Longitudinal β-decay vector coupling of nucleons and nuclear isospin doublets

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β-decay vertex and nucleon form factors

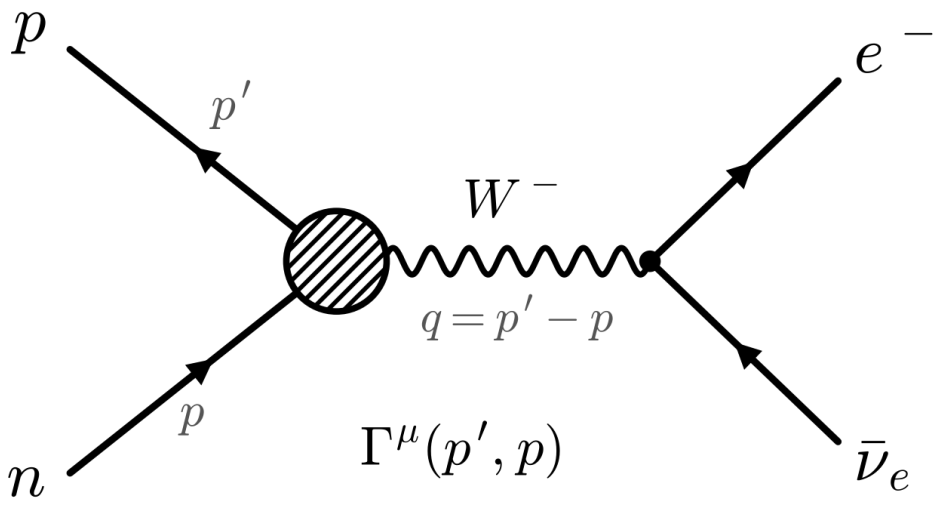

Fig. 1. Neutron β-decay; the vertex $\Gamma^\mu(p',p)$ carries the nucleon structure

$$\langle p|V^\mu|n\rangle = \bar{u}(p')\left[\gamma^\mu g_V + \frac{i\sigma^{\mu\nu}q_\nu}{2m}g_M + \frac{q^\mu}{m}\,g_S\right]u(p)$$

$$\langle p|A^\mu|n\rangle = \bar{u}(p')\left[\gamma^\mu\gamma_5 g_A + \frac{i\sigma^{\mu\nu}q_\nu}{2m}\gamma_5 g_T + \frac{q^\mu}{m}\gamma_5\, g_P\right]u(p)$$

First class: g_V, g_M (vector); g_A, g_P (axial)

Second class (Weinberg, G-odd): $\boldsymbol{g_S}$ (vector, q^μ term) and $\boldsymbol{g_T}$ (axial, tensor)

- **Determining $\boldsymbol{g_S}$ is a nontrivial problem**

Exact isospin symmetry: $g_S = g_T = 0$. Isospin breaking $m_u \neq m_d$: $\boldsymbol{g_S \propto \Delta m}$ — small but nonzero

Ward identity with broken isospin symmetry

$$\partial^\mu j^\alpha_\mu = 2\epsilon^{\alpha\beta\gamma} m^\beta j^\gamma, \qquad \mathbf{m} = \bar{m} + \tfrac{1}{2}\xi^\alpha \tau^\alpha$$

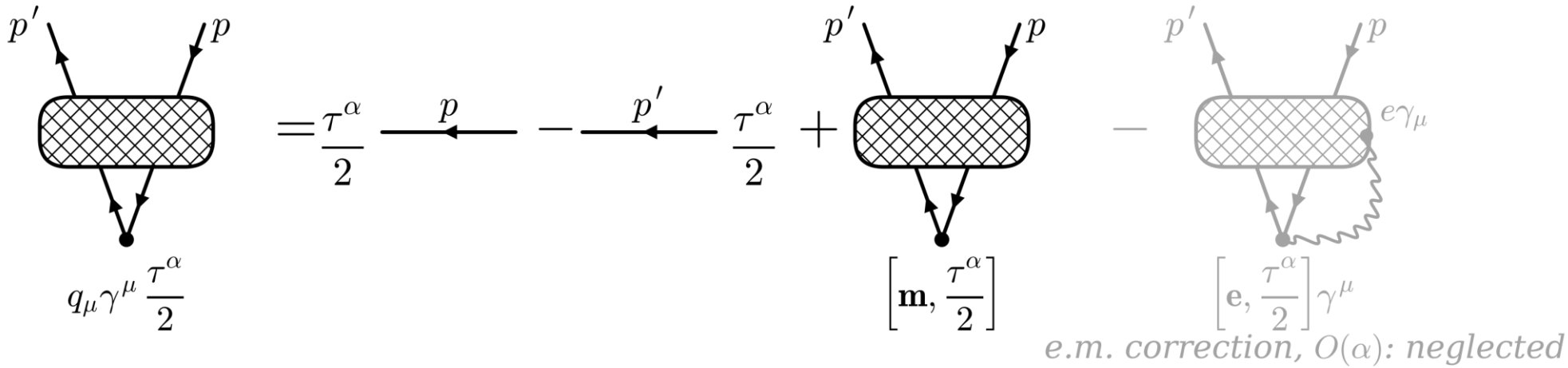

Fig. 2. Blobs: truncated four-fermion Green function (σ, ω exchange); lines: full propagators. Grey: e.m. term, $O(\alpha)$

$$q_\mu \Gamma^{\mu\alpha}(p', p) = S^{-1}(p')\tfrac{1}{2}\tau^\alpha - \tfrac{1}{2}\tau^\alpha S^{-1}(p) - 2i\epsilon^{\alpha\beta\gamma} m^\beta \Gamma^\gamma(p', p)$$

- ξ^α — isospin breaking in the mass matrix: $\xi^\alpha = \delta^{\alpha 3}(m_p - m_n)$
- $[\mathbf{m}, \tau^\alpha] \neq 0 \Rightarrow$ **the current is not conserved**: the scalar vertex Γ^γ appears

Off-shell structure of the vertex functions

Isospin decomposition:

$$\Gamma^{\mu\alpha} = \tfrac{1}{2}\tau^{\alpha}\,\Gamma^{\mu}_{[0]} + [\mathbf{m}, \tfrac{1}{2}\tau^{\alpha}]\,\Gamma^{\mu}_{[1]} + \ldots, \qquad \Gamma^{\alpha} = \tfrac{1}{2}\tau^{\alpha}\,\Gamma_{[0]} + \ldots$$

Off shell: a set of form factors $F_i^{\varkappa'\varkappa}(q^2, p'^2, p^2)$

$$\Lambda^{\varkappa}(p) = \frac{\varkappa\not{p} + \sqrt{p^2}}{2\sqrt{p^2}}, \qquad \varkappa = \pm 1$$

$$\Gamma^{\mu}_{[0]} = \sum_{\varkappa'\varkappa} \Lambda^{\varkappa'}(p') \left(\gamma^{\mu} F_1^{\varkappa'\varkappa} + \frac{i\sigma^{\mu\nu}q_{\nu}}{2\bar{m}} F_2^{\varkappa'\varkappa} + q^{\mu}(p'^2 - p^2) F_3^{\varkappa'\varkappa} \right) \Lambda^{\varkappa}(p)$$

$$\Gamma^{\mu}_{[1]} = \sum_{\varkappa'\varkappa} \Lambda^{\varkappa'}(p') \left(\gamma^{\mu}(p'^2 - p^2) F_4^{\varkappa'\varkappa} + \frac{i\sigma^{\mu\nu}q_{\nu}}{2\bar{m}}(p'^2 - p^2) F_5^{\varkappa'\varkappa} + q^{\mu} F_6^{\varkappa'\varkappa} \right) \Lambda^{\varkappa}(p)$$

$$\Gamma_{[0]} = \sum_{\varkappa'\varkappa} \Lambda^{\varkappa'}(p')\, F_0^{\varkappa'\varkappa}\, \Lambda^{\varkappa}(p), \qquad F_i^{\varkappa'\varkappa} = F_i^{\varkappa'\varkappa}(q^2, p'^2, p^2)$$

$\Lambda^{\varkappa}(p)$: projectors onto positive ($\varkappa = +1$) and negative ($\varkappa = -1$) energy states

All F_i are **isovector** form factors ⇒ isovector radii, e.g. $\langle r^2\rangle = \langle r^2\rangle_p - \langle r^2\rangle_n$

Constraints on the form factors

Dirac hermiticity + G-parity (with isospin breaking) give

$$F_i^{\varkappa'\varkappa}(q^2, p'^2, p^2) = F_i^{\varkappa\varkappa'}(q^2, p^2, p'^2) \in \mathbb{R} \quad (q^2 < 0)$$

Consequences

- Antisymmetric structures only with the factor $(p'^2 - p^2) \to F_3, F_4, F_5$
- $\Gamma^\mu_{[0]}$: first-class current; $\Gamma^\mu_{[1]}$: second-class current, $\propto [\mathbf{m}, \tau^\alpha]$

> On shell, the longitudinal part comes **only from $q^\mu F_6$ in $\Gamma^\mu_{[1]} \Rightarrow \propto \Delta m$**

On-shell Ward identity and g_S

On shell ($\varkappa' = \varkappa = +$):

$$F_1(q^2) + 2\bar{m}\, q^2 F_3(q^2) = 1, \qquad F_0(q^2) - q^2 F_6(q^2) = 1$$

Longitudinal part of the vector vertex:

$$\Gamma^{\mu\alpha}_{\parallel} = [\mathbf{m}, \tfrac{1}{2}\tau^{\alpha}]\, q^{\mu}\left(2\bar{m}F_3 + F_6\right), \qquad 2\bar{m}F_3 + F_6 = F_0'(0) - F_1'(0) + O(q^2)$$

$$\boxed{\frac{g_S}{M} = \Delta m \left(F_0'(0) - F_1'(0)\right)}$$

$$\frac{g_S}{M} = \frac{\Delta m}{6}\left(\langle r_s^2\rangle - \langle r^2\rangle\right)$$

$$\Delta m = m_{I3\,=\,+1/2} - m_{I3\,=\,-1/2}$$

$6F_1'(0)$, $6F_0'(0)$ — isovector vector and scalar mean-square radii $\Rightarrow$ **g_S is fixed by their difference**

Relativistic corrections to isovector densities

Nonrelativistic wave functions + corrections to $O(1/c^2)$ (Foldy–Wouthuysen)

Vector density:

$$\rho^\alpha(\mathbf{x}) = \left(F_1(0) + \tfrac{1}{6}R^2\Delta\right)\psi^\dagger \tfrac{1}{2}\tau^\alpha\psi, \qquad \tfrac{1}{6}R^2 = F_1'(0) + \frac{F_2(0)}{4M^2} + \frac{F_1(0)}{8M^2}$$

Scalar density:

$$\rho_s^\alpha(\mathbf{x}) = Q_s\left(F_0(0) + \tfrac{1}{6}R_s^2\Delta\right)\psi^\dagger \tfrac{1}{2}\tau^\alpha \frac{M}{E}\psi, \qquad \tfrac{1}{6}R_s^2 = F_0'(0)$$

- **Vector:** Darwin term → $+3/(4M^2)$ in the radius
- **Scalar:** no Darwin term; M/E suppresses the scalar charge
- **Lorentz contraction:** only $O(q^4)$ — radii unchanged

Isovector charges and radii of composite systems

Density of a composite system = sum over constituents (quarks or nucleons)

Vector: charge Q and radius

sum over constituents →

$$\langle I_3'|\sum_k \tfrac{1}{2}\tau_k^\alpha|I_3\rangle = \langle I_3'|\tfrac{1}{2}\tau^\alpha|I_3\rangle$$

$$\langle I_3'|\sum_k \tfrac{1}{2}\tau_k^\alpha\left(r_k^2+\mathcal{R}_k^2\right)|I_3\rangle = \langle I_3'|\tfrac{1}{2}\tau^\alpha|I_3\rangle\, R^2$$

← *system as a whole*

Scalar: charge Q_s and radius

sum over constituents →

$$\langle I_3'|\sum_k \tfrac{1}{2}\tau_k^\alpha \frac{m_k}{E_k}\mathcal{Q}_{sk}|I_3\rangle = \langle I_3'|\tfrac{1}{2}\tau^\alpha|I_3\rangle\, Q_s$$

$$\langle I_3'|\sum_k \tfrac{1}{2}\tau_k^\alpha \frac{m_k}{E_k}\mathcal{Q}_{sk}\left(r_k^2+\mathcal{R}_{sk}^2\right)|I_3\rangle = \langle I_3'|\tfrac{1}{2}\tau^\alpha|I_3\rangle\, Q_s R_s^2$$

← *system as a whole*

Link to experiment:

$$R^2 = \langle r_E^2\rangle_\uparrow - \langle r_E^2\rangle_\downarrow + \frac{3}{4M^2}$$

- Q_s<1: suppressed by constituent motion (m_k/E_k)
- The nucleon requires **a quark model**

The MIT bag model

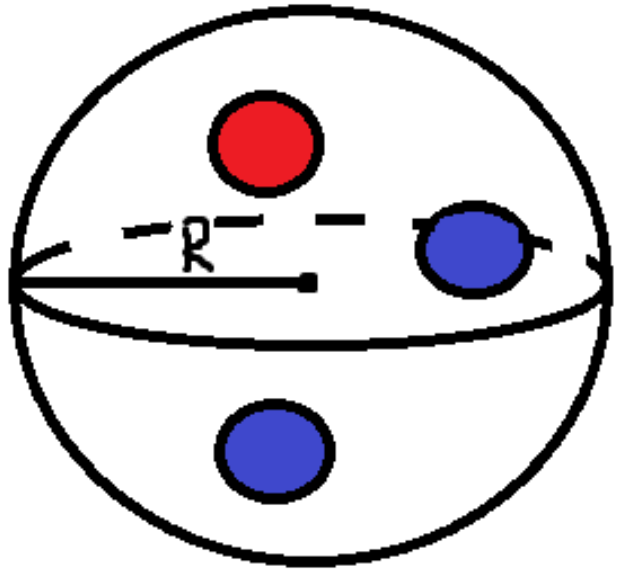

Fig. 3. Three quarks in a bag of radius R

- Massless quarks, Dirac equation
- Boundary condition $i \not{n} \psi = \psi$

Results

$$Q_s = 0.479, \qquad \langle r_E^2 \rangle = 0.531\, R^2, \qquad Q_s \langle r_s^2 \rangle = 0.182\, R^2$$

$$\rho = \frac{\langle r_s^2 \rangle}{\langle r_E^2 \rangle} = 0.716, \qquad \langle r^2 \rangle = 0.577\ \mathrm{fm}^2, \qquad \langle r_s^2 \rangle \simeq 0.589\ \mathrm{fm}^2$$

- $Q_s \approx 0.48$: maximal relativistic effect (massless quarks)
- R fixed by $\langle r_E^2 \rangle_p - \langle r_E^2 \rangle_n$ (PDG)
- Moving system: $Q'_s = Q_s/\gamma$

Massless quarks give an **upper bound** on relativistic effects in the scalar charge

Nonrelativistic oscillator quark model

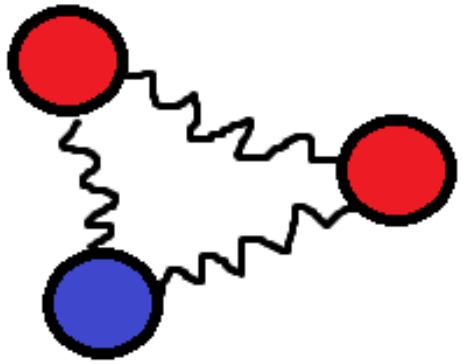

Fig. 4. Quarks in an oscillator potential

- Constituent quarks, $m = m_N/3$
- Harmonic oscillator
- Schrödinger eq. $+\,O(1/c^2)$

Ground-state Wigner function:

$$W_{\mathbf{P}}(\mathbf{p}_i,\mathbf{q}_i) = (2\pi)^3\delta^3\Big(\mathbf{P}-\sum_i \mathbf{p}_i\Big)\,2^6\exp\Big(-m\omega\sum_{i<j}(\mathbf{q}_i-\mathbf{q}_j)^2-\frac{1}{9m\omega}\sum_{i<j}(\mathbf{p}_i-\mathbf{p}_j)^2\Big)$$

Main results:

$$Q_s = \Big\langle\frac{m}{E}\Big\rangle = \sqrt{\frac{8z^3}{\pi}}\,e^z\big(K_1(z)-K_0(z)\big) = \sqrt{2z}\,U\big(\tfrac{1}{2},0,2z\big), \qquad z=\frac{m}{4\omega}$$

$$\langle r^2\rangle = \frac{1}{3m\omega}, \qquad \langle r_s^2\rangle = \langle r^2\rangle$$

$$\frac{1}{3m\omega}+\frac{3}{4m^2} = \langle r_E^2\rangle_p - \langle r_E^2\rangle_n + \frac{3}{4m_N^2}, \qquad m=\frac{m_N}{3}$$

$$\omega = 0.0745\ \text{GeV}, \quad Q_s = 0.791, \quad \langle r_s^2\rangle = 0.557\ \text{fm}^2, \quad \langle r_s^2\rangle/\langle r_E^2\rangle = 0.677$$

Isospin doublet ^{3}H – ^{3}He

Two levels: **quarks in nucleons + nucleons in the nucleus;** $\Psi = \Psi_{\text{nucl}} \prod_i \psi_{N_i} \Rightarrow Q_s^{\text{nucl}} = Q_s\, Q_{sN}$

Method

- Oscillator model applied to nucleons: $m \to m_N, \omega \to \omega_N$
- ω_N from the ^{3}He, ^{3}H charge radii
- Nucleon Q_s, $\langle r_s^2 \rangle$: MIT and NRQM

Results

$\omega_N = 3.67$ MeV, $\quad Q_{sN} = 0.994$

$Q_{sN} \langle r^2_{s,\text{nuc}} \rangle = 3.763$ fm^2

$g_s^{\text{bag}} = 1.52 \times 10^{-4}, \quad g_s^{\text{osc}} = 1.70 \times 10^{-4}$

$g_s^{\text{exp}} = -0.005 \pm 0.040$ [1]

Both estimates agree with experiment

[1] D. Gazit, Phys. Lett. B 666, 472 (2008), doi:10.1016/j.physletb.2008.08.008

Nuclear isodoublets within DRHBc

Deformed relativistic Hartree–Bogoliubov theory in continuum (DRHBc) [2]

- **Fully relativistic:** nucleons are Dirac spinors in scalar (σ) and vector (ω, ρ) meson mean fields
- Upper and lower spinor components ⇒ **vector and scalar densities differ** in the nuclear interior
- Pairing: Hartree–Bogoliubov; axial quadrupole deformation
- Continuum and weakly bound states; Dirac–Woods–Saxon basis
- Output: self-consistent vector ρ_p, ρ_n and scalar ρ_{sp}, ρ_{sn} densities; isovector charges and radii from the constituent sums

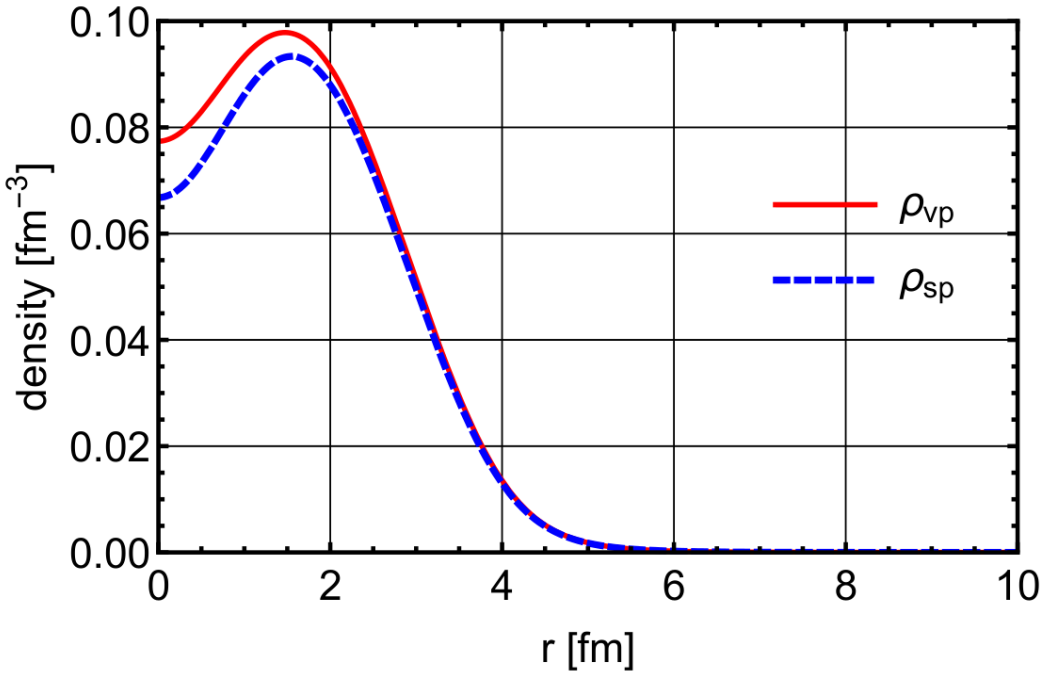

Fig. 5. Vector and scalar proton densities of ^{29}P

[2] P. Guo et al. (DRHBc Mass Table Collab.), At. Data Nucl. Data Tables 158, 101661 (2024)

g_s for nucleons and nuclear isodoublets

$I_3 = -1/2$	$I_3 = +1/2$	Δm [MeV]	$g_{s\,\mathrm{bag}}$ (2015)	$g_{s\,\mathrm{bag}}$	$g_{s\,\mathrm{osc}}$	$g_{s\,\mathrm{exp}}$
${}^{1}_{0}\mathrm{n}$	${}^{1}_{1}\mathrm{p}$	-1.293	1.06×10^{-4}	-7.04×10^{-6}	1.15×10^{-5}	–
${}^{3}_{1}\mathrm{H}$	${}^{3}_{2}\mathrm{He}$	-1.274	4.35×10^{-5}	1.52×10^{-4}	1.70×10^{-4}	-0.005 ± 0.040
${}^{13}_{6}\mathrm{C}$	${}^{13}_{7}\mathrm{N}$	1.710	-1.82×10^{-4}	3.39×10^{-4}	3.14×10^{-4}	–
${}^{15}_{7}\mathrm{N}$	${}^{15}_{8}\mathrm{O}$	2.244	-2.26×10^{-4}	2.92×10^{-4}	2.60×10^{-4}	–
${}^{19}_{9}\mathrm{F}$	${}^{19}_{10}\mathrm{Ne}$	2.730	-2.66×10^{-4}	9.37×10^{-6}	-2.98×10^{-5}	–
${}^{29}_{14}\mathrm{Si}$	${}^{29}_{15}\mathrm{P}$	4.432	-4.06×10^{-4}	7.33×10^{-4}	6.69×10^{-4}	–
${}^{31}_{15}\mathrm{P}$	${}^{31}_{16}\mathrm{S}$	4.889	-4.43×10^{-4}	7.32×10^{-4}	6.62×10^{-4}	–
${}^{33}_{15}\mathrm{P}$	${}^{33}_{16}\mathrm{S}$	-0.758	6.70×10^{-6}	-1.16×10^{-4}	-1.05×10^{-4}	–
${}^{69}_{30}\mathrm{Zn}$	${}^{69}_{31}\mathrm{Ga}$	-1.416	9.03×10^{-5}	-6.25×10^{-4}	-6.04×10^{-4}	–

Table 1. g_s normalized to m_μ (as in exp. [1]); bag (2015) [3]: geometric nuclear radius; bag, osc: MIT / NRQM nucleon + DRHBc nucleus (oscillator for ^{3}H–^{3}He)

- $|g_s| \sim 10^{-5}$–10^{-3}, scale set by Δm
- **^{3}H – ^{3}He: consistent with experiment**
- Nuclei: bag ≈ osc ⇒ nuclear structure dominates
- Nucleon: model-dependent, even the sign

[3] F. Šimkovic, S. Kovalenko, M.I. Krivoruchenko, AIP Conf. Proc. 1686 (2015); arXiv:1507.08823

Conclusion

Main results

- The **generalized Ward identity** was derived for the isovector vector current with explicitly broken isospin symmetry
- Dirac hermiticity and G-parity fix the off-shell structure of the vertex functions: symmetric, real form factors; second-class terms only $\propto[\mathbf{m},\tau^{\alpha}]$
- **g_S is expressed through the difference of isovector scalar and vector mean-square radii** and is proportional to the mass splitting in the doublet
- Relativistic corrections to the isovector densities to $O(1/c^2)$: Darwin and Foldy terms for the vector density, kinematic factor m/E for the scalar one
- Nucleon scalar charge and radius from the MIT bag and NRQM; nuclear contribution from self-consistent DRHBc densities. **^{3}H–^{3}He estimates agree with experiment**

Outlook

- **Exchange currents** (meson-exchange currents) in the scalar form factors of nucleons and nuclei
- Other sources of isospin symmetry breaking: meson coupling constants and mass differences
- Constraints on extensions of the Standard Model predicting enhanced g_s
- Deformed nuclei
- Theoretical support for experiments measuring g_s

Thank You for your attention!