CONSTRAINTS ON NEUTRON SKIN THICKNESS AND SYMMETRY ENERGY

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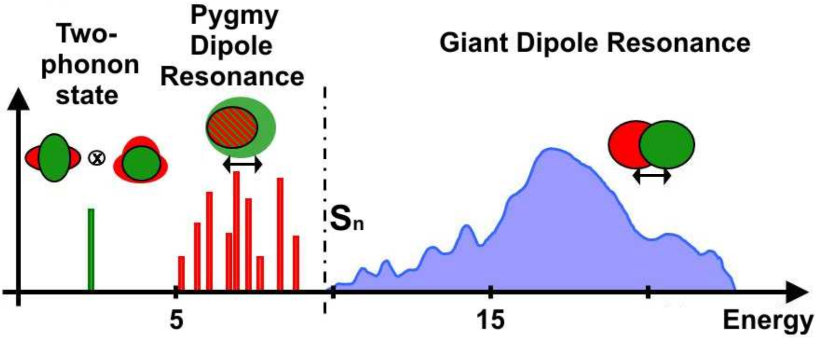
Outline

Introduction

$E1$ strength in (spherical) atomic nuclei



Courtesy: N. Pietralla

Introduction: Relevance of the PDR

1. The PDR might play an important role in nuclear astrophysics. For example, the occurrence of the PDR could have a pronounced effect on neutron-capture rates in the *r*-process nucleosynthesis, and consequently on the calculated elemental abundance distribution.

S. Goriely, **Phys. Lett.** B436, **10 (1998)**.

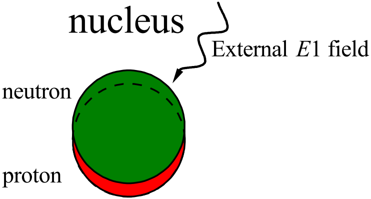
2. The study of the pygmy $E1$ strength is expected to provide information on the symmetry energy term of the nuclear equation of state. This information is very relevant for the modeling of neutron stars.

C. J. Horowitz and J. Piekarewicz, **Phys. Rev. Lett.** 86, **5647 (2001)**.

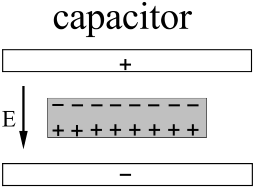
3. New type of nuclear excitation: these resonances are the low-energy tail of the GDR, or if they represent a different type of excitation, or if they are generated by single-particle excitations related to the specific shell structure of nuclei with neutron excess.

N. Paar, D. Vretenar, E. Khan, G. Colò, **Rep. Prog. Phys.** 70, **691 (2007)**.

Introduction: Electric dipole polarizability



Electric dipole
polarizability



Electric dipole
polarization

External field
↕ Balanced $\Longrightarrow$ Displacement dipole polarizability
Restoring force

$$\alpha_D = \frac{8\pi}{9} \sum_{\nu} \frac{B(E1; 0^+_{\text{g.st.}} \to 1^-_\nu)}{E_{1^-_\nu}}$$

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MAIN INGREDIENTS OF THE MODEL

Realization of QRPA

The starting point of the method is the HF-BCS calculations of the ground state, where spherical symmetry is assumed for the ground states. The continuous part of the single-particle spectrum is discretized by diagonalizing the HF Hamiltonian on a harmonic oscillator basis.

J. P. Blaizot and D. Gogny, **Nucl. Phys. A284, 429 (1977).**

We employ the effective Skyrme interaction in the particle-hole channel

$$V(\vec{r}_1,\vec{r}_2)=t_0\left(1+x_0\hat{P}_\sigma\right)\delta(\vec{r}_1-\vec{r}_2)+\frac{t_1}{2}\left(1+x_1\hat{P}_\sigma\right)\left[\delta(\vec{r}_1-\vec{r}_2)\vec{k}^2+\vec{k}'^2\delta(\vec{r}_1-\vec{r}_2)\right]$$
$$+t_2\left(1+x_2\hat{P}_\sigma\right)\vec{k}'\cdot\delta(\vec{r}_1-\vec{r}_2)\vec{k}+\frac{t_3}{6}\left(1+x_3\hat{P}_\sigma\right)\delta(\vec{r}_1-\vec{r}_2)\rho^\alpha\left(\frac{\vec{r}_1+\vec{r}_2}{2}\right)$$
$$+iW_0\left(\vec{\sigma}_1+\vec{\sigma}_2\right)\cdot\left[\vec{k}'\times\delta(\vec{r}_1-\vec{r}_2)\vec{k}\right].$$

T. H. R. Skyrme, **Nucl. Phys. 9, 615 (1959).**

D. Vautherin and D. M. Brink, **Phys. Rev. C5, 626 (1972).**

Realization of QRPA

The Hamiltonian includes the pairing correlations are generated by the density-dependent zero-range force in the particle-particle channel

$$V_{pair}(\vec{r}_1, \vec{r}_2) = V_0 \left(1 - \eta \frac{\rho(r_1)}{\rho_{sat}}\right) \delta(\vec{r}_1 - \vec{r}_2) \,,$$

where ρ_{sat} is the nuclear saturation density; η and V_0 are model parameters. For example, $\eta=0$ and $\eta=1$ are the case of a volume interaction and a surface-peaked interaction, respectively.

A. P. Severyukhin, V. V. Voronov, N. V. Giai, Phys. Rev. C77, 024322 (2008).

The residual interaction in the particle-hole channel V_{res}^{ph} and in the particle-particle channel V_{res}^{pp} can be obtained as the second derivative of the energy density functional $\mathcal{H}$ with respect to the particle density ρ and the pair density $\tilde{\rho}$, respectively.

$$V_{res}^{\mathrm{ph}} \sim \frac{\delta^2 \mathcal{H}}{\delta\rho_1 \delta\rho_2} \qquad V_{res}^{\mathrm{pp}} \sim \frac{\delta^2 \mathcal{H}}{\delta\tilde{\rho}_1 \delta\tilde{\rho}_2} \,.$$

G. T. Bertsch and S. F. Tsai, Phys. Rep. 18, 125 (1975).

Realization of QRPA

We introduce the phonon creation operators

$$Q^{+}_{\lambda\mu i}=\frac{1}{2}\sum_{jj'}\left[X^{\lambda i}_{jj'}\,A^{+}(jj';\lambda\mu)-(-1)^{\lambda-\mu}Y^{\lambda i}_{jj'}\,A(jj';\lambda-\mu)\right],$$

$$A^{+}(jj';\lambda\mu)=\sum_{mm'}C^{\lambda\mu}_{jm\,j'm'}\alpha^{+}_{jm}\alpha^{+}_{j'm'}\,.$$

The index λ denotes total angular momentum and μ is its z-projection in the laboratory system. One assumes that the ground state is the QRPA phonon vacuum $|0\rangle$ and one-phonon excited states are $Q^{+}_{\lambda\mu i}|0\rangle$ with the normalization condition

$$\langle 0|[Q_{\lambda\mu i},Q^{+}_{\lambda\mu i'}]|0\rangle=\delta_{ii'}\,.$$

Making use of the linearized equation-of-motion approach one can get the QRPA equations

$$\begin{pmatrix}\mathcal{A} & \mathcal{B}\\ -\mathcal{B} & -\mathcal{A}\end{pmatrix}\begin{pmatrix}X\\ Y\end{pmatrix}=\omega\begin{pmatrix}X\\ Y\end{pmatrix}.$$

Solutions of this set of linear equations yield the one-phonon energies ω and the amplitudes X, Y of the excited states.

D. J. Rowe, **Nuclear Collective Motion (Methuen, London 1970).**

RESULTS AND DISCUSSION

Details of calculations

The dipole transition probabilities can be expressed as

$$B(E1; 0^+_{gs} \to 1^-_i) = \left| e^{(\mathrm{n})}_{eff} \langle i | \hat{M}^{(\mathrm{n})} | 0 \rangle + e^{(\mathrm{p})}_{eff} \langle i | \hat{M}^{(\mathrm{p})} | 0 \rangle \right|^2 ,$$

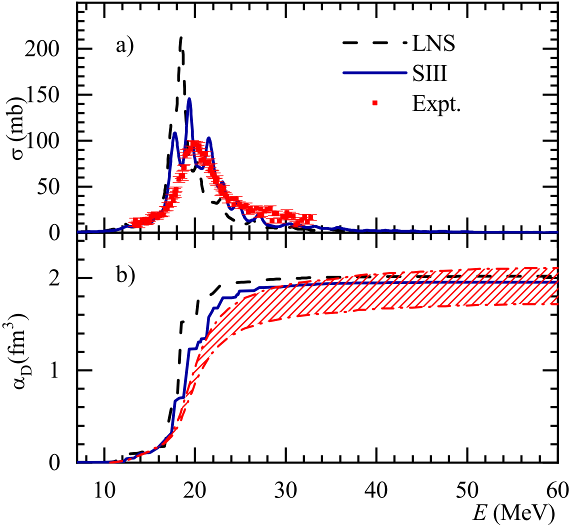
where $\hat{M}^{(\mathrm{p})} = \sum_i^Z r_i Y_{1\mu}(\hat{r}_i)$ and $\hat{M}^{(\mathrm{n})} = \sum_i^N r_i Y_{1\mu}(\hat{r}_i)$. The spurious 1^- state is excluded from the excitation spectra by introduction of the effective neutron $e^{(\mathrm{n})}_{eff} = -Z/A\, e$ and proton $e^{(\mathrm{p})}_{eff} = N/A\, e$ charges.

A. Bohr and B. Mottelson, **Nuclear Structure Vol. II (Benjamin, New York 1975).**

For our systematic analysis of the electric dipole polarizability, we employ 21 Skyrme interactions: LNS, SAMi, SGII, SIII, SK255, SkI2, SkI3, SkI5, SkM*, SkP, SkT5, SkT6, SkT7, SkX, SLy4, SLy5, SVbas, SVmas07, SVmas08, SVmas10, and SVmin. The choice of these parameterizations is due to the large range of values for the effective nucleon mass $m^* = 0.58-1.00$ and the symmetry energy at saturation density $J = 26.8-37.4$ MeV.

M. Dutra et al., **Phys. Rev. C85, 035201 (2012).**

The photoabsorption cross section for ^{40}Ca



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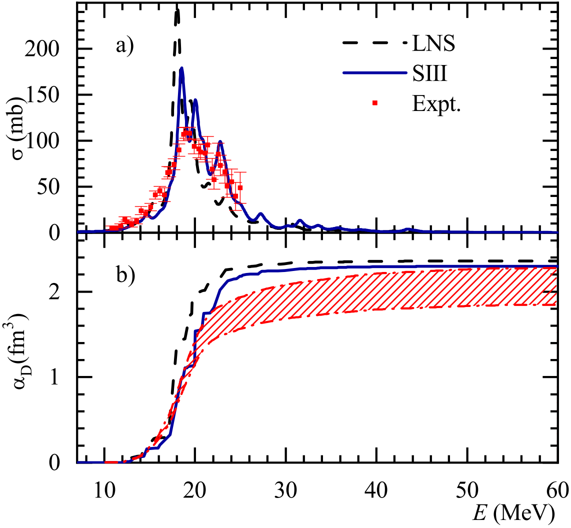
R. W. Fearick et al., **Phys. Rev. Res. 5, L022044 (2023).**

The photoabsorption cross section for ^{48}Ca



N. E. Solonovich, N. N. Arsenyev, A. P. Severyukhin, Phys. Part. Nucl. Letters. 19, 473 (2022)

J. Birkhan et al., PRL 118, 252501 (2017).

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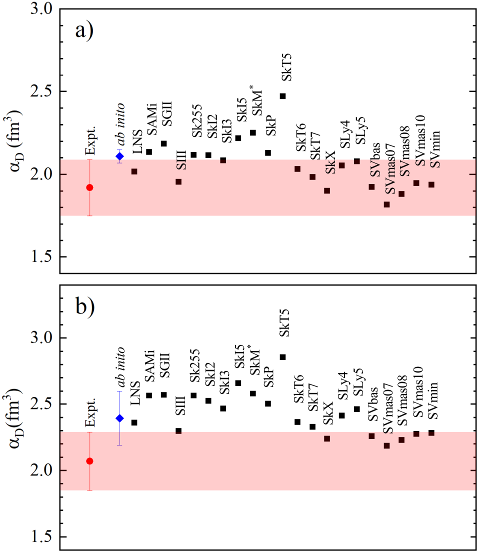
Electric dipole polarizability α_D in $^{40,48}Ca$



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J. Birkhan et al., PRL 118, 252501 (2017).

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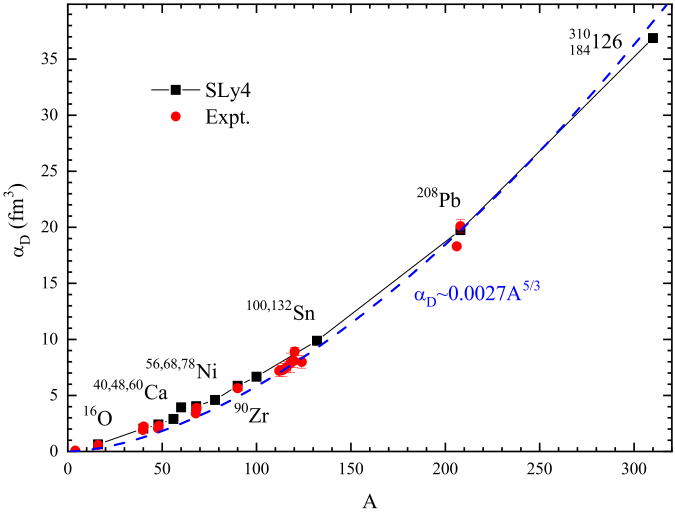
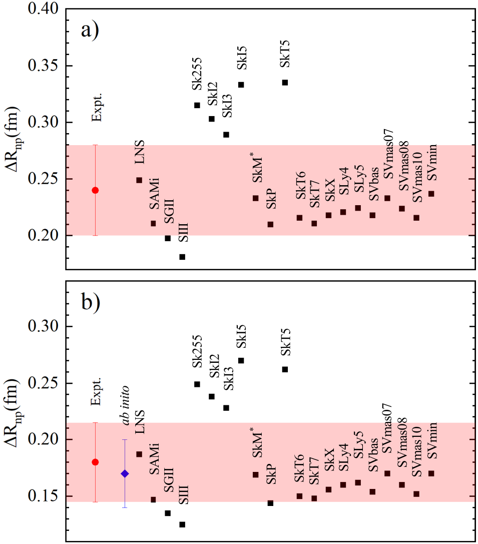
G. Hagen et al., Nature Phys. 12, 186 (2016).

Electric dipole polarizability: Extp. vs Theory



N. N. Arsenyev, A. P. Severyukhin, in preparation.

Neutron skin thickness $\Delta R_{\rm np}$ in ^{132}Sn and ^{208}Pb



N. N. Arsenyev and A. P. Severyukhin, Moscow Univ. Phys. Bull. 79, 200 (2024).

A. Klimkiewicz et al., Phys. Rev. C76, 051603 (2007).

B. Hu et al., Nature Phys. 18, 1196 (2022).

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Pearson correlation coefficient

In statistics, the Pearson correlation coefficient (r) is a measure of linear correlation between two sets of data: $(a_1, b_1), \ldots, (a_N, b_N)$. The Pearson coefficient calculate as

$$r = \frac{\sum_{i=1}^{N}(a_i - \bar{a})(b_i - \bar{b})}{\sigma_a \sigma_b},$$

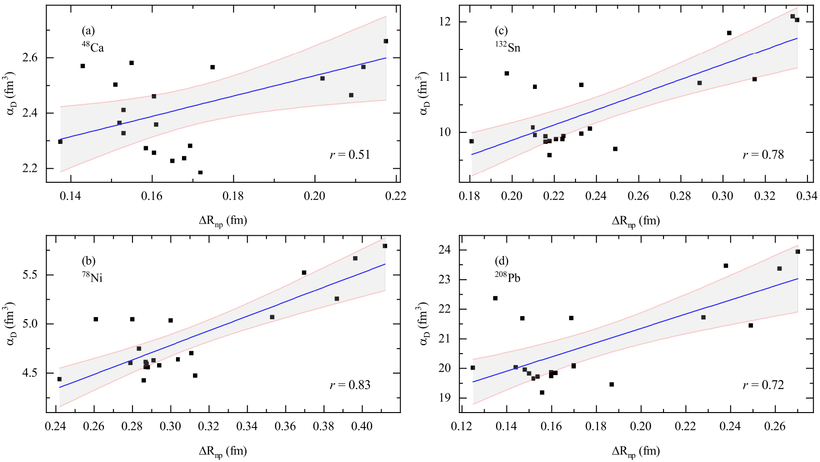
where $\bar{a}$, $\bar{b}$ represent the mean values and σ_a, σ_b the root mean square of the sample. The coefficient r always has a value between -1 and $+1$. Correlations equal to -1 or $+1$ correspond to data points lying exactly on a line. In fact, we may transform a_k to $\tilde{a}_k$, where

$$\tilde{a}_k = r\frac{\sigma_a}{\sigma_b}(b_k - \bar{b}) + \bar{a}\,.$$

For any value of the coefficient r, the value of $\tilde{a}_k$ is the best linear approximation for a_k.

N. Draper and H. Smith, **Applied Regression Analysis (Wiley, New York, 1998).**

Correlations: α_D vs ΔR_{np}



***N. N. Arsenyev and A. P. Severyukhin*, Moscow Univ. Phys. Bull. 79, 200 (2024).**

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Nuclear symmetry energy J

One of the key properties of nuclear matter is the symmetry energy (J), which is particularly important in modeling nuclear matter and finite nuclei because it probes the isospin part of the Skyrme interaction. It is defined as

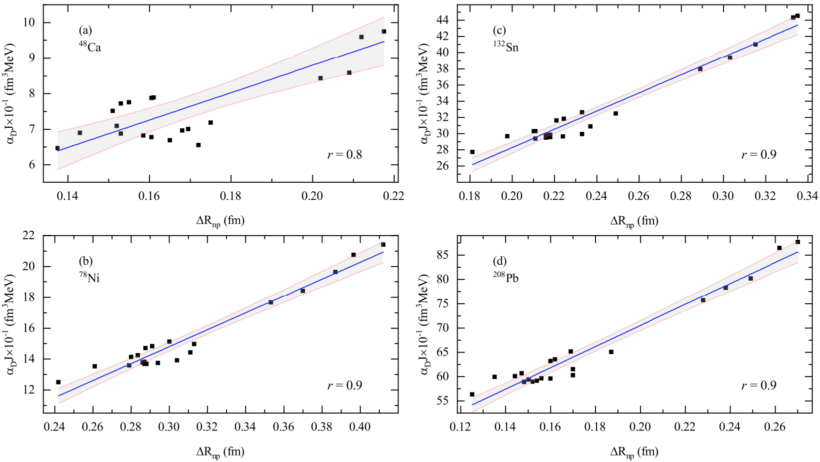
$$J(\rho) = \frac{\hbar^2}{6m}\left(\frac{3\pi^2}{2}\right)^{2/3}\rho^{2/3} - \frac{1}{8}t_0(2x_0+1)\rho - \frac{1}{24}\left(\frac{3\pi^2}{2}\right)^{2/3}\rho^{2/3}\left[3t_1x_1 - t_2(4+5x_2)\right] - \frac{1}{48}t_3(2x_3+1)\rho^{\alpha+1}.$$

Saturation properties of a few Skyrme parametrizations used in this work

	SLy4	SAMi	SIII	SGII	SkM*	SkP	SkI2	SkT5	SK255	LNS
J	32.00	28.00	28.16	26.83	30.03	30.00	33.37	37.00	37.40	33.43
ρ_{sat}	0.160	0.159	0.145	0.158	0.160	0.163	0.158	0.164	0.157	0.175

M. Dutra et al., Phys. Rev. C85, 035201 (2012).

Correlations: α_D vs ΔR_{np} and J



N. N. Arsenyev and A. P. Severyukhin, **Moscow Univ. Phys. Bull. 79, 200 (2024).**

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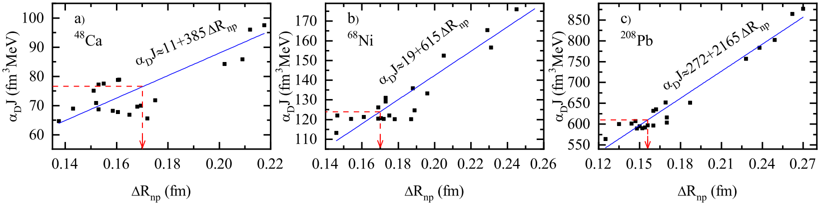
Estimation of the symmetry energy J

We carried out a theoretical analysis of the recently measured α_D and ΔR_{np} in ^{48}Ca, ^{68}Ni, and ^{208}Pb to extract information about the symmetry energy, by using a strong correlation between $\alpha_D J$ and ΔR_{np}. Combining the experimental data and the RPA theory constraints yields the interval of $J = 30 - 37$ MeV.



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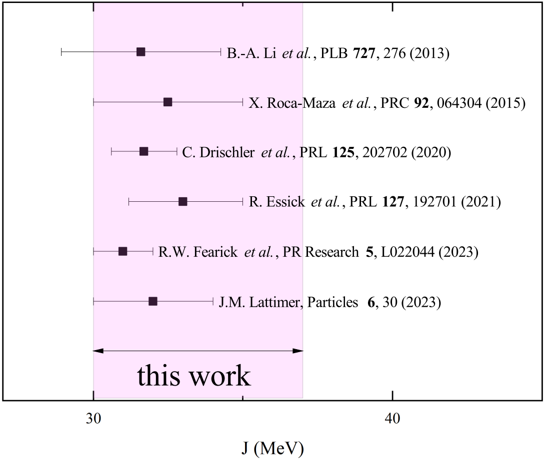
A. Tamii et al., **PRL 107, 062502 (2011).**

D. M. Rossi et al., **PRL 111, 242503 (2013).**

J. Birkhan et al., **Phys. Rev. Lett. 118, 252501 (2017).**

Constraints of the symmetry energy J



N. N. Arsenyev and A. P. Severyukhin, **Moscow Univ. Phys. Bull. 79, 200 (2024).**

Conclusions

We have computed the nuclear dipole polarizability (α_{D}) and neutron skin thickness (ΔR_{np}) of the magic nuclei using a broad set of Skyrme functionals. It is shown that the neutron skin thickness is correlated the product of the electric dipole polarizability and the symmetry energy (J) at saturation density.

We carried out a theoretical analysis of the recently measured α_{D} and ΔR_{np} in ^{48}Ca, ^{68}Ni, and ^{208}Pb to extract information about the symmetry energy, by using a strong correlation between α_{D} and ΔR_{np}. Combining the experimental data and the RPA theory (using a broad set of Skyrme functionals) constraints yields the interval of J=30−37 MeV.

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THE END