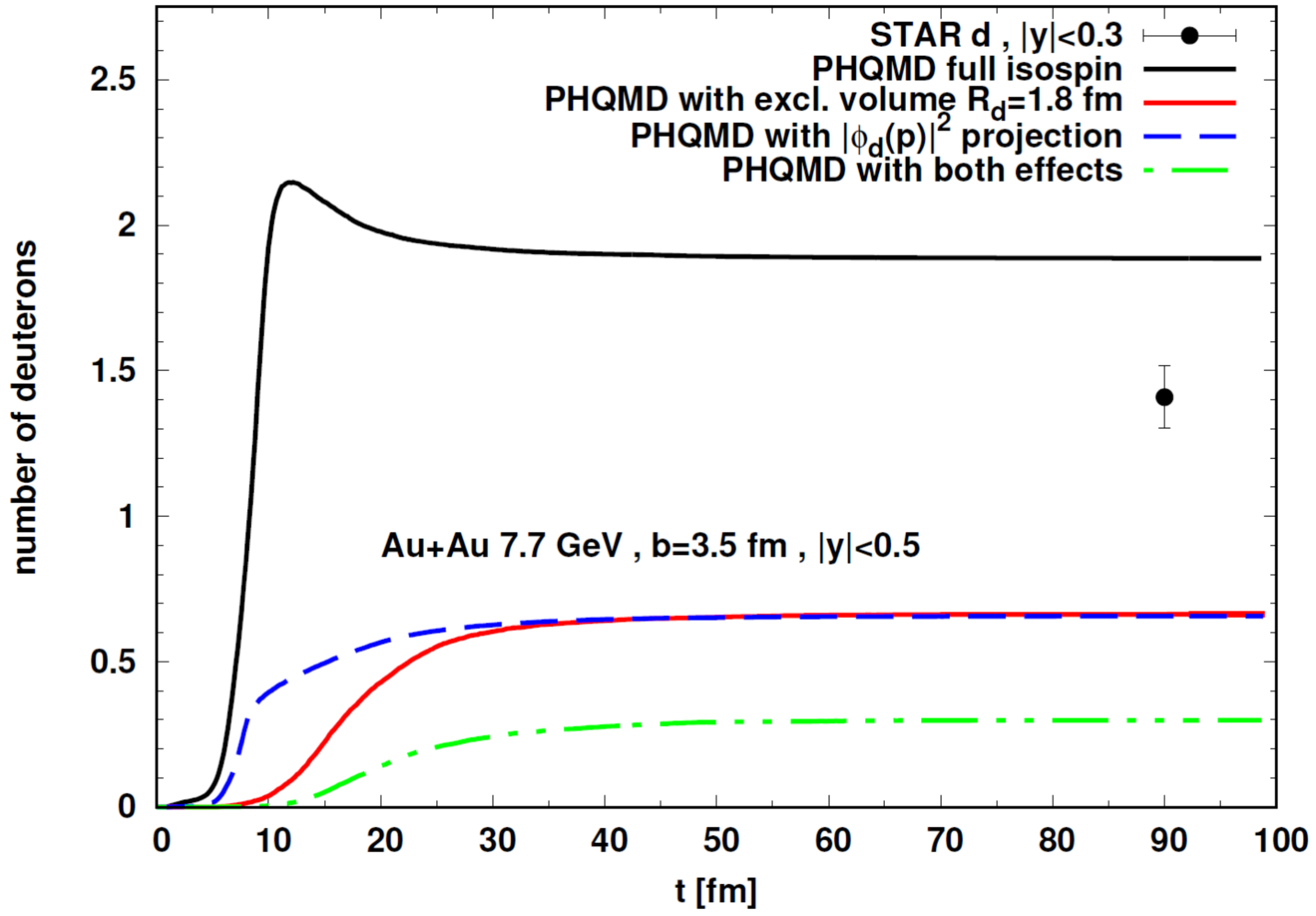
$$\pi^{0} + n + n \leftrightarrow \pi^{-} + d$$

N+N+π inclusion of all possible channels allowed by total isospin T conservation:

Enhance deuteron production

Modelling of the d quantum properties lead to a strong reduction of d production:

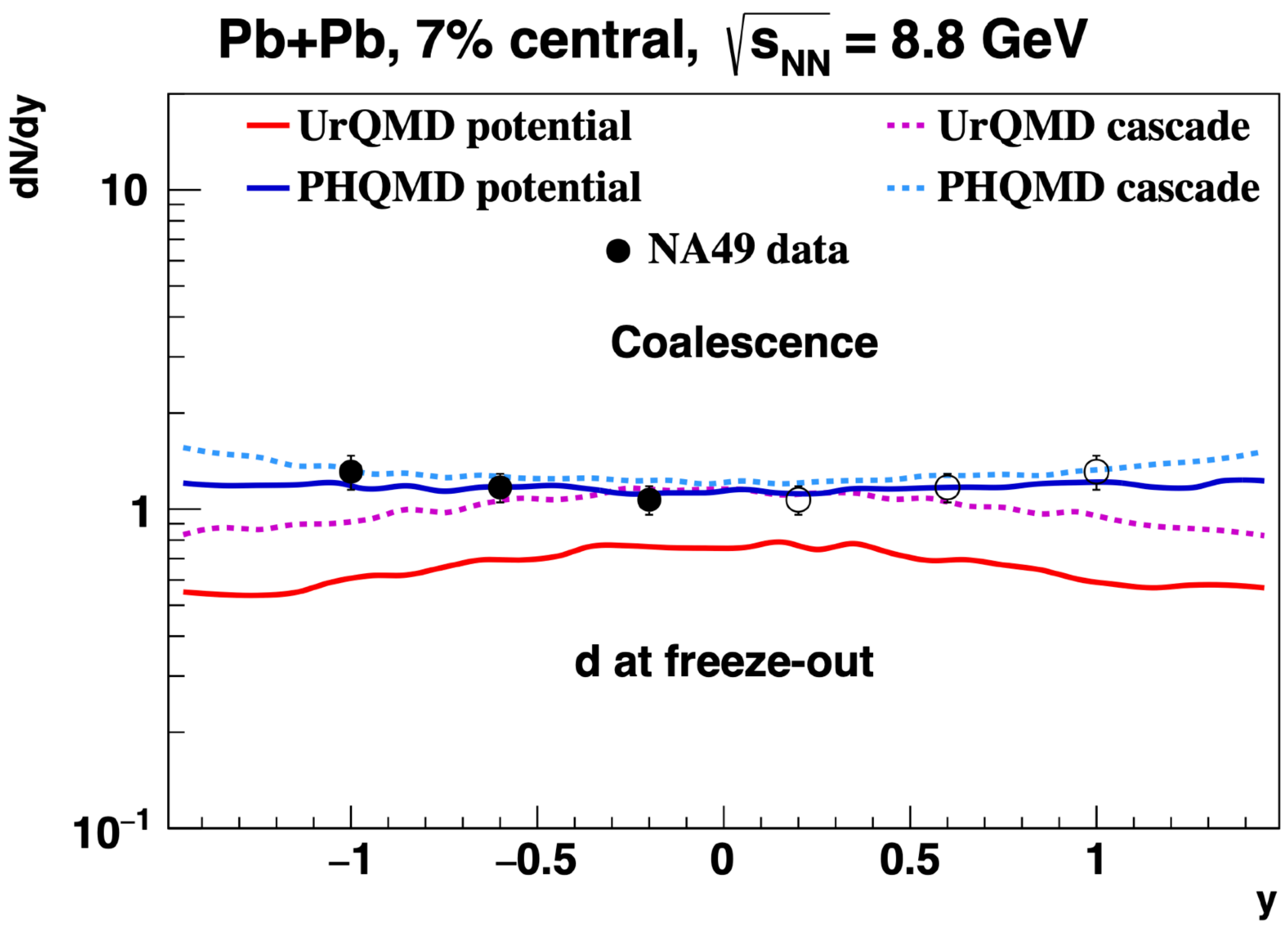
1) The finite-size of *d* in the **coordinate space** (Excluded volume condition): $|\vec{r}(i)^* - \vec{r}(d)^*| < R_d$
2) The momentum correlations of *p* and *n* inside *d* by the projection of the relative momentum of *p*+*n* pair on the *d* wave-function.



Coalescence for deuterons

Statistical description of cluster production, based on proximity in momentum and coordinate space.

- Calculations are performed at the **«freeze-out»**.
- The relative momentum ΔP and distance ΔR between the proton and the neutron are calculated in the p-n CM frame.
- If ΔP < 0.285 GeV and ΔR < 3.575 fm, a deuteron may be formed with the probability Pd = 3/8 (the spin-isospin combinatorial factor).



«**psMST**» library: **MST and coalescence** for any model

V. Kireyeu, Phys.Rev.C 103 (2021) 5, 054905
V. Kireyeu et al., PRC 105 (2022) 044909

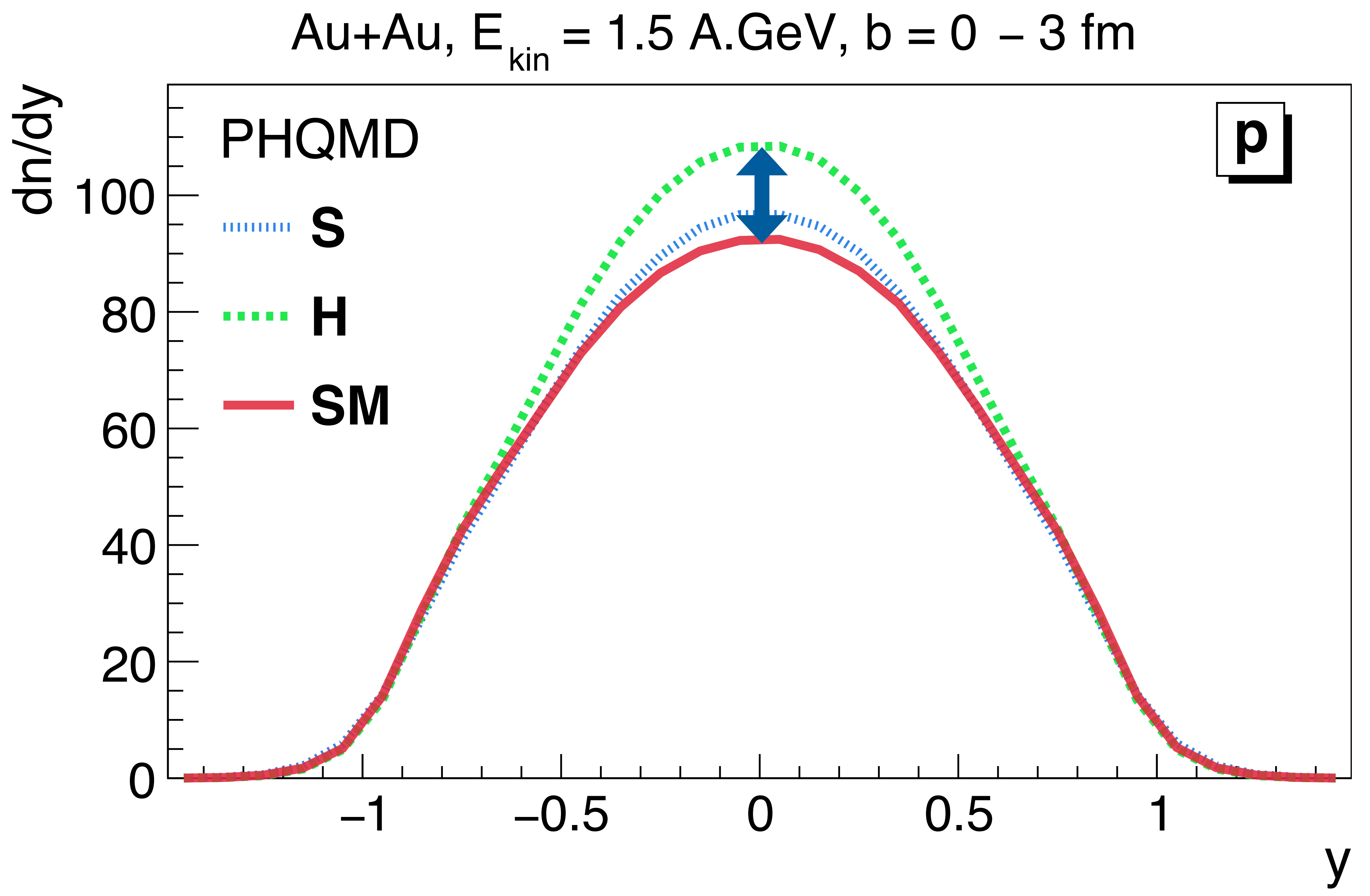
EoS sensitivity

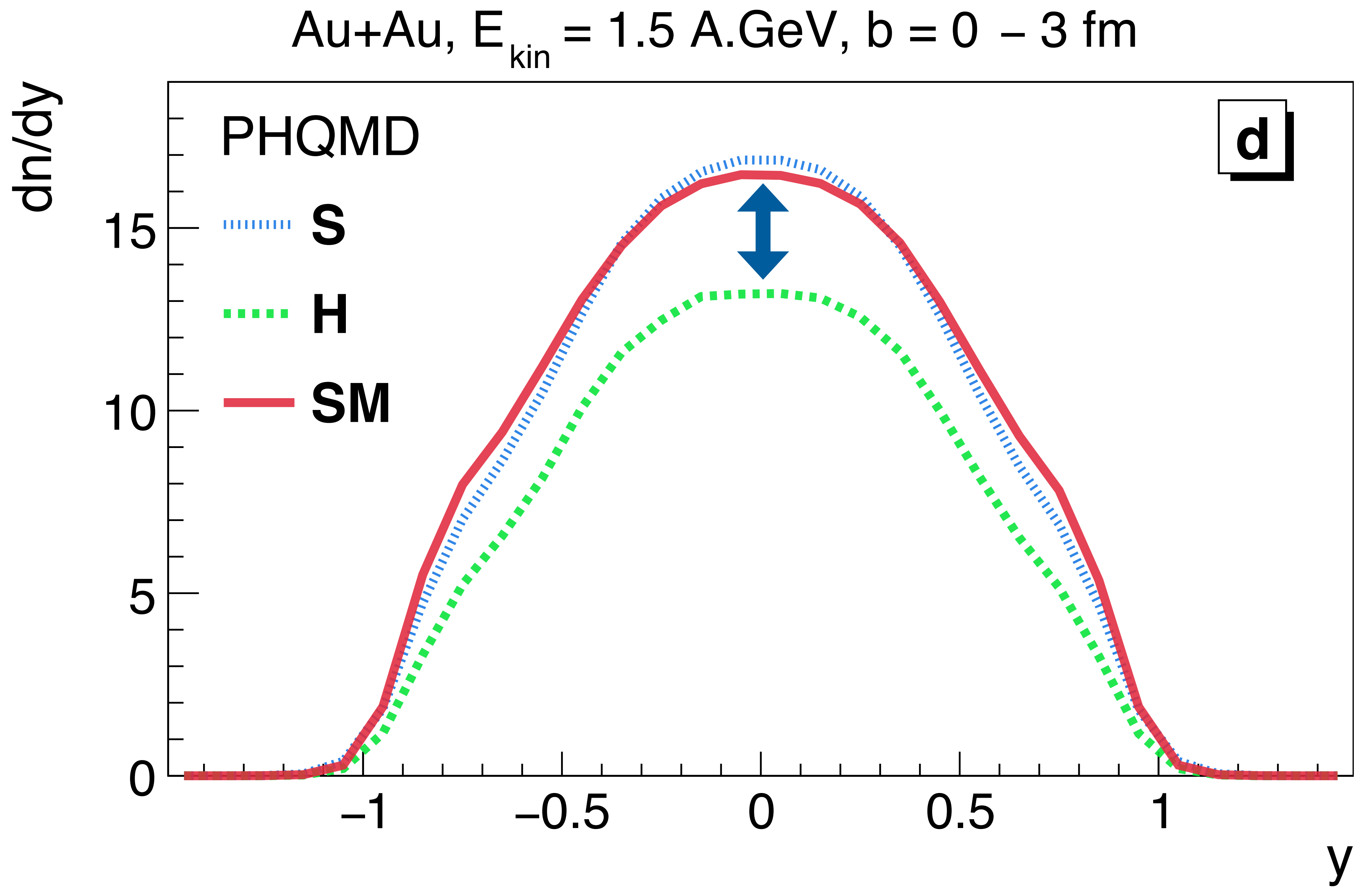
EoS sensitivity

«Hard» (H), «Soft» (S), «Soft mom. dep.» (SM) potentials act differently on different observables:

1. dN/dy yields at mid-rapidity:

- protons: SM = S < H
- deuterons: SM = S > H



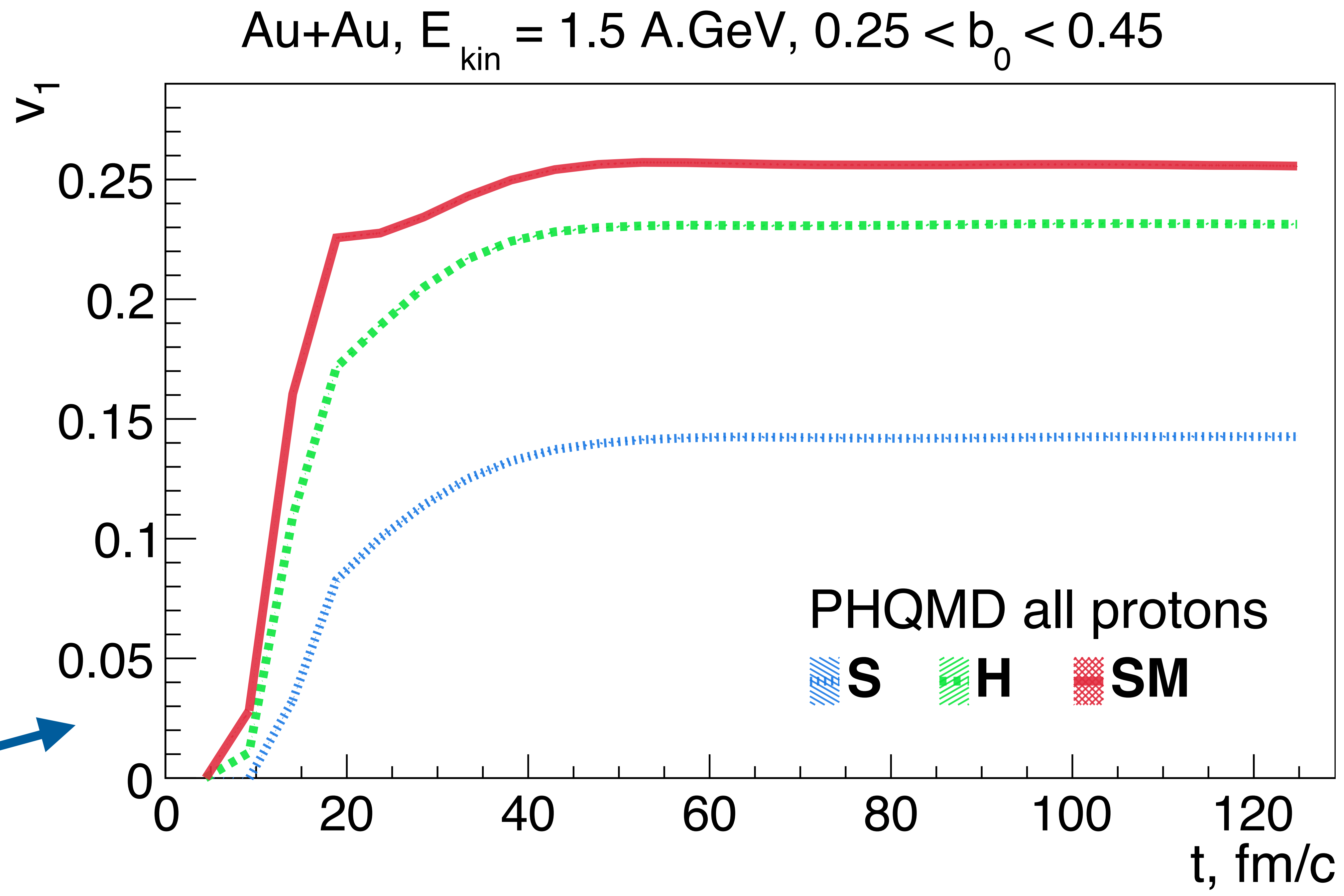


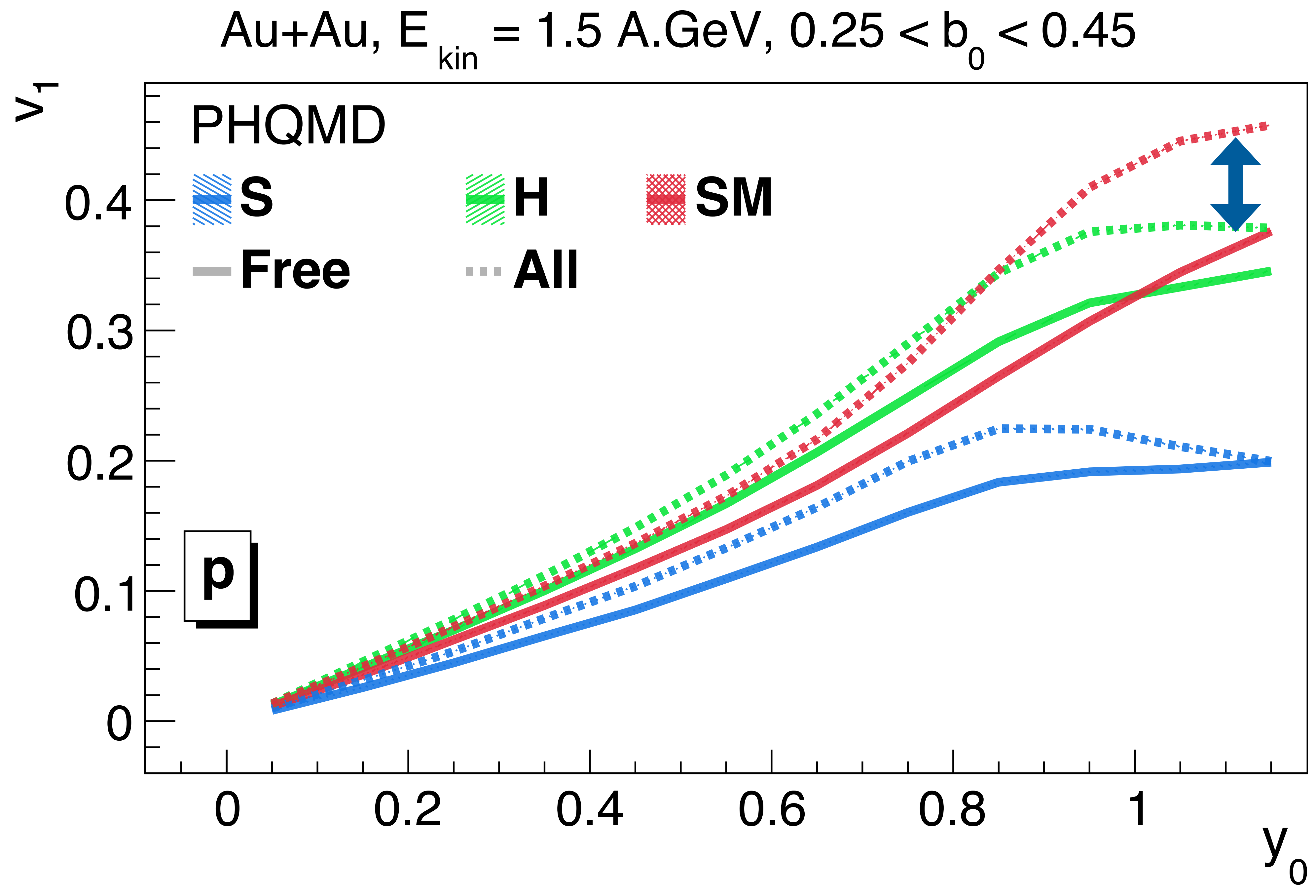
EoS sensitivity

«Hard» (H), «Soft» (S), «Soft mom. dep.» (SM) potentials act differently on different observables:

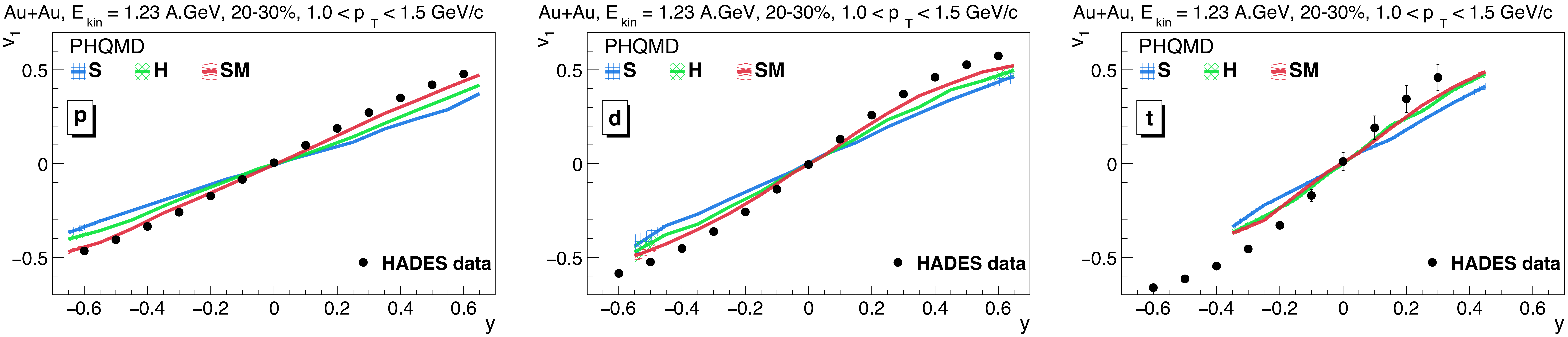
2. Drected flow v_1:

- Protons: SM > H > S
- Flow v_1 with SM EoS develops earlier than for H EoS and much earlier than for S EoS
- $v_1(y)$ of p bound in clusters are larger than of free (unbound) p





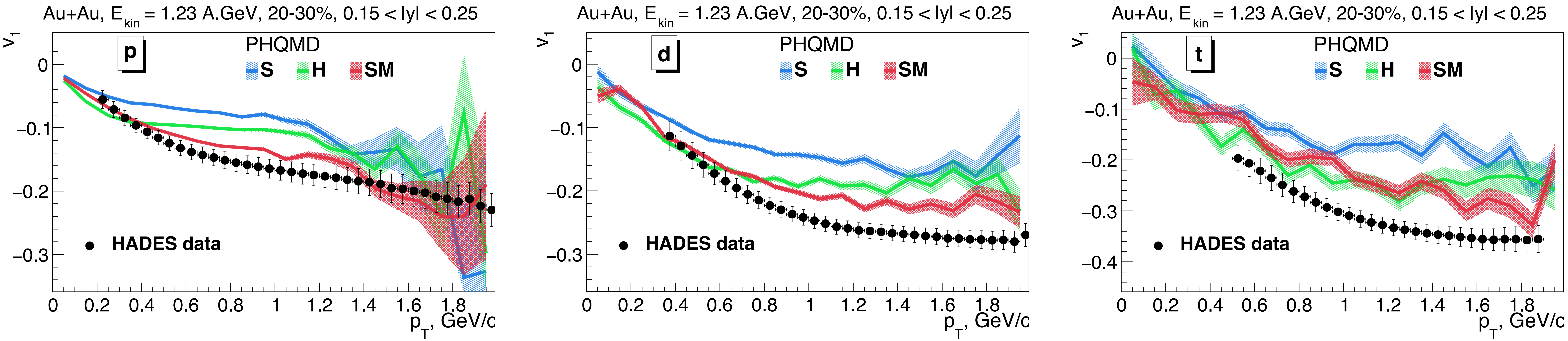
EoS sensitivity



HADES data: Phys. Rev. Lett. 125, 262301 (2020)

- **Strong EoS dependence of v1(y) of protons and deuterons**
- **HADES data favour a soft momentum dependent potential (SM)**

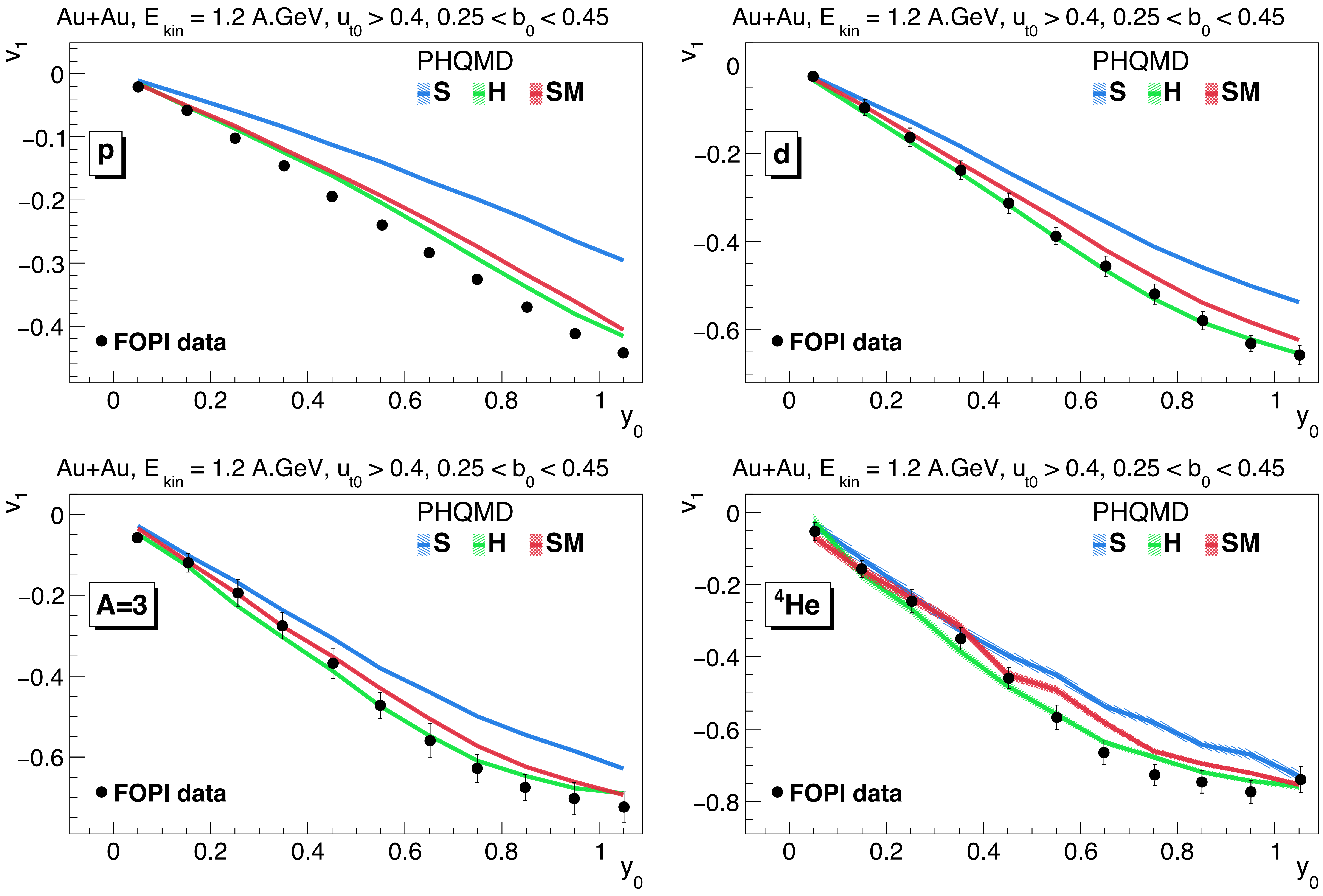
EoS sensitivity



HADES data: Eur. Phys. J. A 59, 80 (2023)

- **Strong EoS dependence of v1(p_T) of protons and deuterons, less for tritons**
- **HADES data favour a soft momentum dependent potential (SM)**

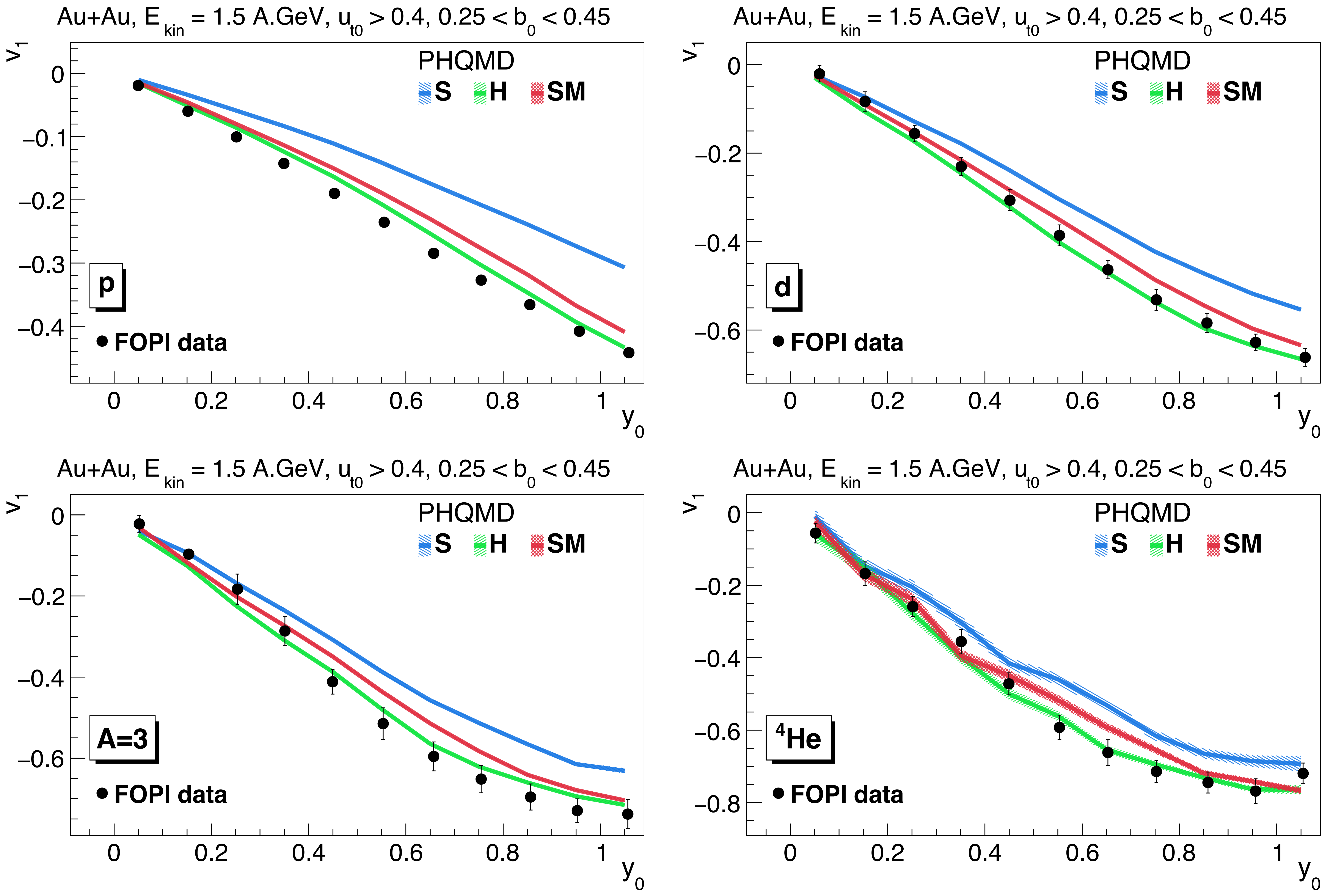
EoS sensitivity



- **Strong EoS dependence of v1(y_0) of protons and deuterons, visible for A=3, ^{4}He**
- **FOPI data favour a hard or soft momentum dependent potential**

FOPI data: Nucl. Phys. A 876, 1 (2012)

EoS sensitivity



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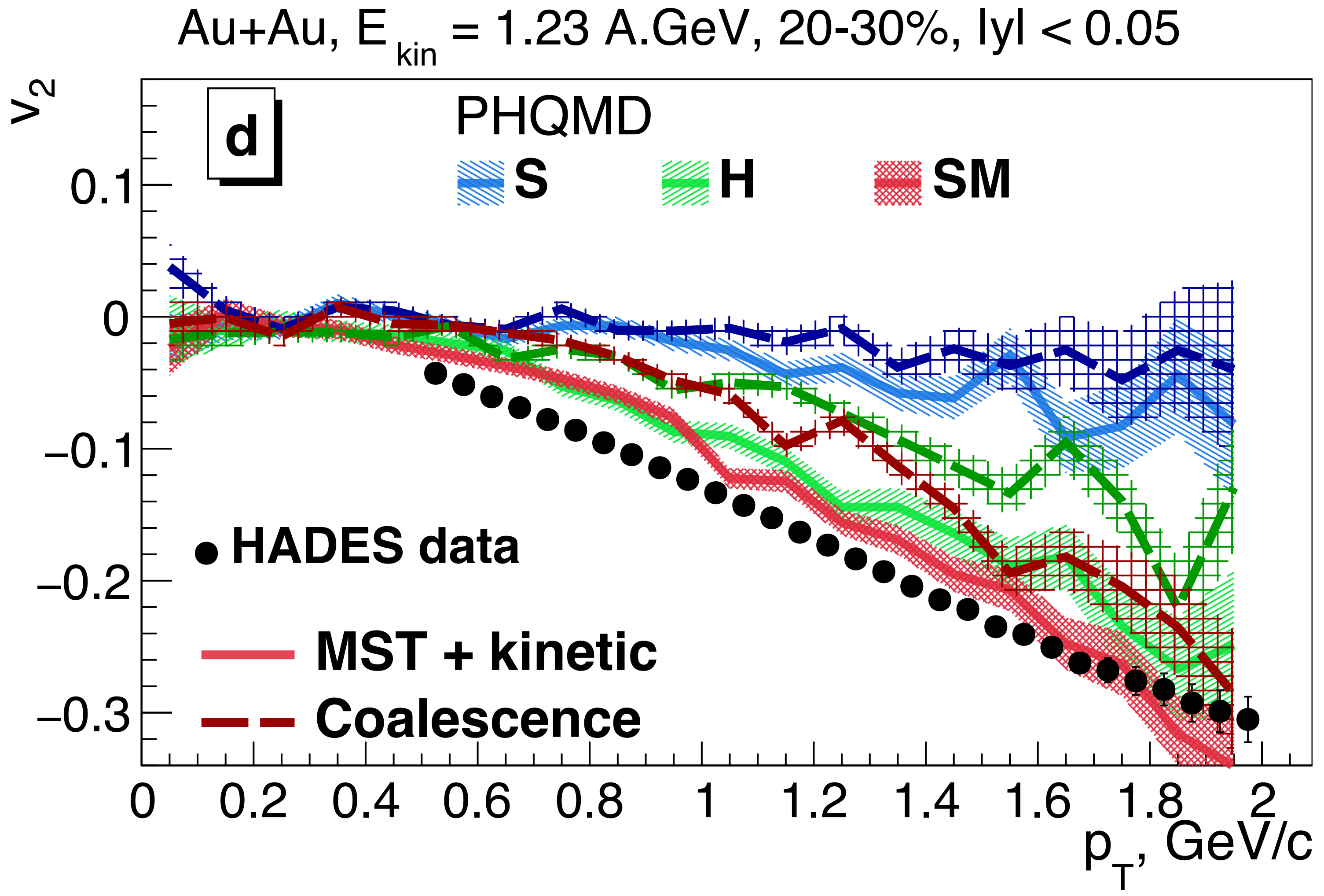
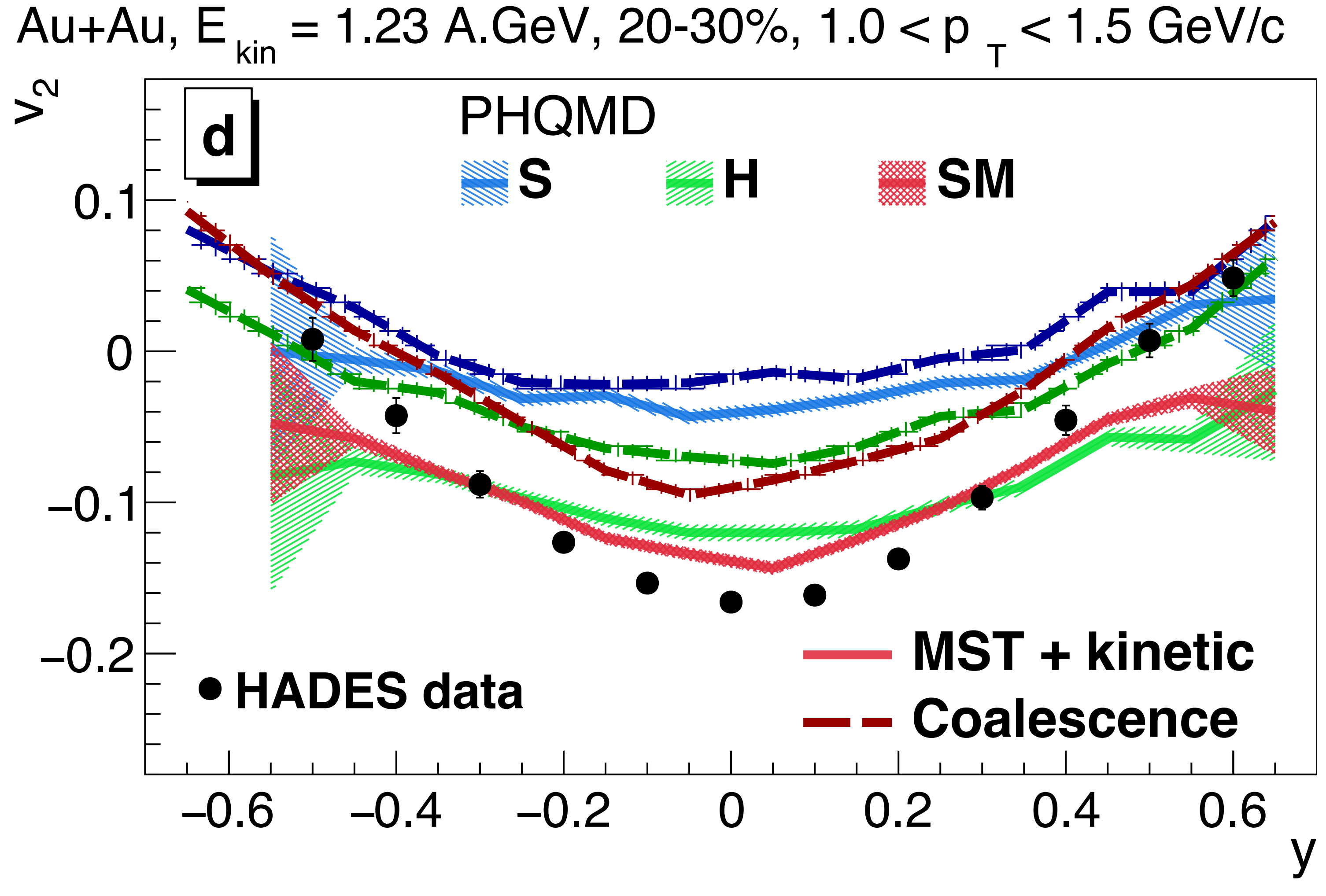
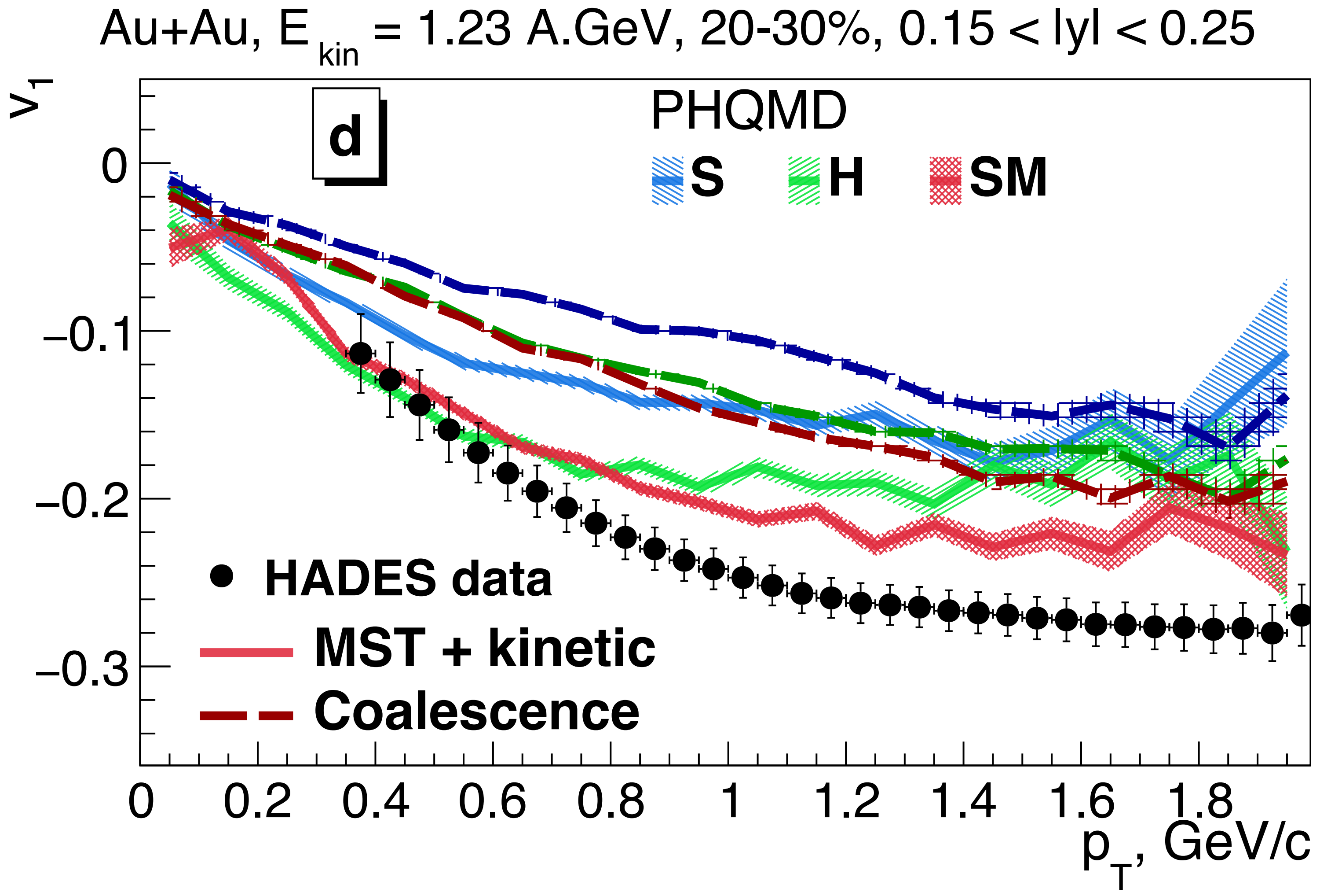
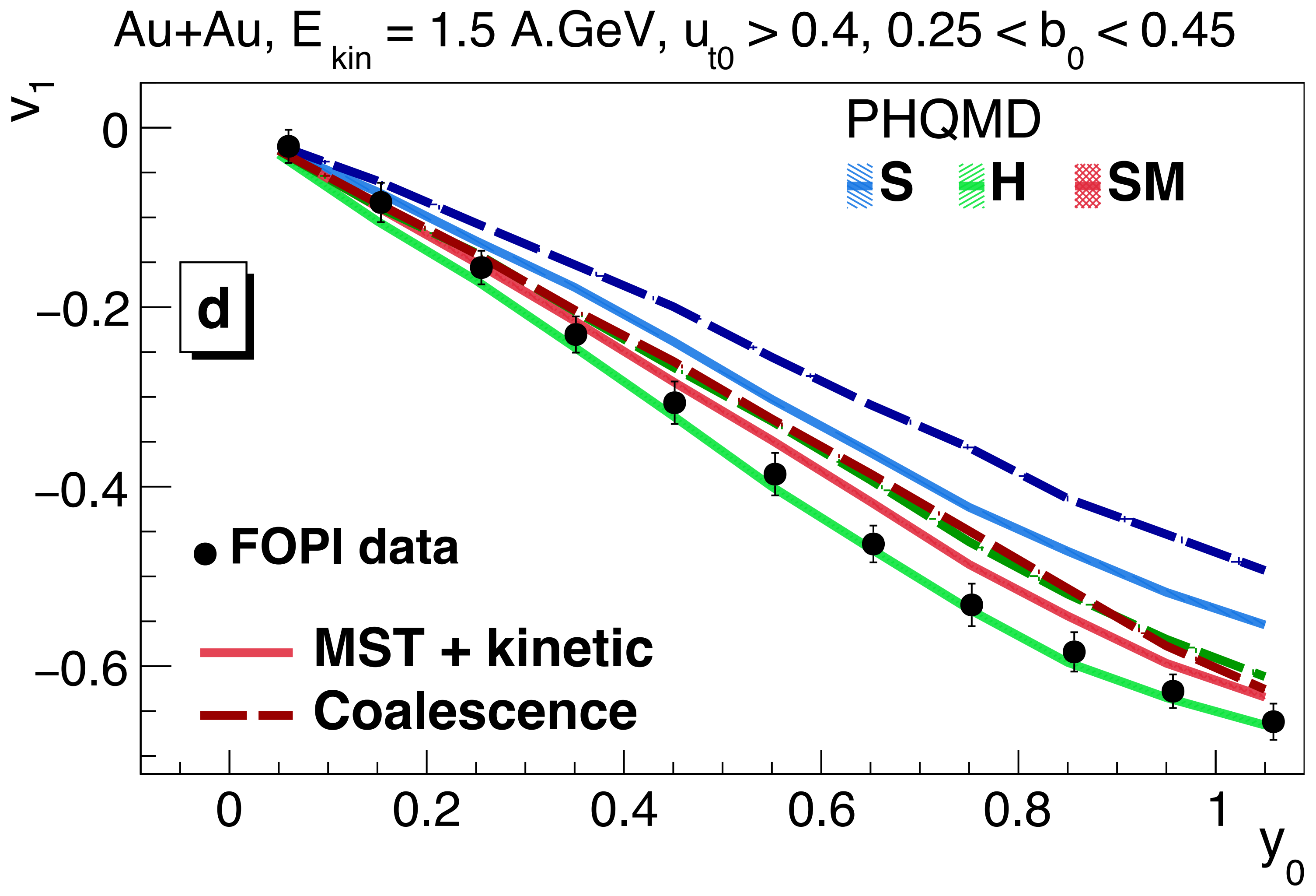
FOPI data: Nucl. Phys. A 876, 1 (2012)

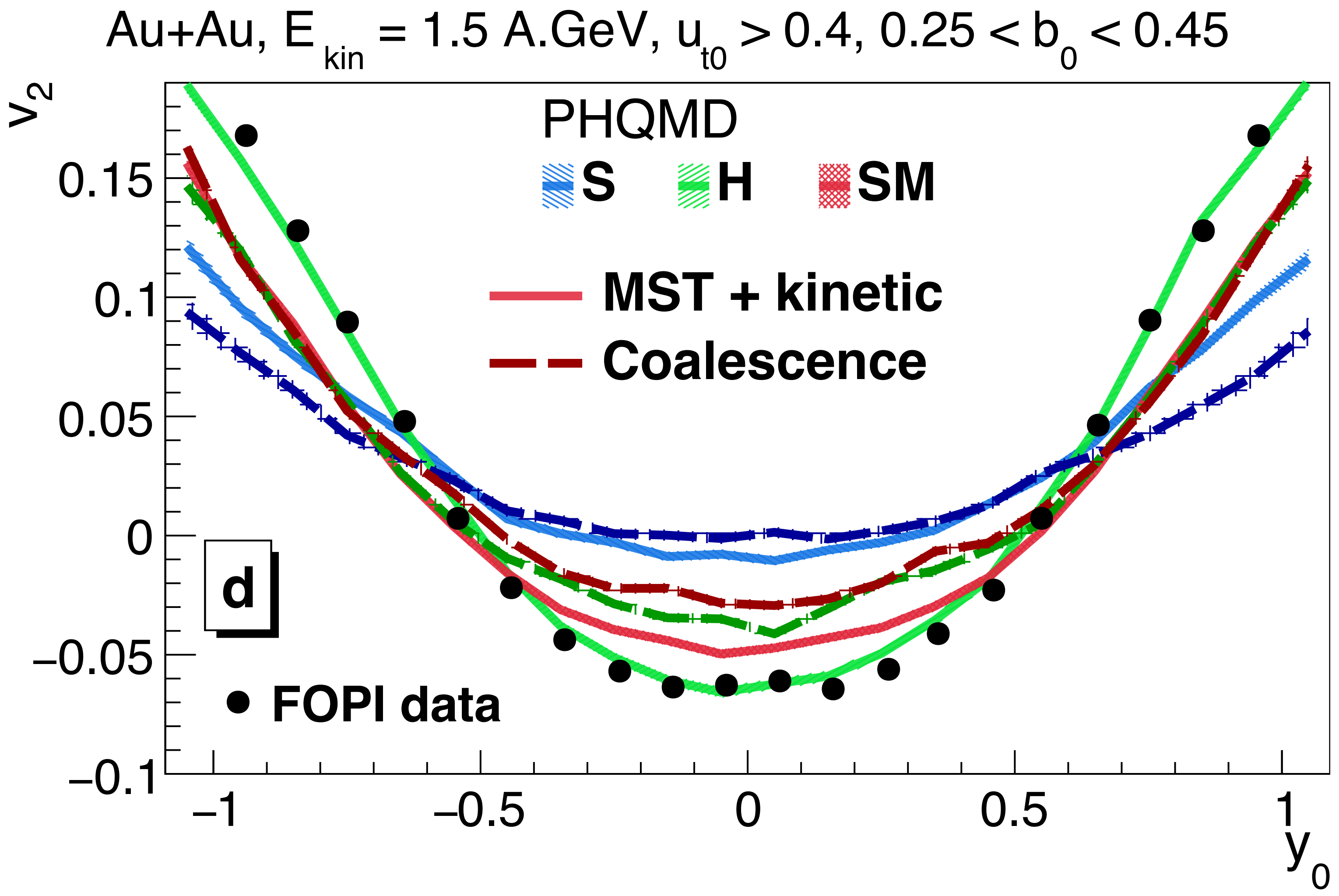
EoS + production mechanism sensitivity











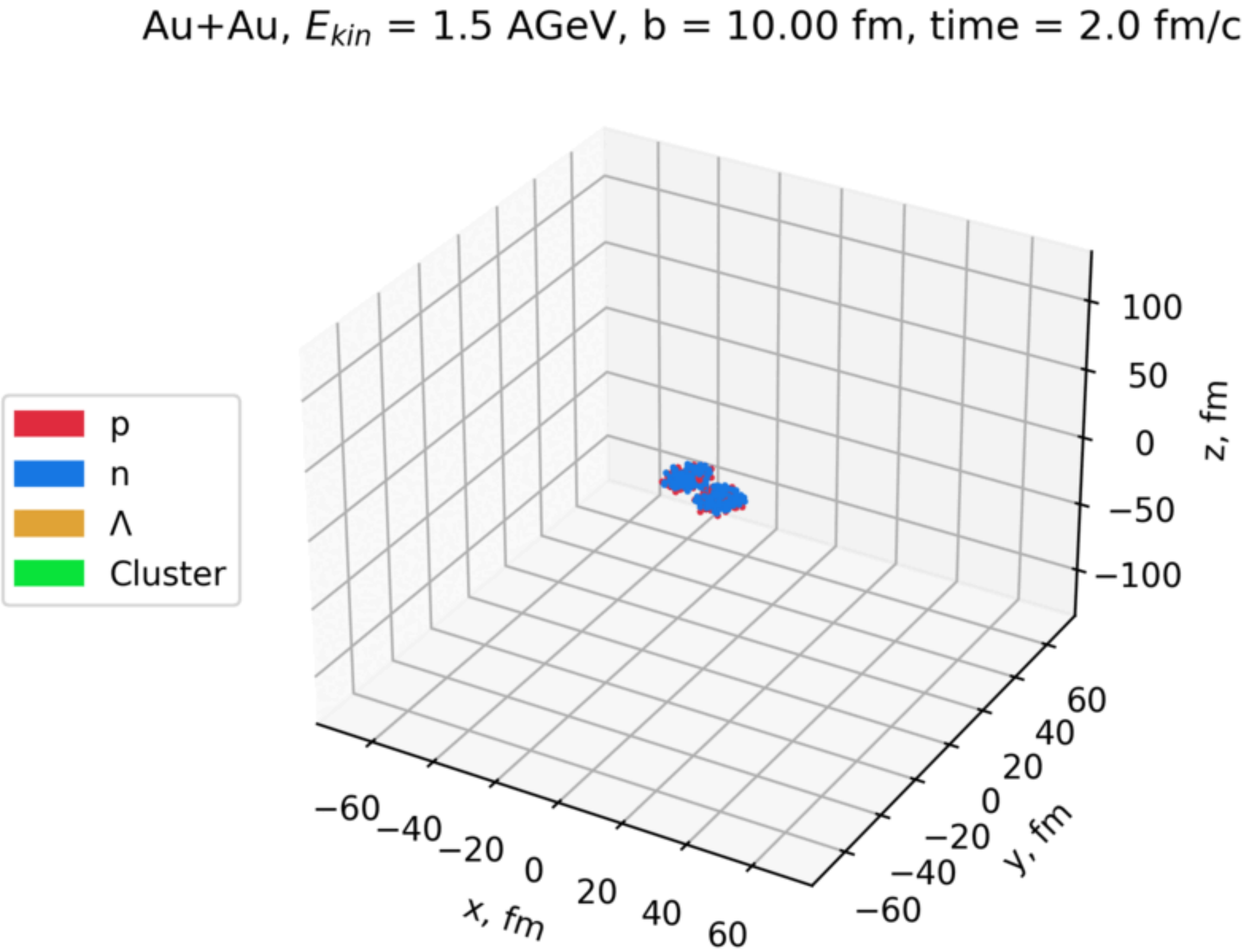
«Cluster formation near midrapidity: How the production mechanisms can be identified experimentally»
V.Kireyeu et al,
Phys. Rev. C 109, 044906 (2024)

In addition to the **strong EoS dependence**, **v1 and v2** (as well as the dN/dy and p_T spectra) **are very sensitive to the production mechanisms** —> can help to **identify experimentally the origin of the deuteron production** in heavy-ion collisions.

Summary

- **The PHQMD** is a microscopic n-body transport approach for the description of heavy-ion dynamics and cluster and hypernuclei formation
- Clusters are formed **dynamically**:
  - by **potential interactions** among nucleons and hyperons
  - by **kinetic mechanism** for deuterons production.
- The PHQMD reproduces cluster and hypernuclei data on dN/dy and dN/dpT as well as ratios d/p and / for heavy-ion collisions from AGS to top RHIC energies.
- **Strong dependence of v_1, v_2 on EoS** — soft, hard, soft-mom. dependent - at SIS energies:
  - HADES and FOPI data data on **v_1, v_2** favour a **soft momentum dependent potential**
  - The hard EoS gives the results for v1,v2 which are close to the **SM EoS**
  - **Soft EoS** is excluded by the HADES and FOPI flow data
- The EoS dependence of on v1 and v2 decreases with increasing size of clusters while it is still quite strong for deuterons
- The flow coefficients **v1 and v2 are sensitive** to the **deuterons production mechanism**, thus they may help to identify which one is realised in nature: **coalescence or dynamical cluster production + kinetic** mechanism for deuterons.

Thank you for your attention!



https://phqmd.gitlab.io/

QMD vs MF

Cluster formation is sensitive to nucleon dynamics

> it's important to keep the nucleon correlations by realistic nucleon-nucleon interactions in transport models:

- **QMD** (quantum-molecular dynamics) – allows to keep correlations
- **MF** (mean-field based models) – correlations are smeared out
- **Cascade** – no correlations by potential interactions

QMD:

🟥 PHQMD + psMST

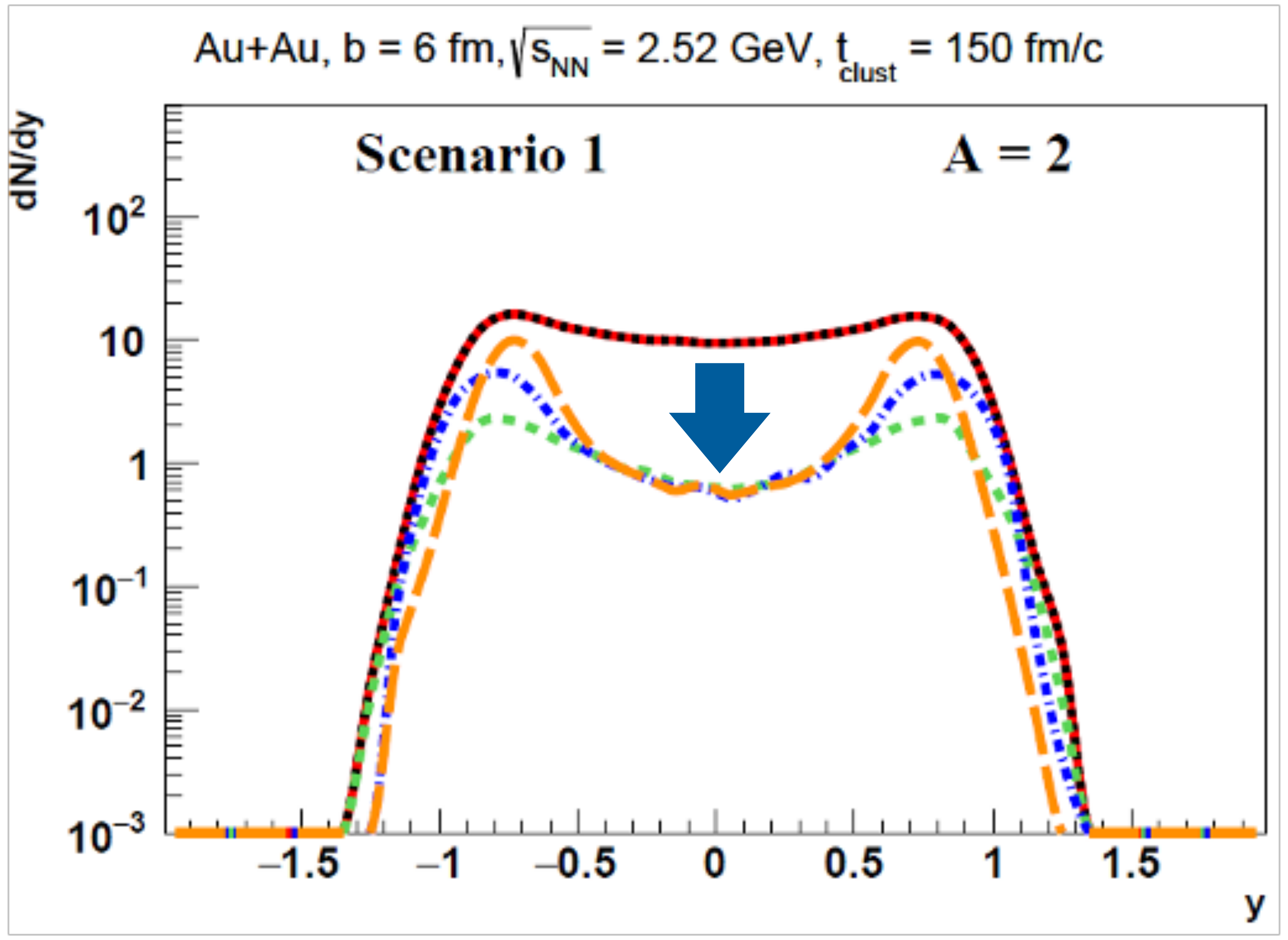
MF:

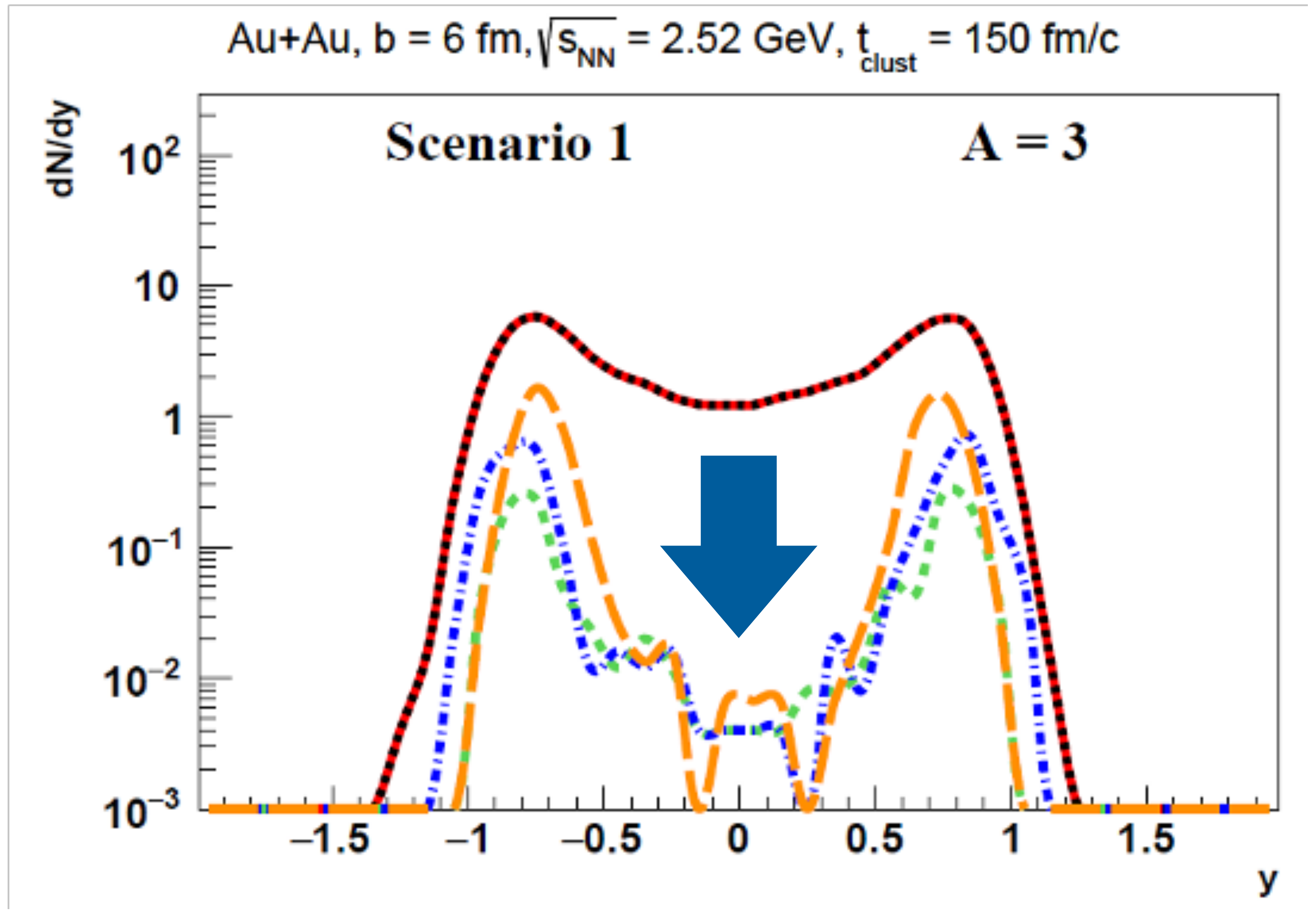
🟧 PHSD + psMST

Cascade:

🟩 🟩 SMASH + psMST

▪ı▪ UrQMD + psMST





V. Kireyeu, Phys.Rev.C 103 (2021) 5

QMD vs MF

When at the end of the expansion two nucleons (two test particles) are close together:

- In **QMD** they interact with the **full nucleon-nucleon force -> cluster get stable**
- In **MF** they interact with the **(full nucleon-nucleon force)/N** which cannot keep the cluster together **-> at large time no clusters anymore**

QMD:

🟥 PHQMD + psMST

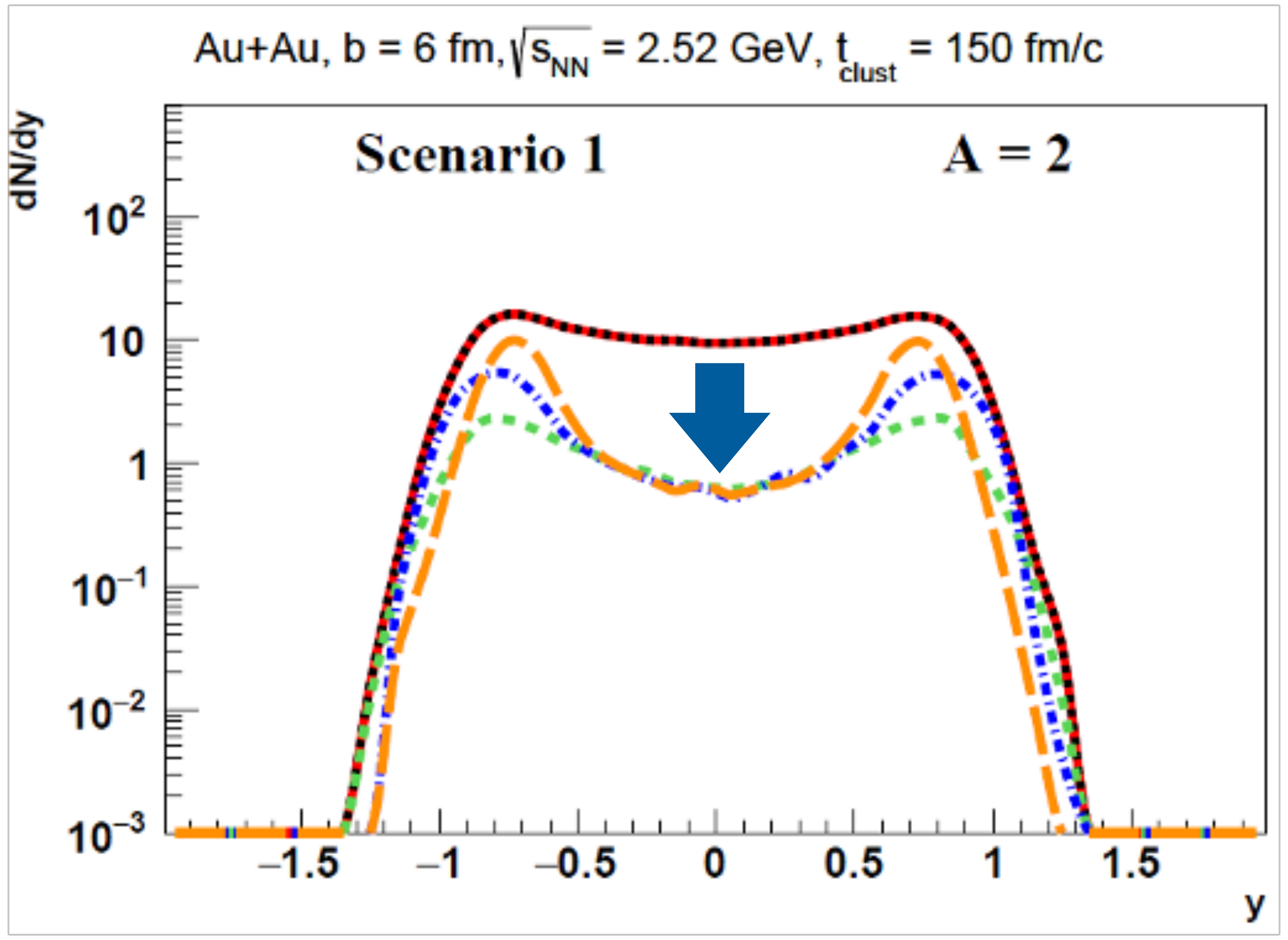
MF:

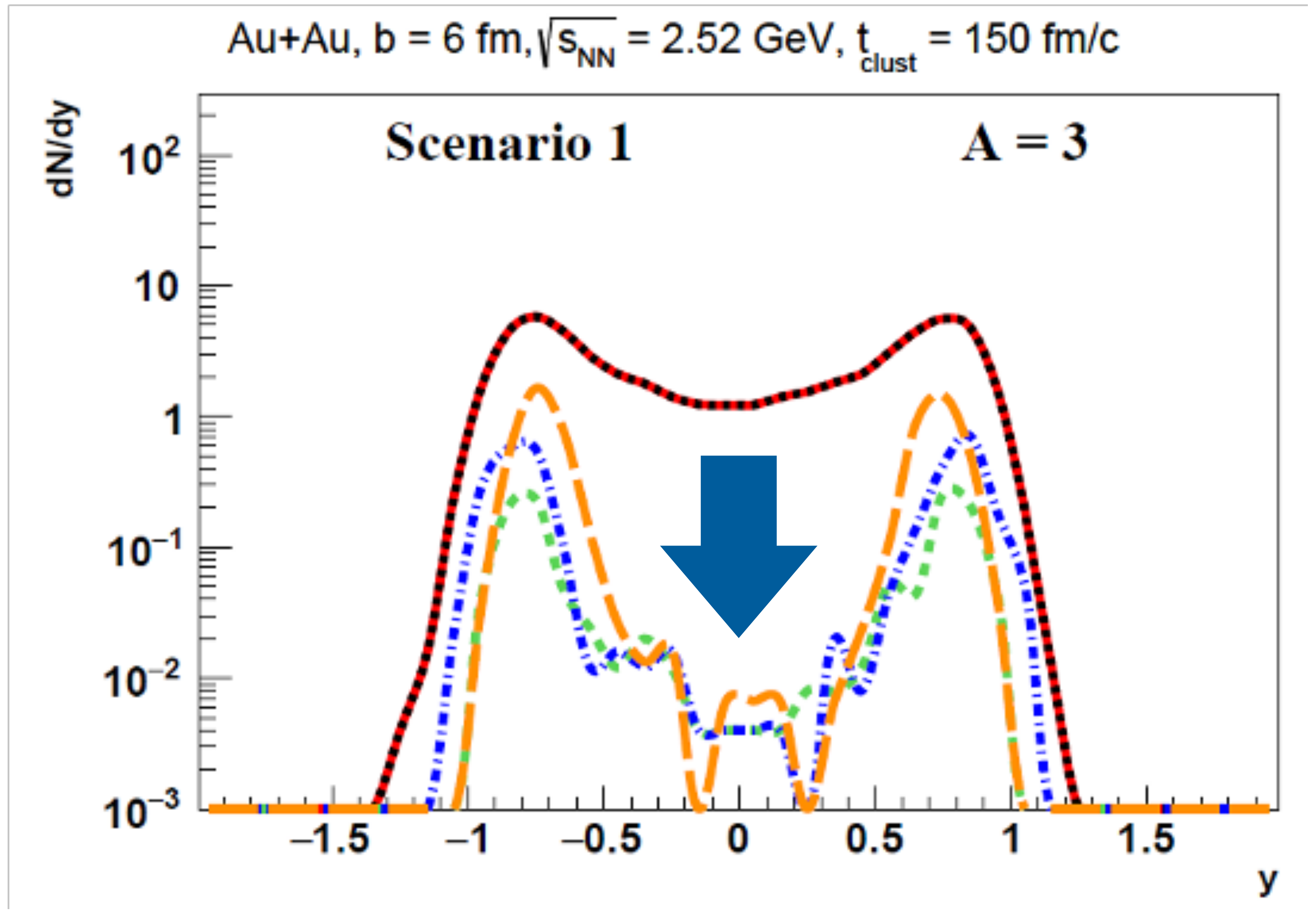
🟧 PHSD + psMST

Cascade:

🟩 🟩 SMASH + psMST

🟦 ▪ 🟦 UrQMD + psMST



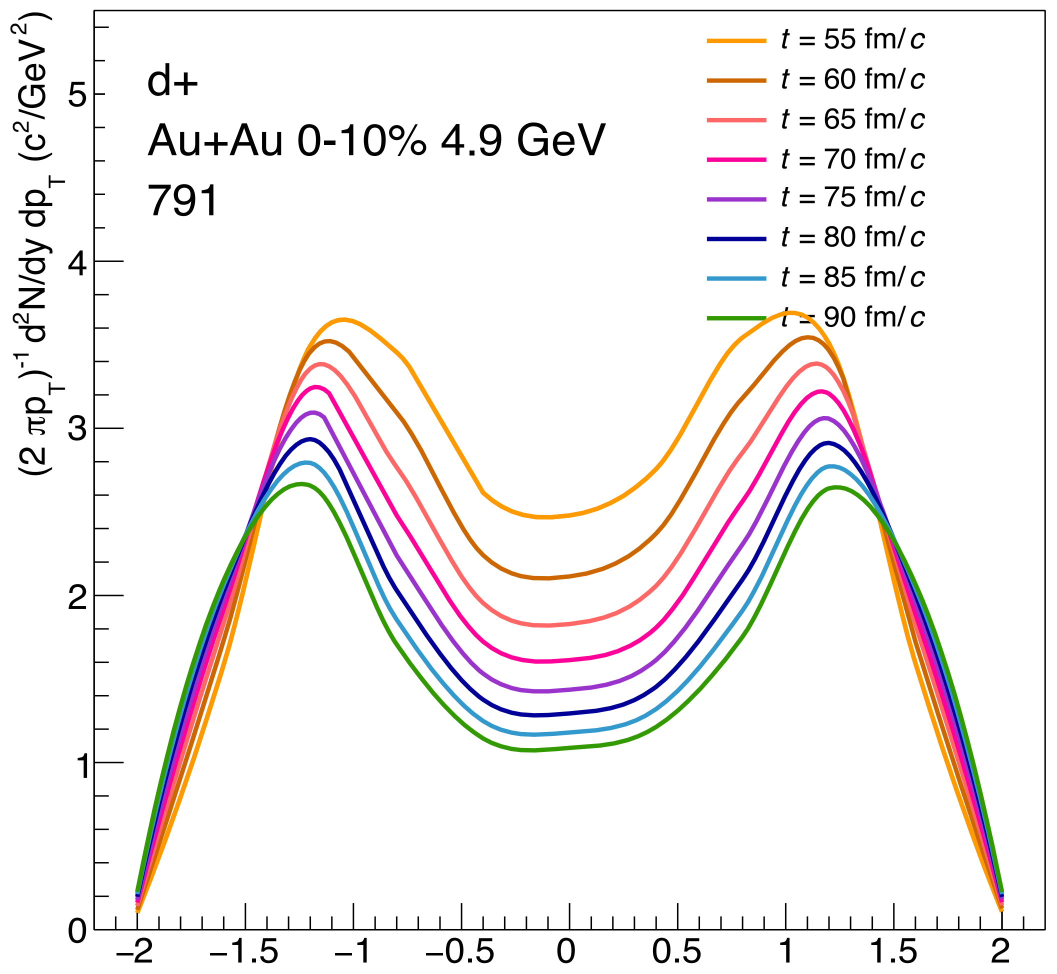


V. Kireyeu, Phys.Rev.C 103 (2021) 5

Cluster stability over time

QMD can not describe clusters as ‘quantum objects’

- the cluster **quantum ground state** has to respect a minimal average kinetic energy of the nucleons while **the semi-classical** (QMD) ground state - not!
- nucleons may still be emitted from the QMD clusters while in the corresponding quantum system this is not possible
- thus, a cluster which is “bound” at time t can **spontaneously** dissolve at t + Δt



= **QMD clusters are not fully stable over time**:

- the multiplicity of clusters is time dependent
- the form of the final rapidity, p_T distribution and ratio of particles do not change with time

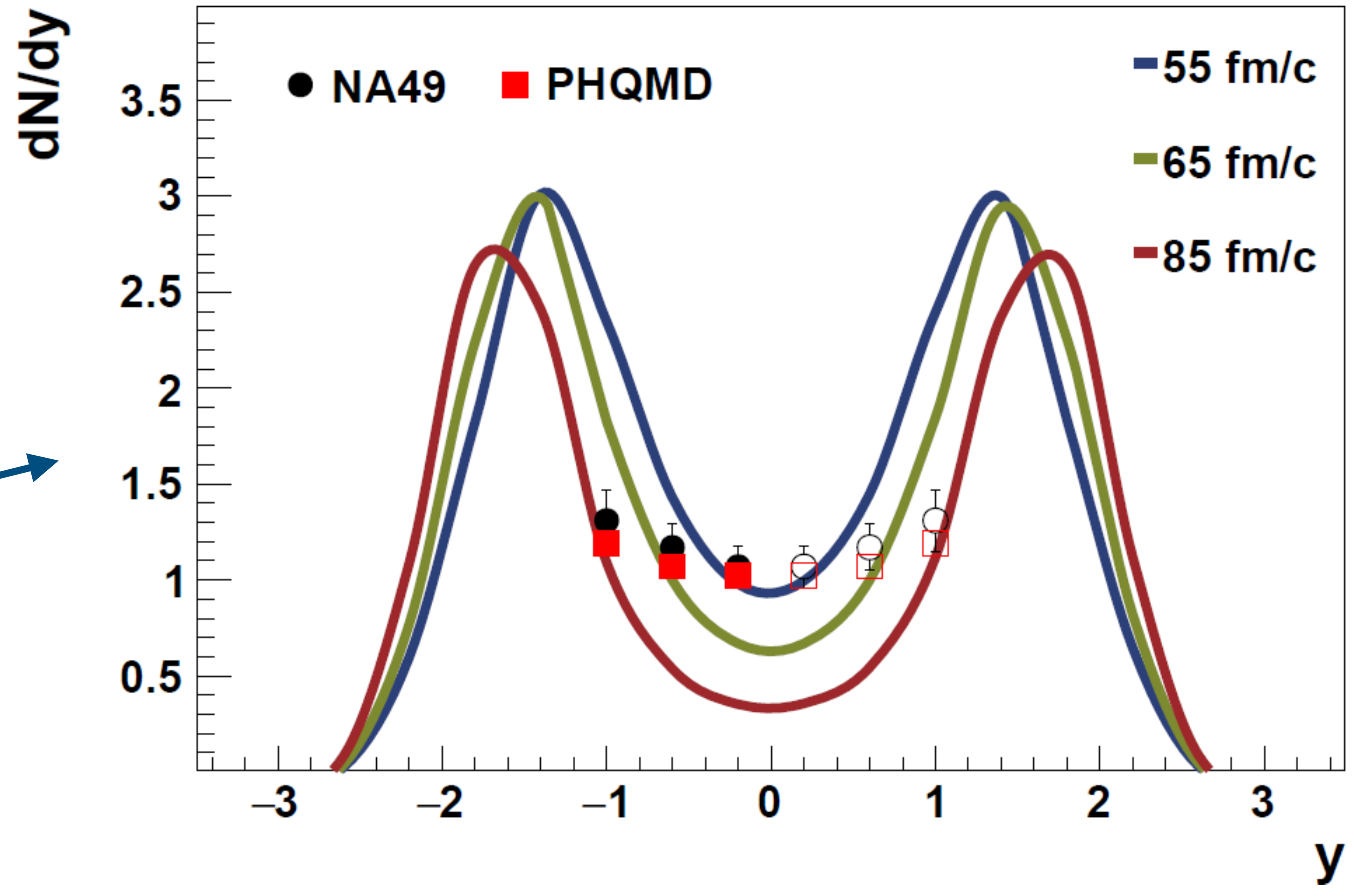
How to stabilize QMD clusters?

Scenario 1: **S. Gläßel et al., PRC 105 (2022) 1, 014908**

PHQMD results are taken at **‘physical time’** :

$$t = t_0 cosh(y)$$

where t_0 is the time selected as a best description of the cluster multiplicity at y=0

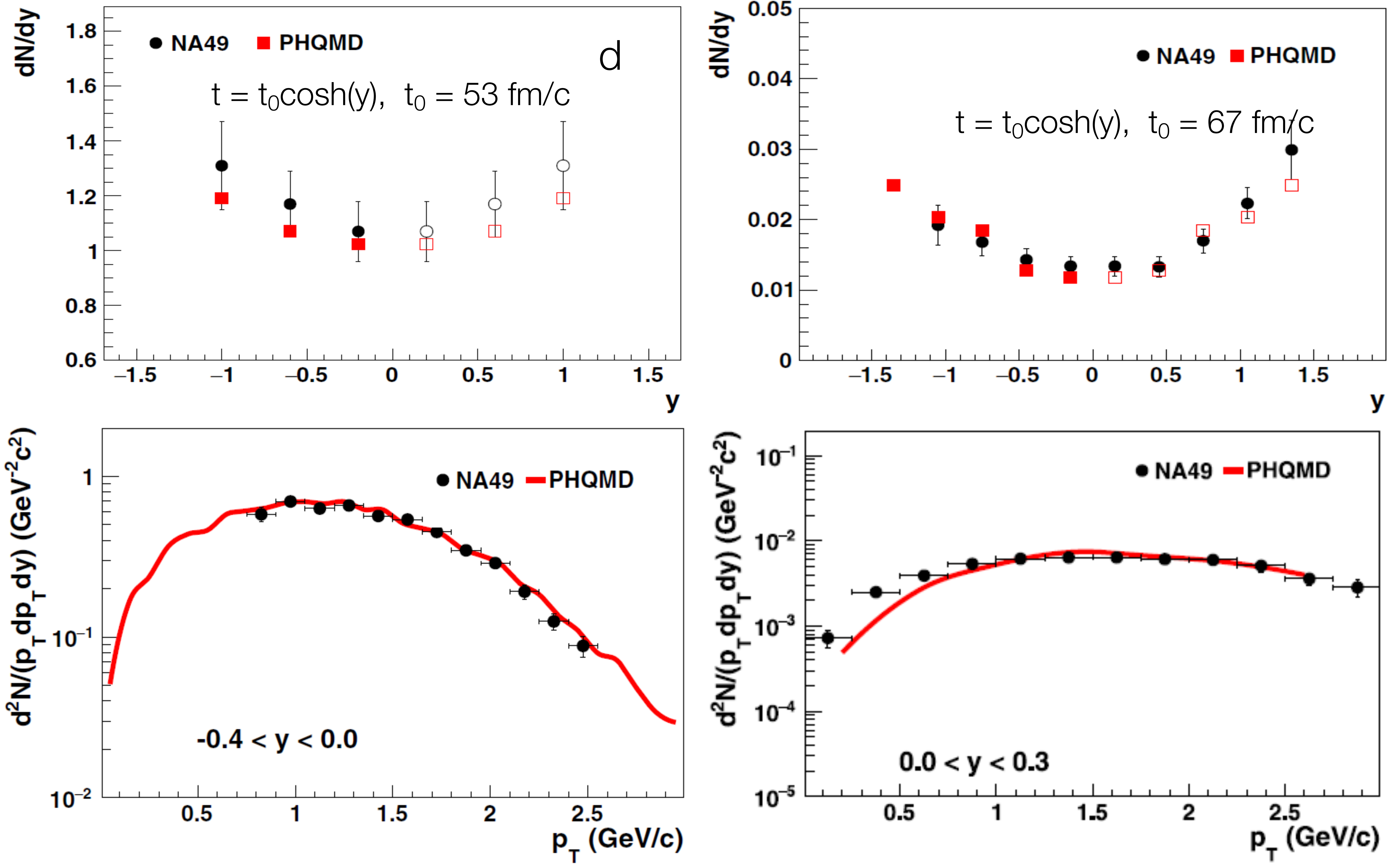


Cluster stability over time

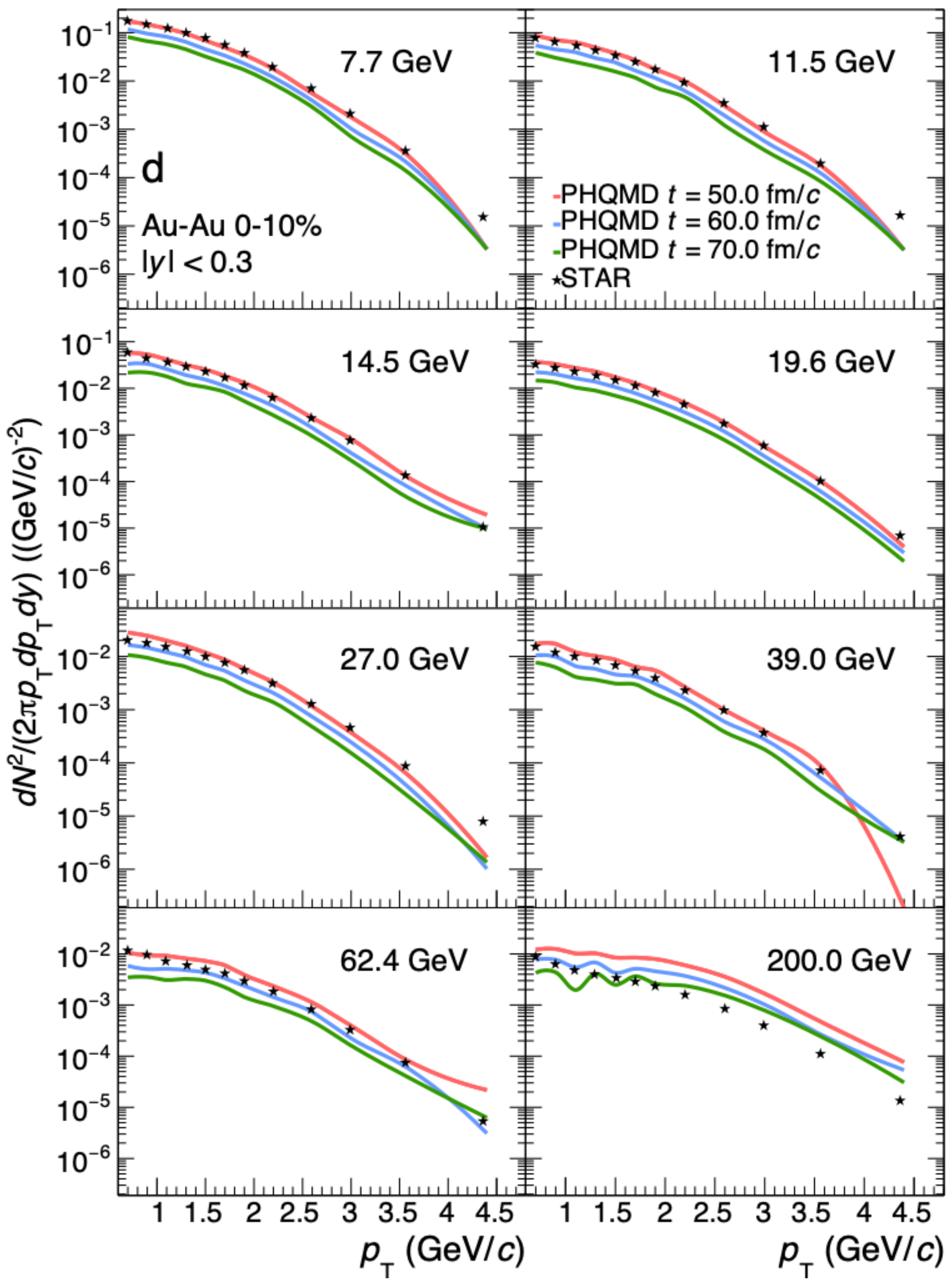
NA49 $\sqrt{s_{NN}}$ = 8.8 GeV

deuterons

^{3}He



STAR $\sqrt{s_{NN}}$= 7.7 GeV – 200 GeV

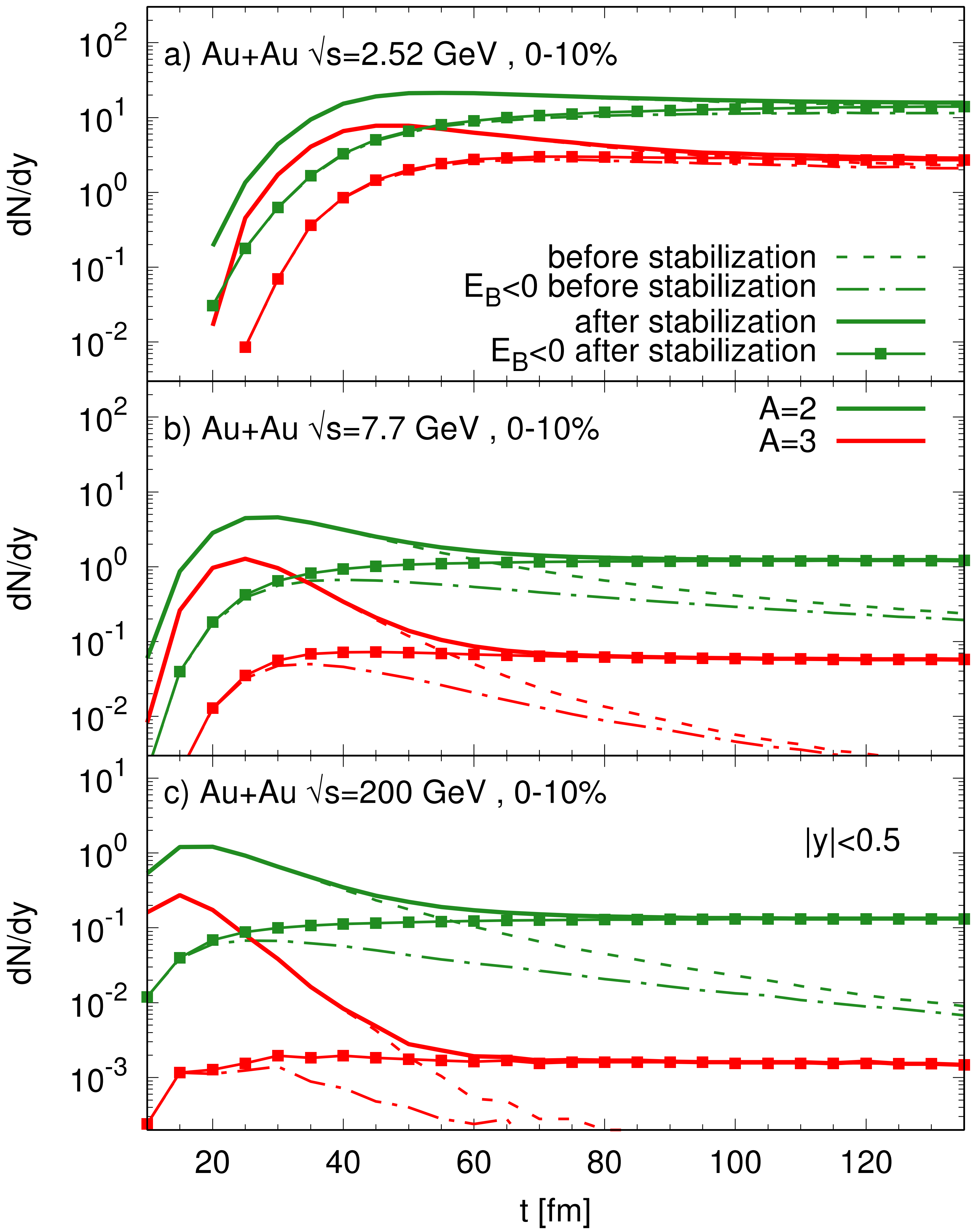


=> The PHQMD results for d and ^{3}He agree with NA49 and STAR data.

S. Gläßel et al., PRC 105 (2022) 1, 014908

T. Anticic et al. (NA49), Phys. Rev. C 94, 044906 (2016) J. Adam et al. (STAR), Phys. Rev. C 99, 064905 (2019)

Cluster stability over time



Scenario 2:
G. Coci et al., PRC 108 (2023) 1, 014902

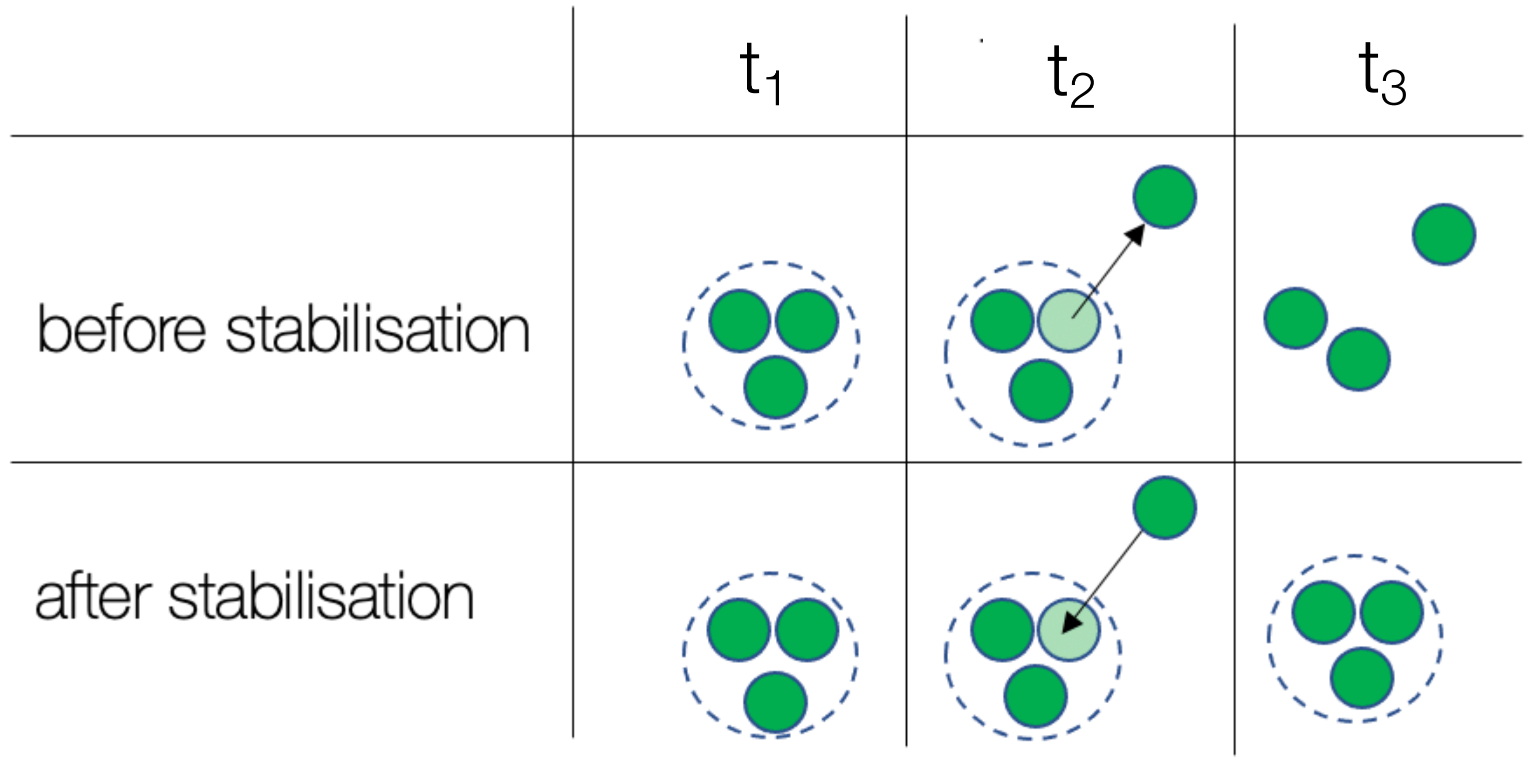
Stabilisation Procedure:

- consider asymptotic state: clusters and free nucleons

- For each nucleon in MST track the **freezout-time** = time at which the last collision occurred

- Recombine nucleons into clusters with **$E_B < 0$** if time of cluster disintegration is larger than nucleon freeze-out time

Allows to recover most of “lost” clusters



Kinetic mechanism for deuteron formation

G. Coci et al., PRC 108 (2023) 1, 014902

$$\pi^{\pm,0} + p + n \leftrightarrow \pi^{\pm,0} + d$$

$$\pi^- + p + p \leftrightarrow \pi^0 + d$$

$$\pi^+ + n + n \leftrightarrow \pi^0 + d$$

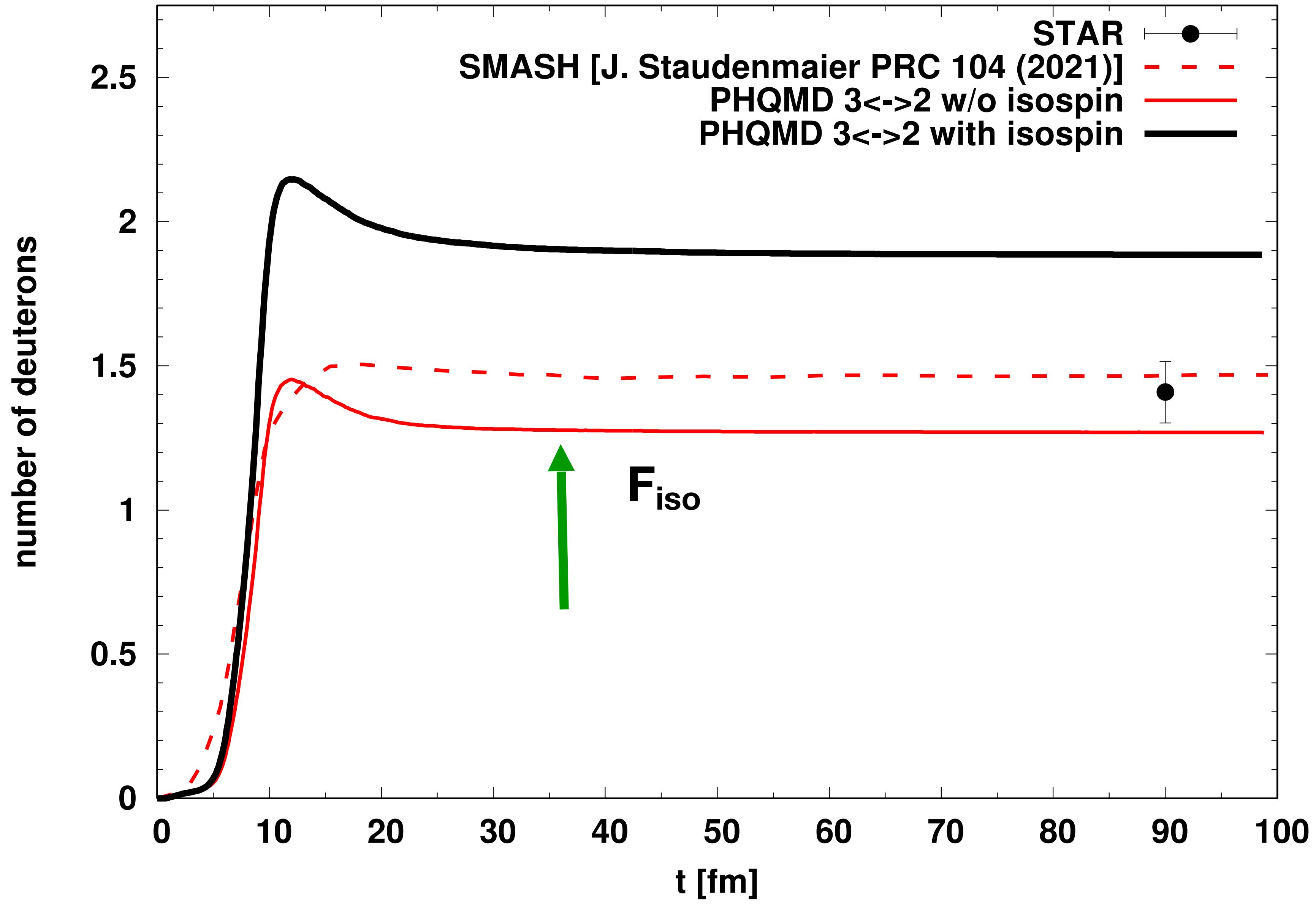
$$\pi^0 + p + p \leftrightarrow \pi^+ + d$$

$$\pi^0 + n + n \leftrightarrow \pi^- + d$$

N+N+π inclusion of all possible channels allowed by total isospin T conservation:

RHIC BES energy $\sqrt{s_{NN}}$ = 7.7 GeV:

- **Hierarchy due to large π abundance**
  π+N+N -> π+d >> N+p+n -> N+d
- **Inclusion of all isospin channels enhances deuteron yield ~ 50%.**
- **p_T slope is not affected**



GSI SIS energy $\sqrt{s_{NN}}$ < 3 GeV :

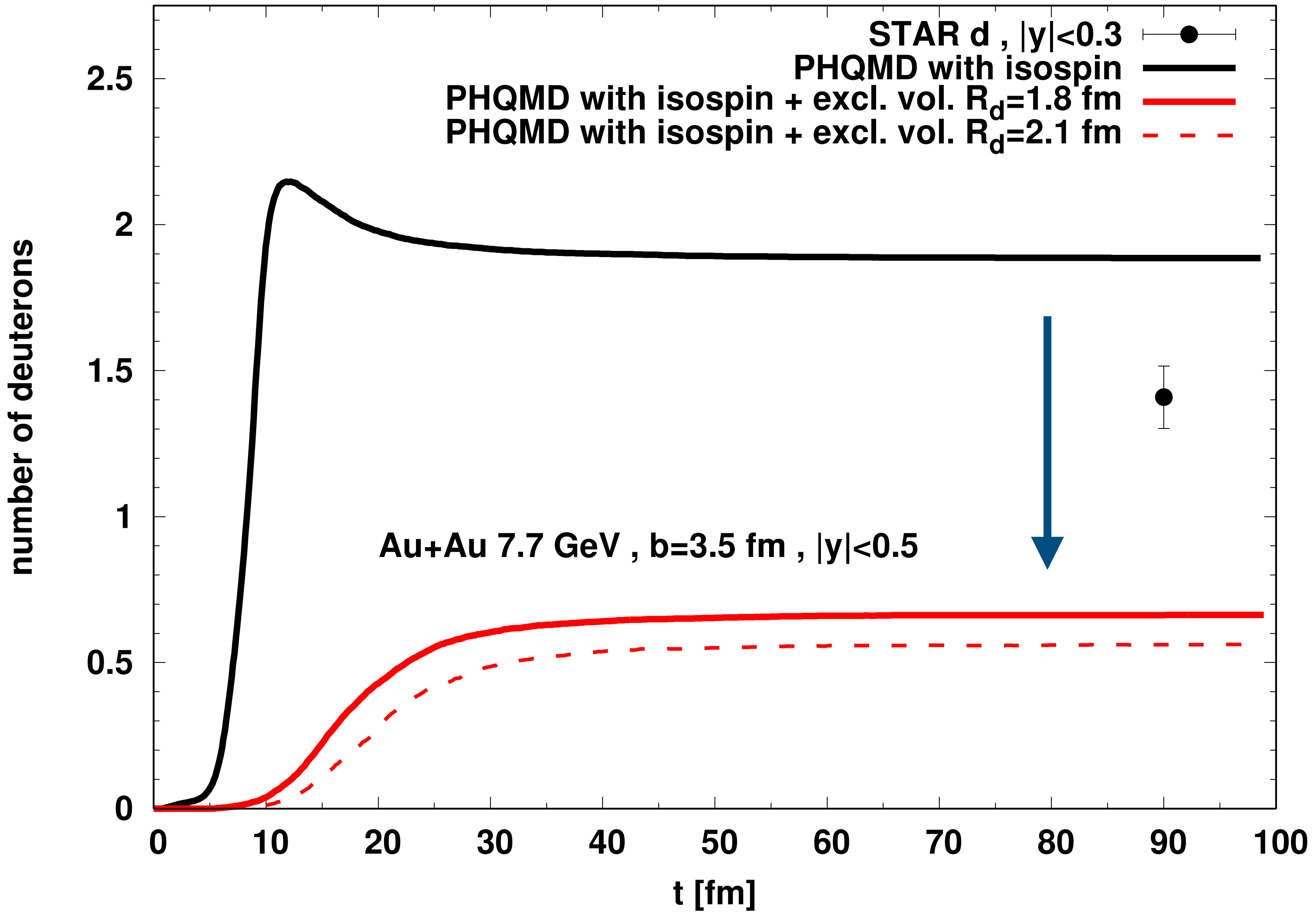
- **Baryon dominated matter**
- **Enhancement due to inclusion of isospin π+N+N channels is negligible**

Kinetic mechanism for deuteron formation

G. Coci et al., PRC 108 (2023) 1, 014902

1) the finite-size of *d* in the **coordinate space** (*d* is not a point-like particle) – for in-medium *d* production: assume that a deuteron can not be formed in a high density region, i.e. if there are other particles (hadrons or partons) inside the 'excluded volume'.

Excluded volume condition: $|\vec{r}(i)^* - \vec{r}(d)^*| < R_d$

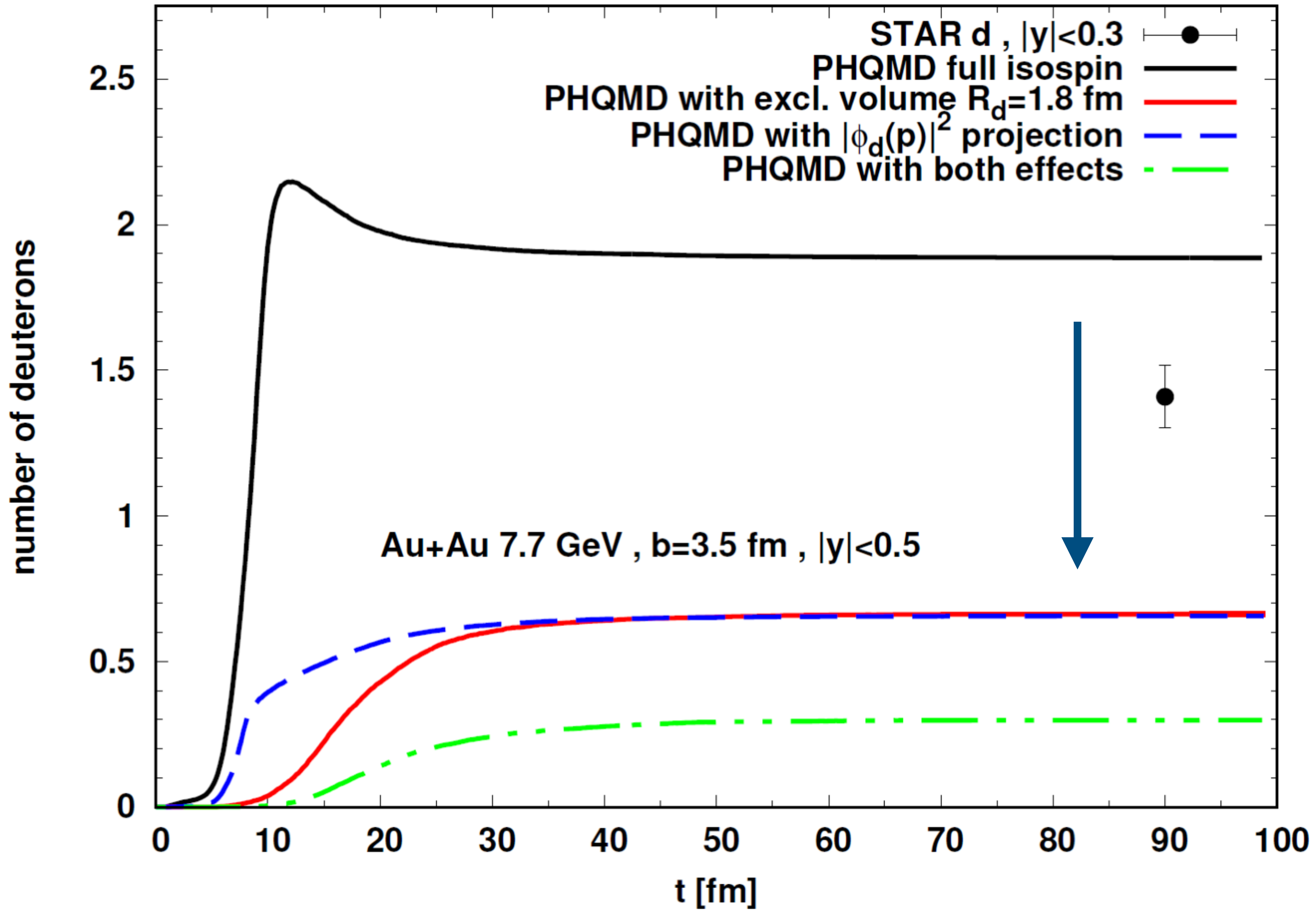


Strong reduction of *d* production

p_T slope is not affected by excluded volume condition

2) the **momentum correlations** of *p* and *n* inside *d*: QM properties of deuteron must be also in momentum space -> **momentum correlations of pn-pair**

- For a "candidate" deuteron calculate the relative momentum p of the interacting pn-pair in the deuteron rest frame
- The probability of the pn-pair to bind into a final deuteron with momentum p is given by the projection on DWF



Strong reduction of *d* production by projection on DWF

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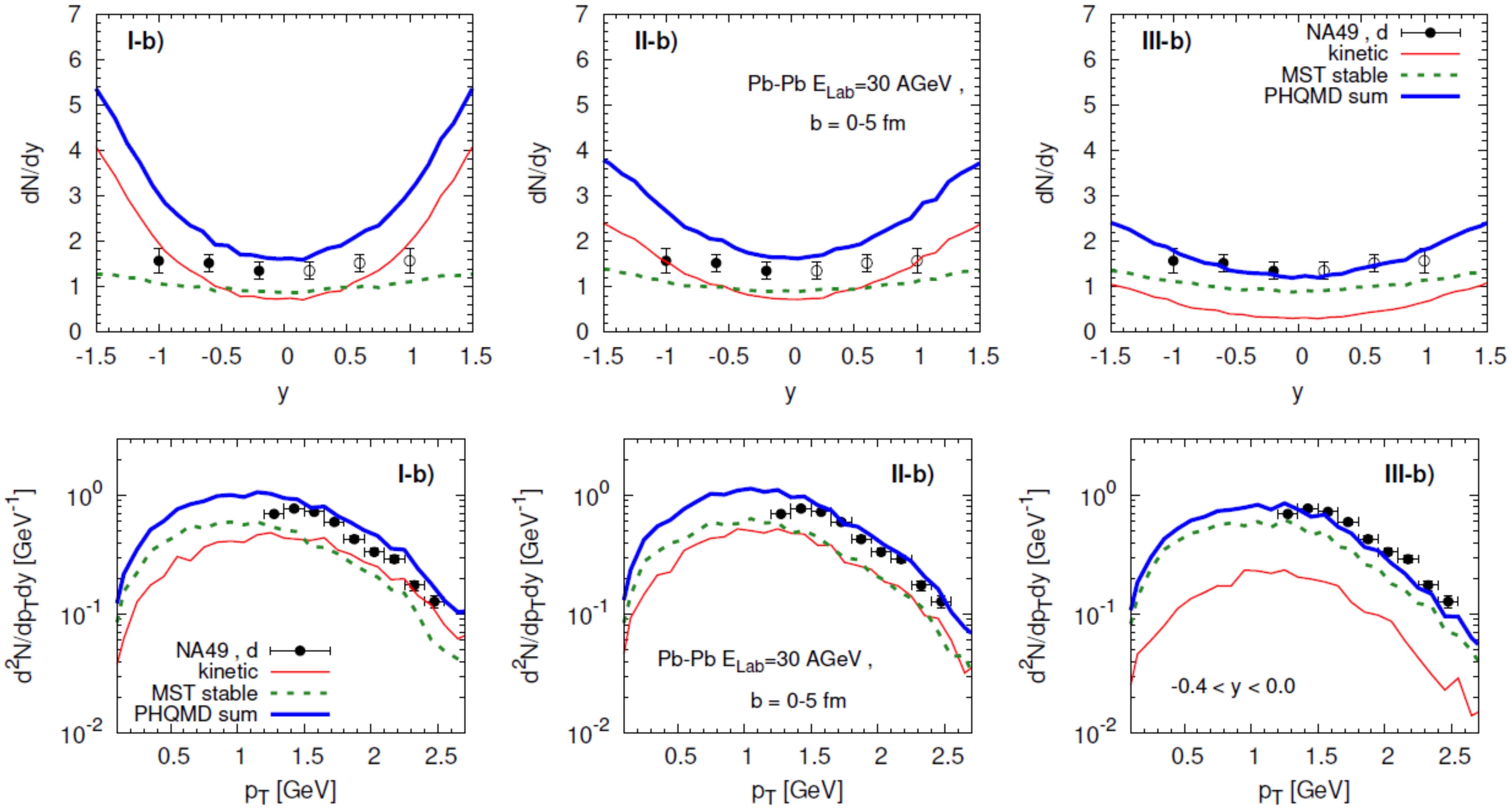
Total deuteron production = **Kinetic mechanism with finite-size effects**
+ **MST (with stabilization)** identification of deuterons (“stable” bound (E_B<0) A=2, Z=1 clusters)

Finite-size effects for kinetic deuterons:


1) excluded-volume

2) Momentum projection

3) both effects

NA49 data [PRC 94 (2016) 04490699]

Kinetic mechanism for deuteron formation

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Total deuteron production = **Kinetic mechanism with finite-size effects**
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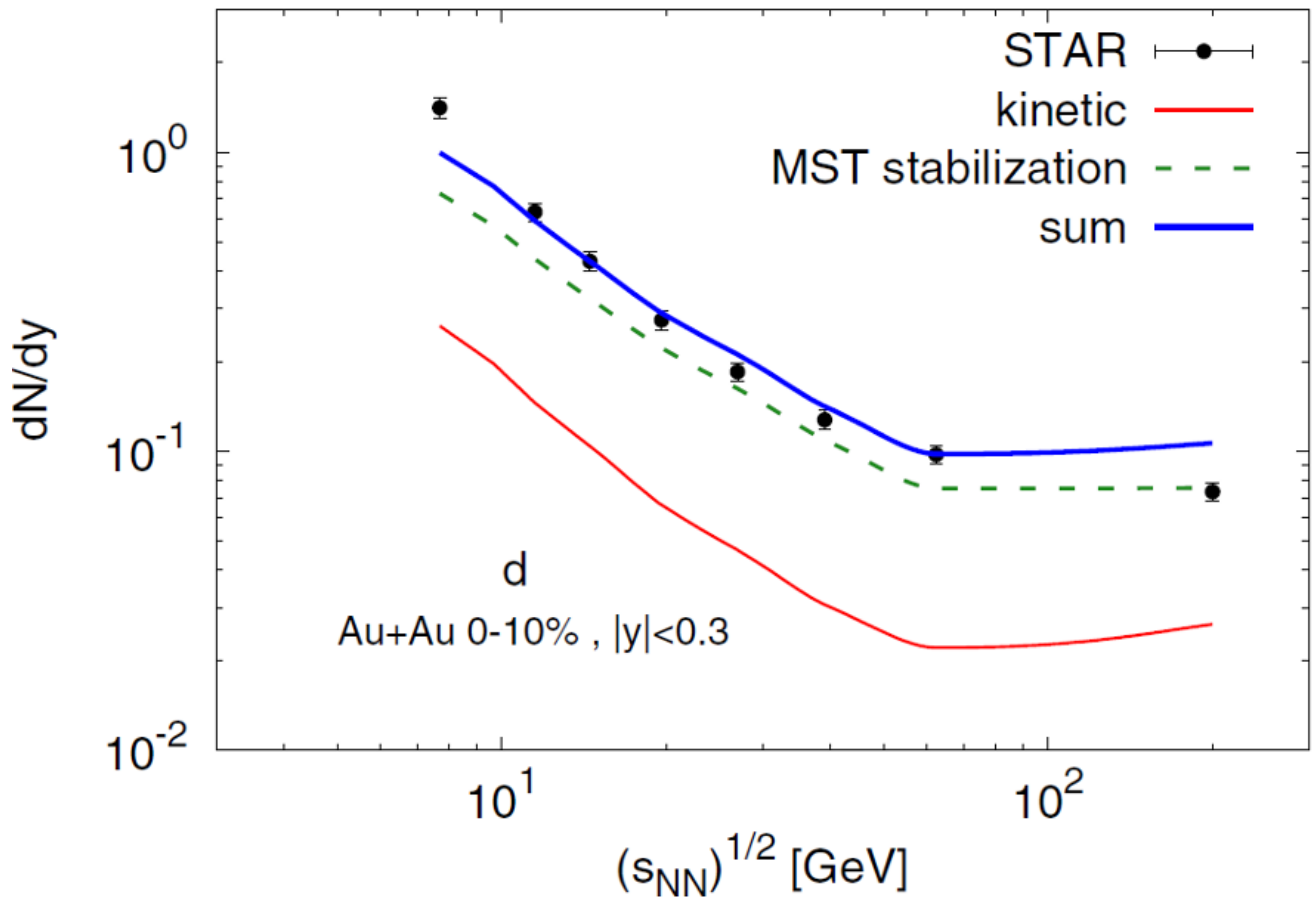
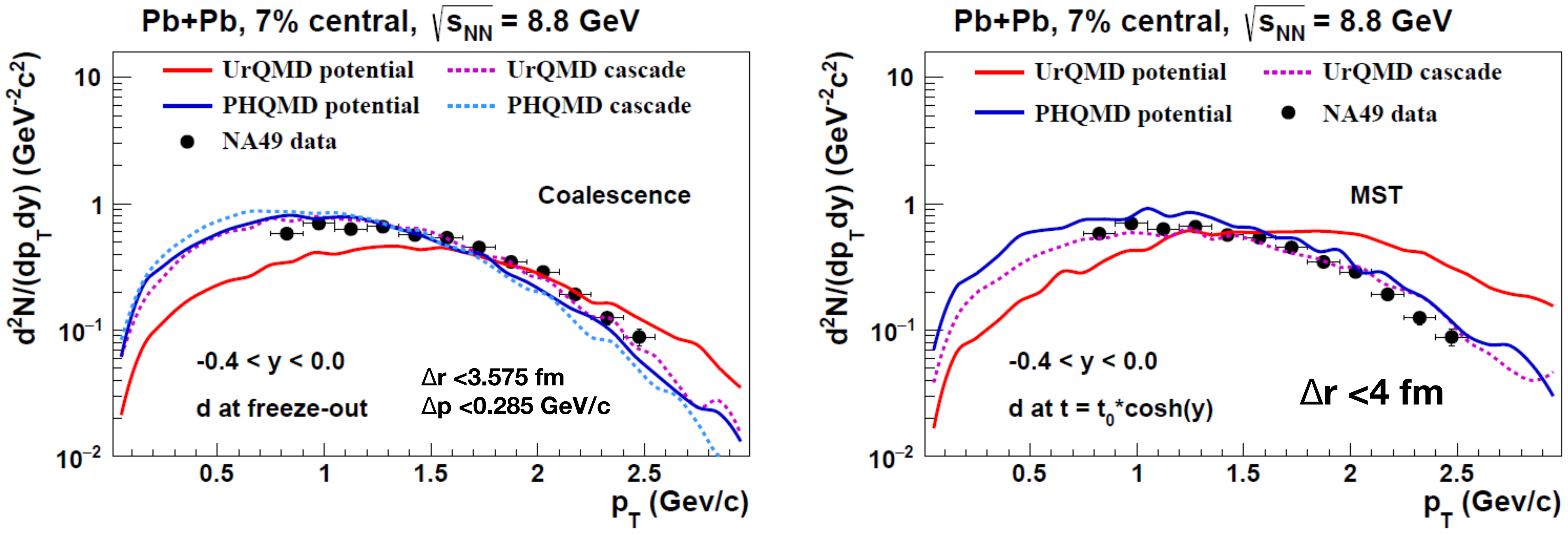
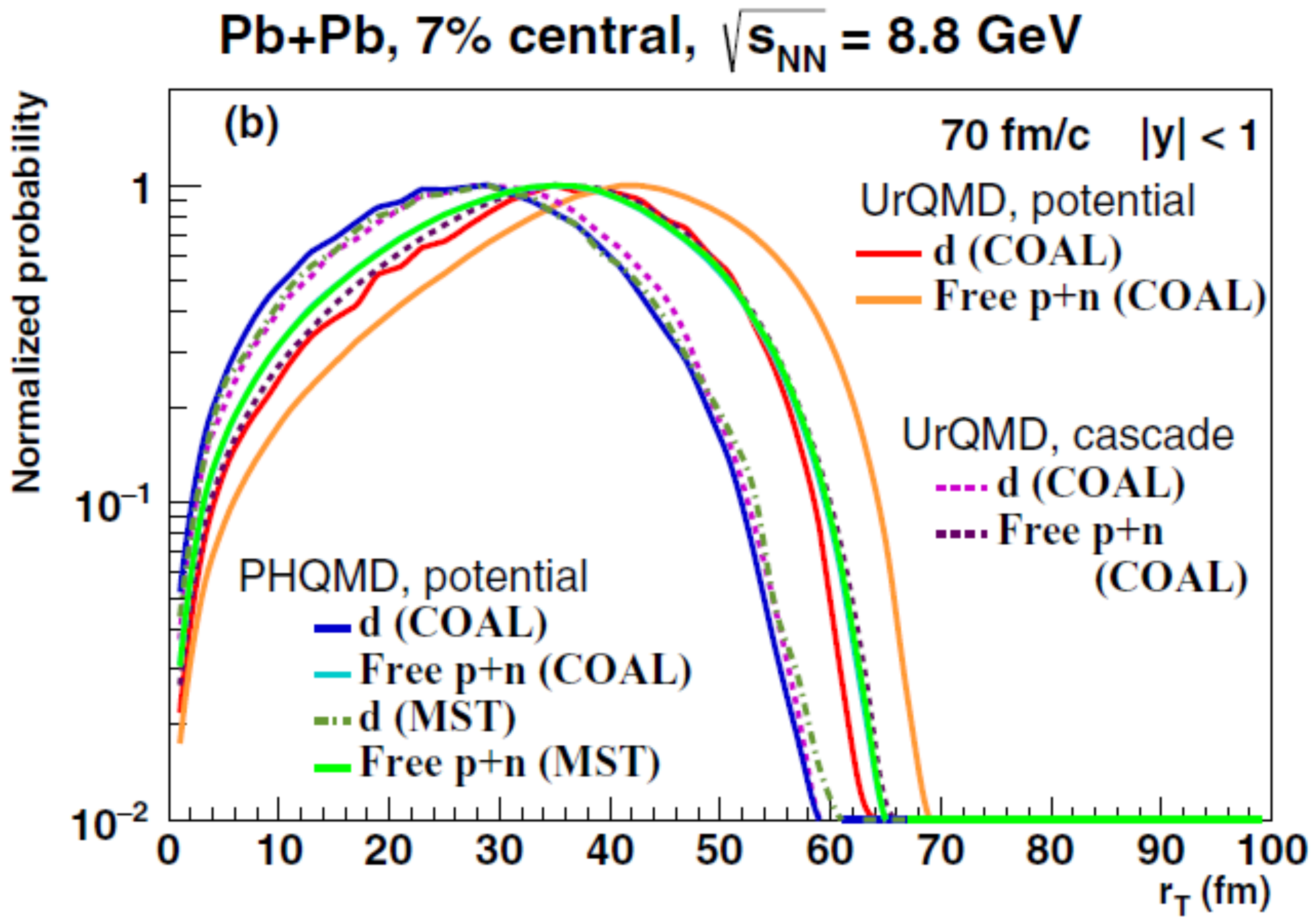
Excitation function dN/dy of deuterons at midrapidity



- **PHQMD provides a good description of STAR data**
- **The potential mechanism is dominant for the deuterons production at all energies!**

Where clusters are formed: coalescence and MST





- **Coalescence and MST** give very similar multiplicities and y- and pT –distributions
- PHQMD and UrQMD results in the cascade mode are very similar
- Deuteron production is sensitive to the realization of potential in transport approaches

Coalescence as well as the MST procedure show that the **deuterons remain in transverse direction closer to the center** of the heavy-ion collision than free nucleons

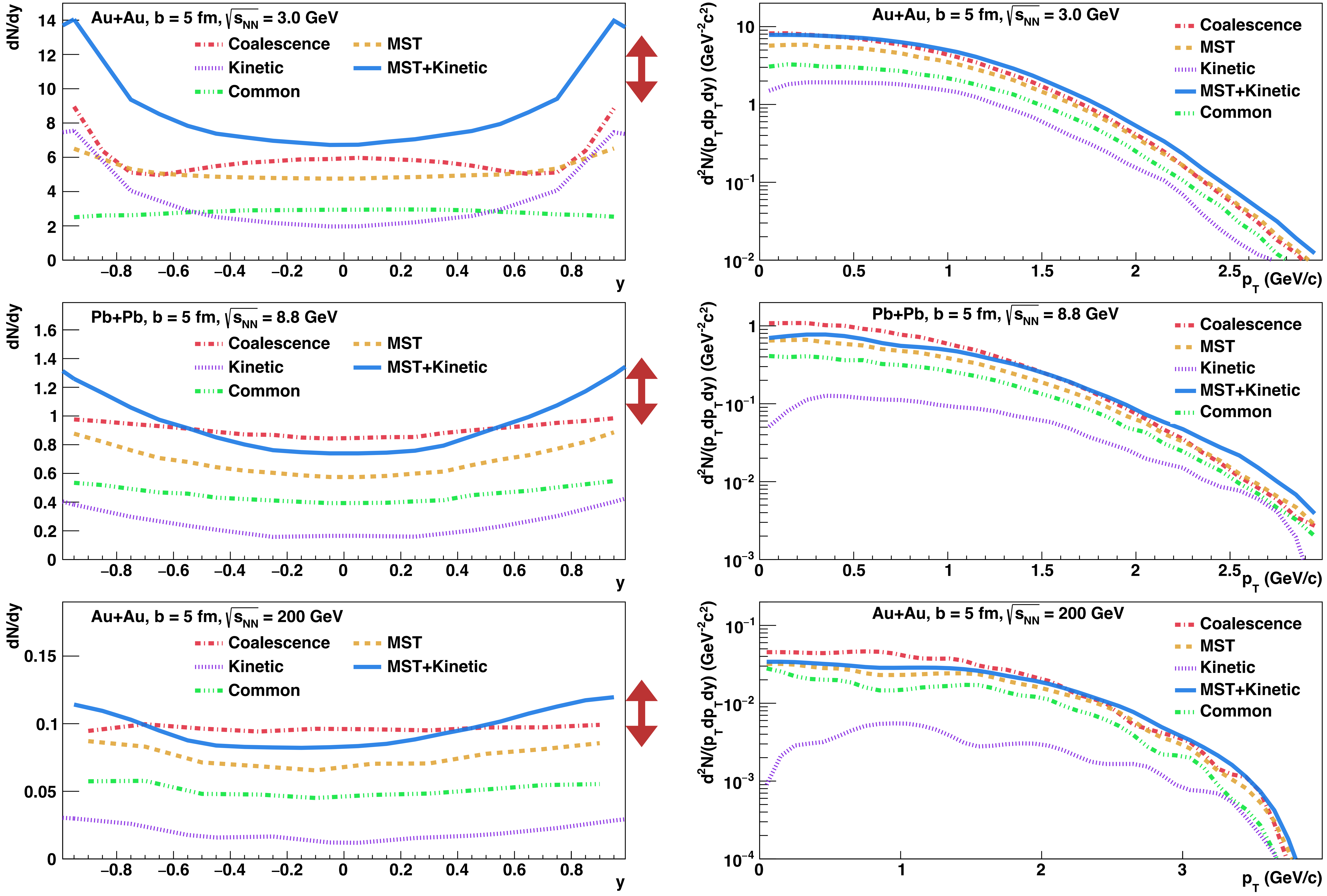
Deuterons are behind the fast nucleons.

«Ice in a fire» puzzle is solved?

V. Kireyeu et al., PRC 105 (2022) 044909

Can the deuteron formation mechanism be identified experimentally?

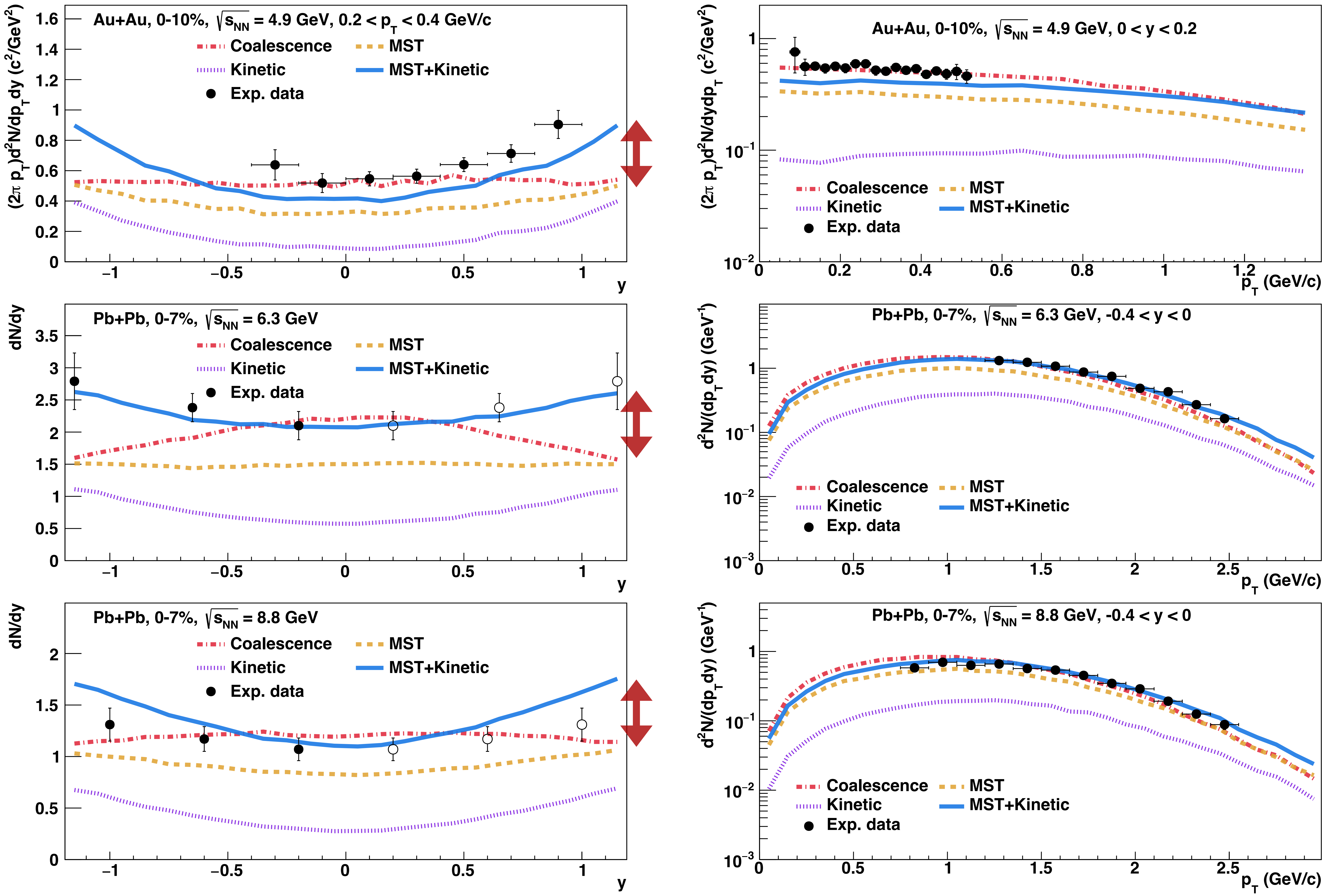
V. Kireyeu et. al, arxiv:2304.12019



- **At mid-rapidity only ~20%** of coalescence deuterons (at freeze-out) are found by MST.
- **Rapidity distribution** has a different shape.
- **Transverse momentum** distributions has different slope at low p_T

Can the deuteron formation mechanism be identified experimentally?

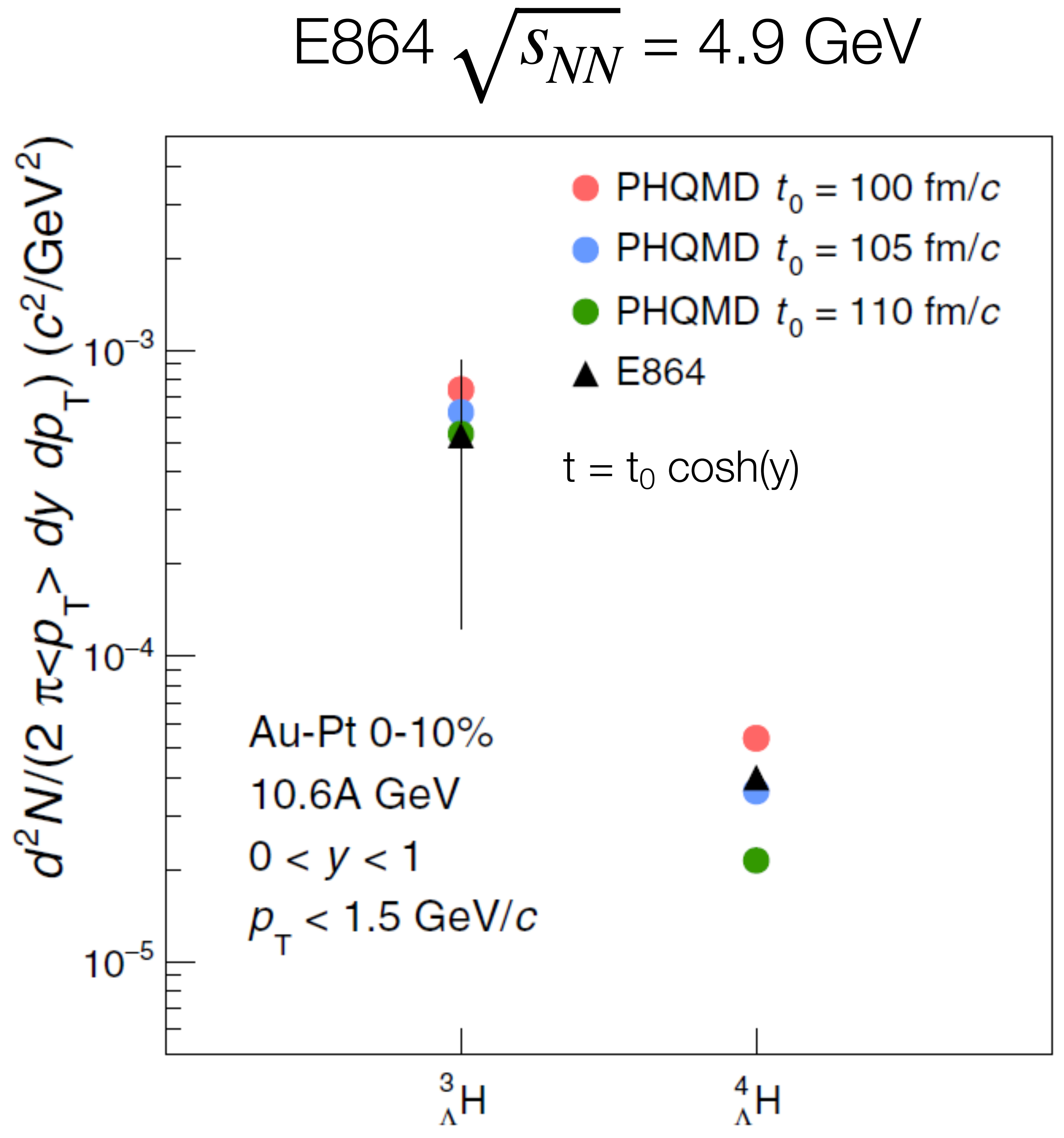
V. Kireyeu et. al, arxiv:2304.12019

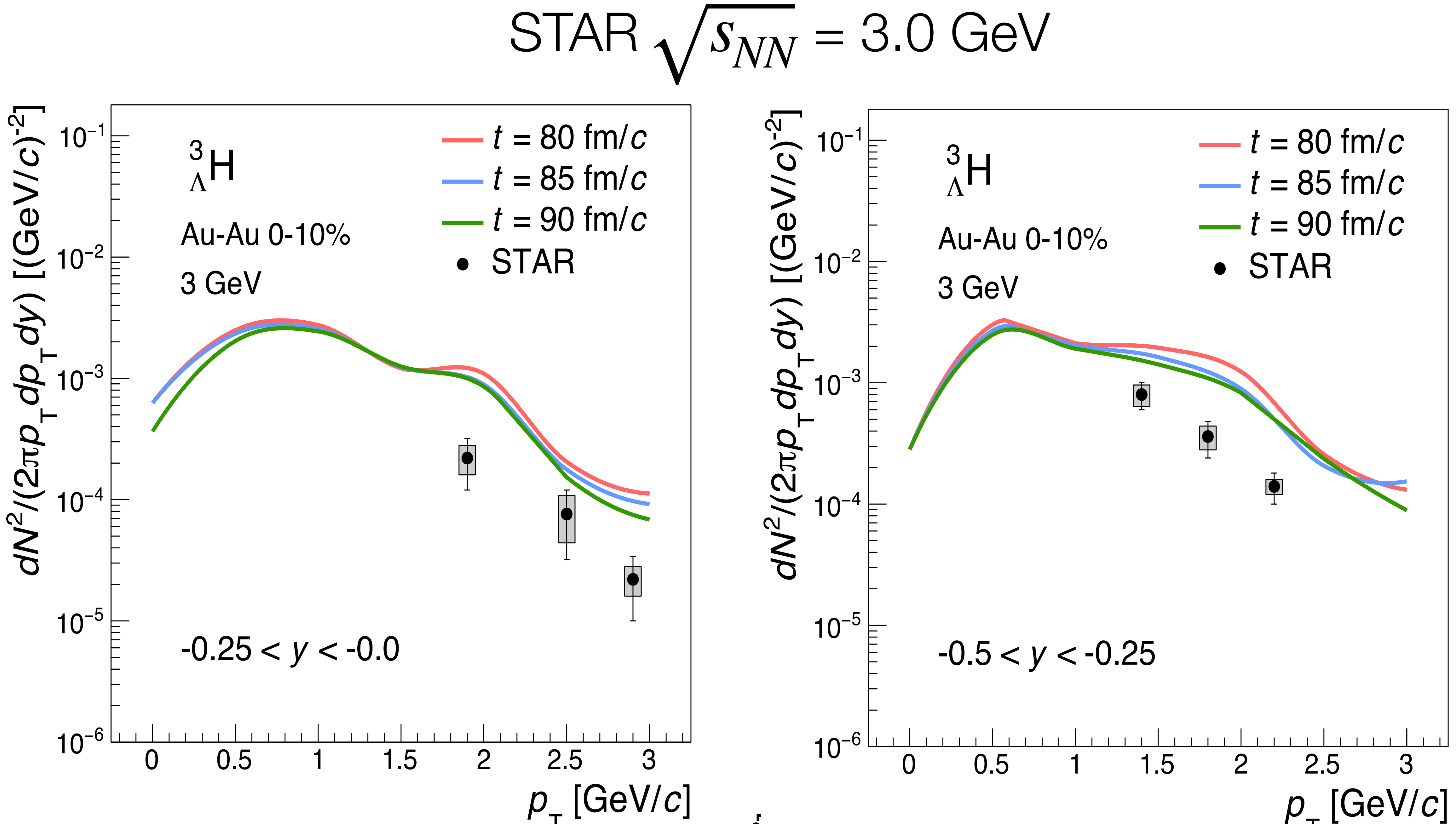


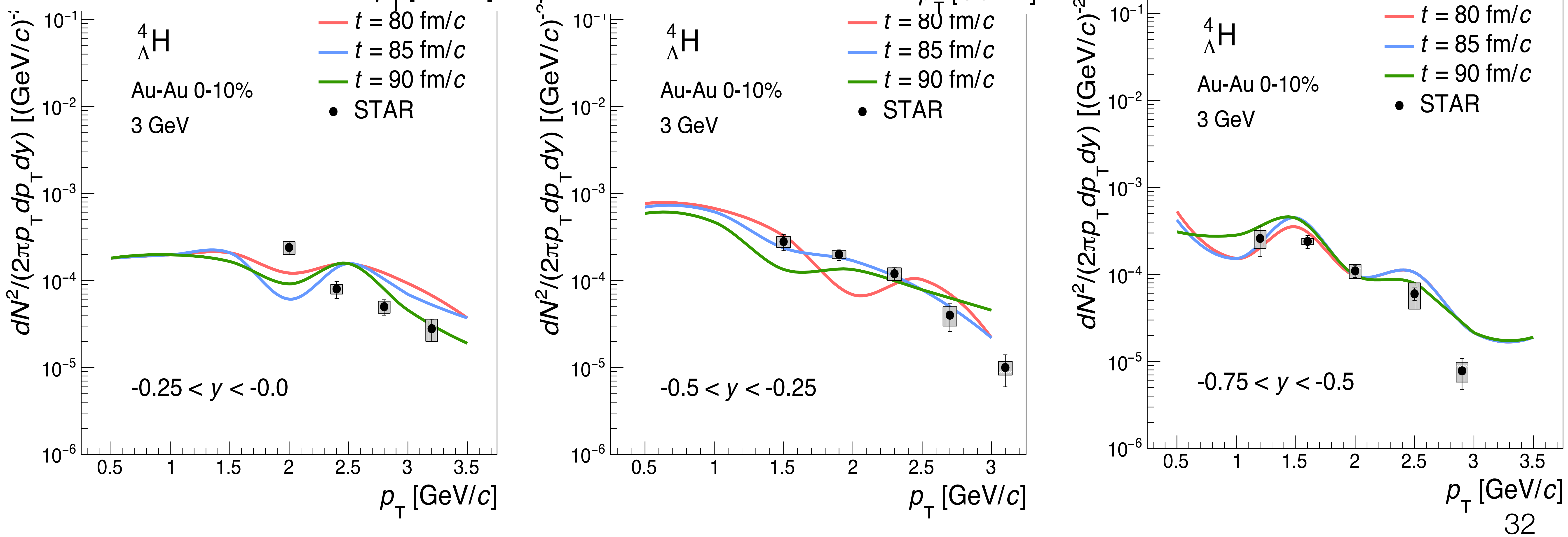
The analysis of the presently available data **points tentatively to the MST + kinetic scenario**, but further experimental data are necessary to establish this mechanism.

Hypernuclei production at $\sqrt{s_{NN}}$ = 3.0 and 4.9 GeV

S. Gläßel et al., PRC 105 (2022) 1, 014908







=> Reasonable description of hypernuclei production at $\sqrt{s_{NN}}$ = 3.0 GeV

*Yue-Hang Leung: First results of H3L & H4L (dN/dy, c_T, v_1) from 3 GeV Au+Au collisions with the STAR detector (CPOD2021)

Assumption for nucleon-hyperon potential: $V_{N\Lambda} = 2/3\ V_{NN}$

=> trend of the experimental STAR* & p_T-spectra at $\sqrt{s_{NN}}$ =3 is produced well

=> yields are slightly overpredicted

Time evolution of the cluster formation