e^+e^- data: no anomaly in the muon $g-2$?

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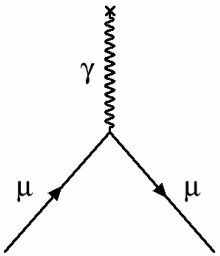
The 8th Int. Conf. on Particle Physics and Astrophysics (ICPPA-2026)
27-30 October 2026, MEPhI, Moscow

Introduction

$$\vec{\mu}_\mu = -g_\mu \frac{e}{2m_\mu}\vec{S}$$

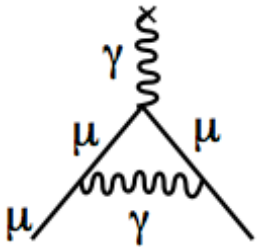
- $a_\mu = (g_\mu - 2)/2$ measured by FNAL Muon g-2 experiment to 124 ppb
- Long standing $\sim$ 4-5σ theory/experiment tension *(resolved by lattice QCD?)*
- $O(100)$ ppb precision SM test, sensitive to multi-TeV scale New Physics
  - Theory uncertainty mostly due to QCD

Tree level



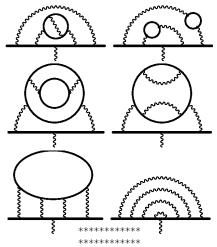
$a_\mu = 0$

Schwinger



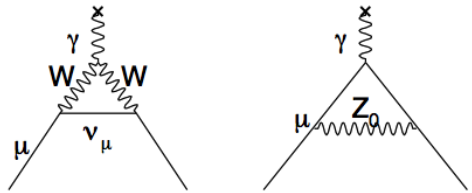
$+\frac{\alpha}{2\pi} = 11614097.3 \times 10^{-10}$

QED 2-5 loops



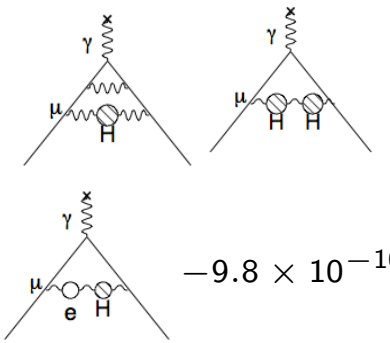
$+44374.6 \times 10^{-10}$

Electroweak



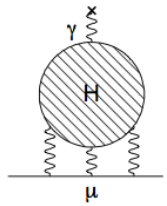
$+15.4 \times 10^{-10}$

Hadron VP NLO



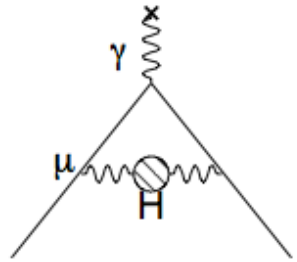
-9.8×10^{-10}

Hadron light-by-light



$+9.2 \times 10^{-10}$

LO hadron VP $\longleftarrow$ *This talk*



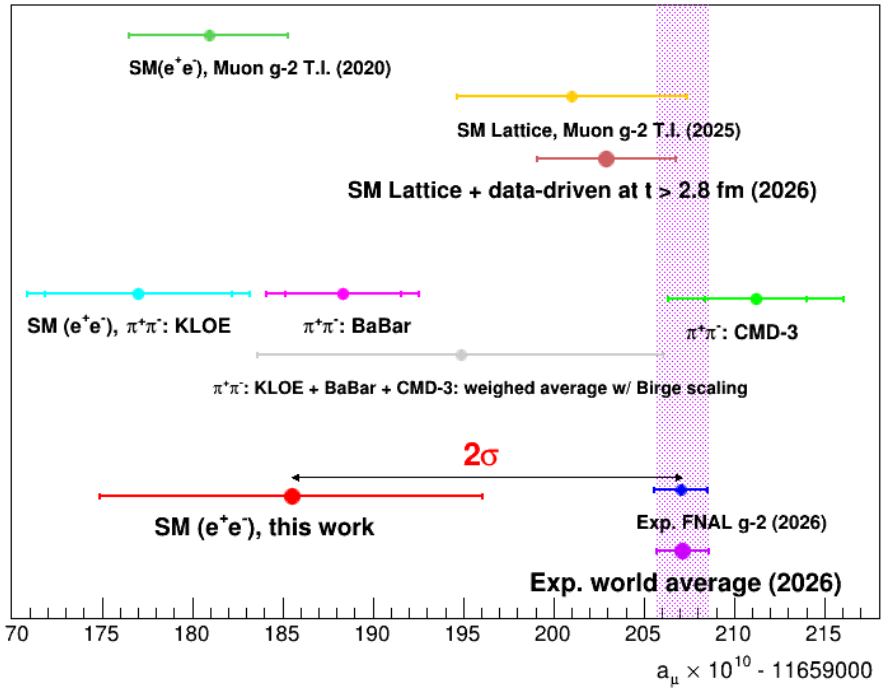
$(\simeq 700 \pm ?) \times 10^{-10}$

Typical momentum running in the loop $|Q| \sim m_\mu < 1$ GeV

- Can't use pQCD.
- Lattice or **dispersive** HVP calculations only.

$g_\mu - 2$: experiment vs theory

- The experimental world average dominated by FNAL Muon g-2, Run 1-6 [*arXiv:2606.17323*]: $a_\mu = 11659207.15(1.45) \times 10^{-10}$ **(124 ppb)**
- SM a_μ uncertainty mostly from $a_\mu^{\mathrm{LO-HVP}}$
- $\sim$ 2020:
  - Lattice QCD: too large uncertainty $\sigma(a_\mu^{\mathrm{LO-HVP}}) \sim 20 \times 10^{-10}$
  - Dispersive HVP calculation using e^+e^- data: $\sigma(a_\mu^{\mathrm{LO-HVP}}) \sim 5 \times 10^{-10}$. ▶ The anomaly: $4 - 5\sigma$ tension with the experiment.
    [*Phys. Rept. 887 (2020) 1*]
- Progress by 2026:
  - Lattice uncertainty improved by factor of 10. Hybrid $a_\mu^{\mathrm{LO-HVP}}$ (lattice + data-driven long range tail): $\sigma(a_\mu^{\mathrm{LO-HVP}}) = 3.4 \times 10^{-10}$, good agreement with the experiment.
    [*Nature 653 (2026) 8114, 373*]
  - Dispersive $a_\mu^{\mathrm{LO-HVP}}$: $\sim 3 - 5\sigma$ discrepancy between estimates using CMD-3 (2024), BaBar (2012) and KLOE (2018) $\sigma_{tot}(e^+e^- \to \pi^+\pi^-)$ data at $\sqrt{s} < 1$ GeV comprising $\simeq 70\%$ of $a_\mu^{\mathrm{LO-HVP}}$.



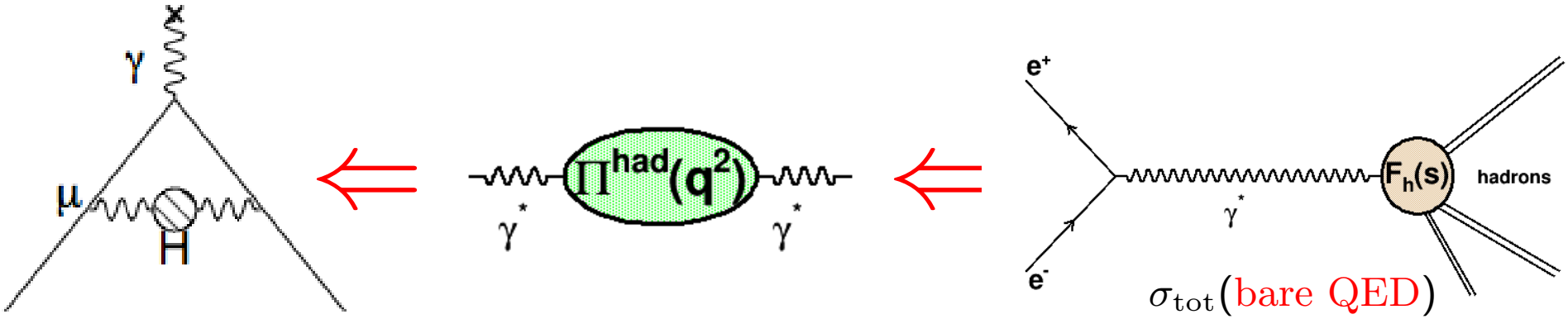
▶ Lattice QCD HVP accuracy has greatly improved. No $g_\mu - 2$ anomaly?

▶ The dispersive $a_\mu^{\mathrm{LO-HVP}}$ using only CMD-3 $\pi^+\pi^-$ data at $\sqrt{s} < 1$ GeV is compatible with the experiment and lattice QCD, contrary to BaBar and KLOE.

▶ The dispersive e^+e^- estimate of $a_\mu^{\mathrm{LO-HVP}}$ is the only independent cross-check of lattice QCD.

$\Rightarrow$ **Try to consistently treat incompatible** $\sigma_{tot}(e^+e^- \to hadrons)$ **inputs.**

Dispersive $a_\mu^{\mathrm{LO-HVP}}$ from $\sigma_{tot}(e^+e^- \to \text{hadrons})$



The dispersion relation [*A. Petermann, Phys. Rev. 105 (1957) 1931*]:

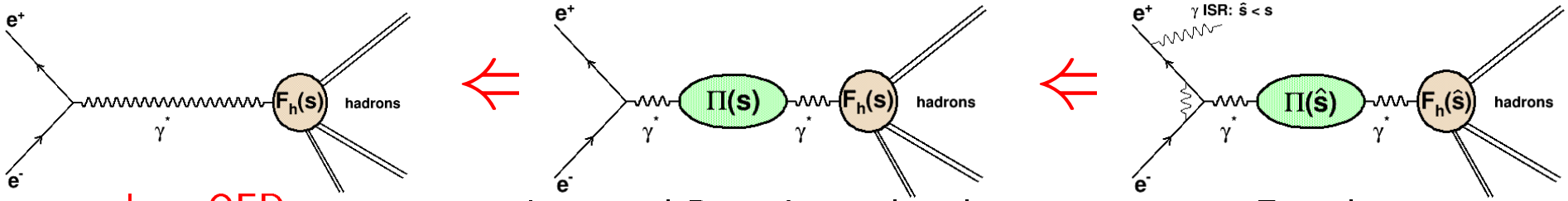
$$a_\mu^{\mathrm{LO-HVP}} = 4\alpha_0^2 \int_{m_\pi^2}^{\infty} \frac{ds}{s} K(s) \frac{1}{\pi} \operatorname{Im} \Pi^{\mathrm{had}}(s) = \frac{\alpha_0^2}{3\pi^2} \int_{m_\pi^2}^{\infty} \frac{ds}{s} K(s) R^{\mathrm{had}}(s)$$

$$R^{\mathrm{had}}(s) = \sigma_{\mathrm{tot}}(e^+e^- \to \textit{hadrons}, \text{bare } e^+e^- \text{ vtx. and } \gamma^*) \Big/ \frac{4\pi\alpha_0^2}{3s}$$

$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)(s/m_\mu^2)} \ .$$

$a_\mu^{\mathrm{LO-HVP}}$ from $\sigma_{tot}(e^+e^- \to \text{hadrons})$

[*Details of the procedure:* *Phys. Part. Nucl. 55 (2024) 6, 1432* *& our previous talk*]

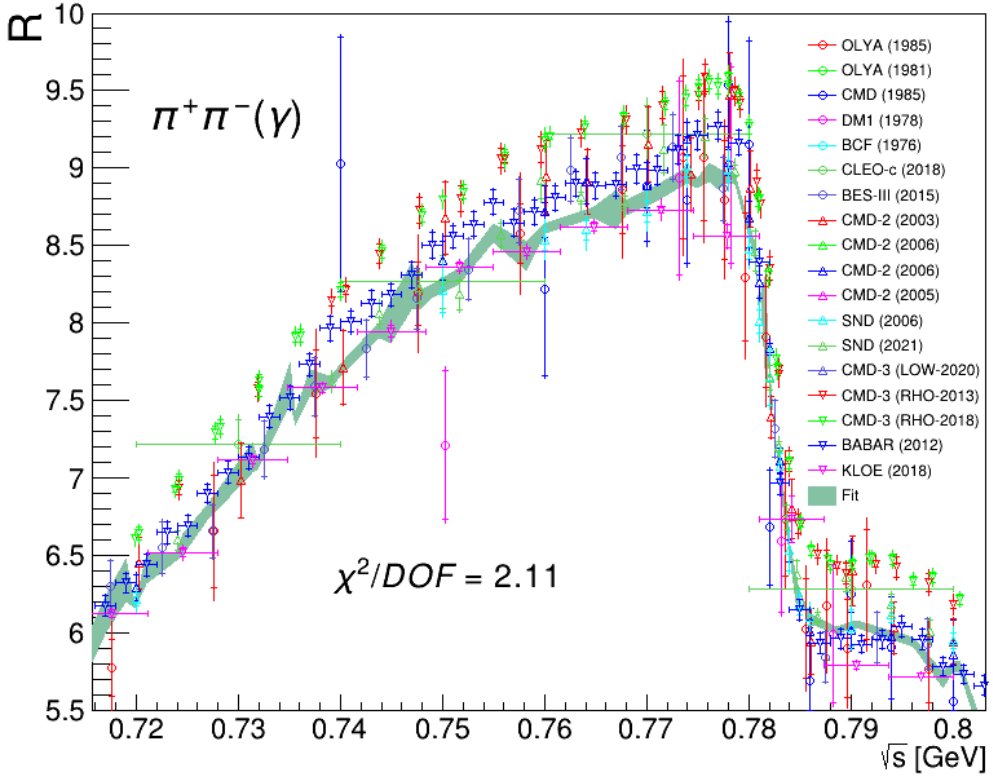


bare QED ⇐ Improved Born Approximation ⇐ Experiment

- We need first to uniformly rescale all published measurements to bare QED cross section.
- All experiments publish cross sections corrected for ISR + e^+e^- vertex loops. The $\sigma_{\mathrm{tot}}(e^+e^- \to \textit{hadrons})$ compilation is taken from our CrossSection database.
- Some experiments correct for photon VP, others leave the VP correction to readers.
- FSR correction. Additional hard γ's are rejected in the event selection to suppress backgrounds from other final states. Experimentalists then 'undress' the cross section, i.e. correct it for soft FSR using certain FSR model. Need to add the FSR back: $\sigma(\mathrm{hadrons}(+\gamma'\mathrm{s})) = \sigma(\mathrm{hadrons})C_{\mathrm{FSR}} = \sigma(\mathrm{hadrons})\left[1+\eta(s)\frac{\alpha}{\pi}\right]$, with $\eta(s)$ from scalar QED for $\pi^\pm$, $K^\pm$. The approximation: $C_{\mathrm{FSR}} = (1+0.004\pm0.004)^{N_{\mathrm{charged}}}$.
- $\sigma_{\mathrm{tot}}(e^+e^- \to \textit{hadrons})$ is measured mostly inclusively at $\sqrt{s} > 2$ GeV and for (semi)exclusive final states at $\sqrt{s} < 2$ GeV. Most final states are measured by multiple experiments.
- Parameterize bare QED cross section in each final state by continuous piecewise linear function w/ positions of the nodes in $\sqrt{s}$ given by clusterization of experimental points.
- Perform a standard χ^2 fit of the parameterized cross section taking into account correlated uncertainties within each experiment and between experiments *[details in backup]*. The fitted $\sigma_{\mathrm{tot}}(\mathrm{bare\ QED})$ can be plugged into the dispersion integral to find the contribution to photon VP and thus $a_\mu^{\mathrm{LO-HVP}}$ at $0.3 < \sqrt{s} < 11.2$ GeV. Outside this range use analytic parameterizations of $\sigma_{\mathrm{tot}}(\mathrm{bare\ QED})$.

The problem: incompatible $\sigma(e^+e^- \to \text{hadrons})$ data

- Most dramatic tensions are seen → in the $\pi^+\pi^-$ channel comprising $\simeq 70\%$ of $a_\mu^{\text{LO-HVP}}$. CMD-3 (2024), KLOE (2018) and BaBar (2012) have $\sim 1\%$ systematic uncertainties but deviate at $\sim 3-5\sigma$ level
  - Preliminary SND data support CMD-3 and disagree with BaBar and KLOE
  - The preliminary BaBar result with 460/fb and a new analysis technique confirms their 2012 results
- At glance, tensions result in a poor fit → which makes dispersive estimates of $a_\mu^{\text{LO-HVP}}$ senseless until the origin of the discrepancies is pinned down.
  - Muon $g-2$ Theory Initiative drops dispersive $a_\mu^{\text{LO-HVP}}$ in the '2025 White Paper and quotes only the lattice QCD result.
  - Possible origins of BaBar/KLOE/CMD-3 tensions are widely discussed in the literature, still inconclusively.
- Still, what can we get from the e^+e^- data alone?



▶ The assumption: incompatible measurements are affected by some *uncontrolled systematics* which could not be evaluated in the experiments *in situ* but can be estimated by comparing different experiments. ⇒ **Infer missing systematics from the data and obtain a realistic uncertainty for the fitted cross section and hence for $a_\mu^{\text{LO-HVP}}$**

[*arXiv:2607.06352*]

Quantifying the tensions in $\pi^+\pi^-$

- Minimize the χ^2 with original covariance matrices parameterized for each experiment as
$$C = \mathrm{diag}(\sigma_{stat\,1}^2, \ldots, \sigma_{stat\,n}^2) + v_{sys} \otimes v_{sys}^T, \quad v_{sys} = (\sigma_{sys\,1}, \ldots, \sigma_{sys\,n})$$
- *Is χ^2 distributed uniformly over degrees of freedom?*
- For each experiment, find unit eigenvectors of its covariance matrix C, project the discrepancy $\Delta(s_i) = R_{exp}(s_i) - R_{fit}(s_i)$ onto the eigenvectors and find the contribution to χ^2 from each projection to identify direction of the tension.
  - Simplification: take $n_{sys} = v_{sys}/|v_{sys}|$ as the approximation for the first eigenvector, find the projection of the discrepancy $\Delta_i = R_{exp\,i} - R_{fit\,i}$ onto it: $\Delta_{sys} = n_{sys}(n_{sys}^T \Delta)$.
  - Compute $\Delta\chi^2_{sys} = \Delta_{sys}^T C^{-1} \Delta_{sys}$.
  - Then find the residual discrepancy $\Delta_{res} = \Delta - \Delta_{sys}$, and compute $\Delta\chi^2_{res} = \Delta_{res}^T C^{-1} \Delta_{res}$.

Exp.	Year	n_p	$\frac{\Delta\chi^2}{n_p}$	$\Delta\chi^2_{sys}$	$\frac{\Delta\chi^2_{res}}{n_p}$	Pull
CMD-3	2020	13	4.08	44.06	0.95	0.048
	2018	114	2.00	93.80	1.12	0.071
	2013	82	1.79	54.15	1.00	0.072
BaBar	2012	323	1.32	13.45	1.23	0.020
KLOE	2018	85	2.97	1.94	3.00	-0.009
BES-III	2015	60	0.86	0.29	0.85	0.002
CLEO-c	2018	35	0.67	1.36	0.63	0.016
CMD-2	2003	43	0.90	8.37	0.81	0.018
	2005	36	0.75	0.46	0.66	0.071
	2006	10	1.71	8.10	1.35	-0.052
	2006(ρ)	29	1.14	1.83	1.05	0.011
SND	2006	45	1.54	0.01	1.55	-0.001
	2021	36	2.15	2.73	2.05	0.013
CMD	1985	24	1.56	0.14	1.58	-0.007
OLYA	1985	79	0.97	0.00	0.97	0.004
DM1	1978	16	0.82	1.49	0.75	-0.042

For a good fit one expects $\Delta\chi^2 \sim 1$ for all degrees of freedom.
⟵ The tension: CMD-3, BaBar, CMD-2 have $\Delta\chi^2_{sys} \gg 1$ for projections onto "systematic" eigenvectors while $\Delta\chi^2_{res} \sim 1$ for the residual degrees of freedom. KLOE features more peculiar tension pattern. (Incorrect parameterization of the error matrix? The original one seems to be lost.) Statistical errors are not severely underestimated as the residuals give $\Delta\chi^2_{res} \sim 1$ per d.o.f.; $\Delta\chi^2_{sys} \gg 1$ for the "systematic" d.o.f. indicates that only systematic errors could be underestimated ⇒ modify only them.

Missing systematic uncertainties:

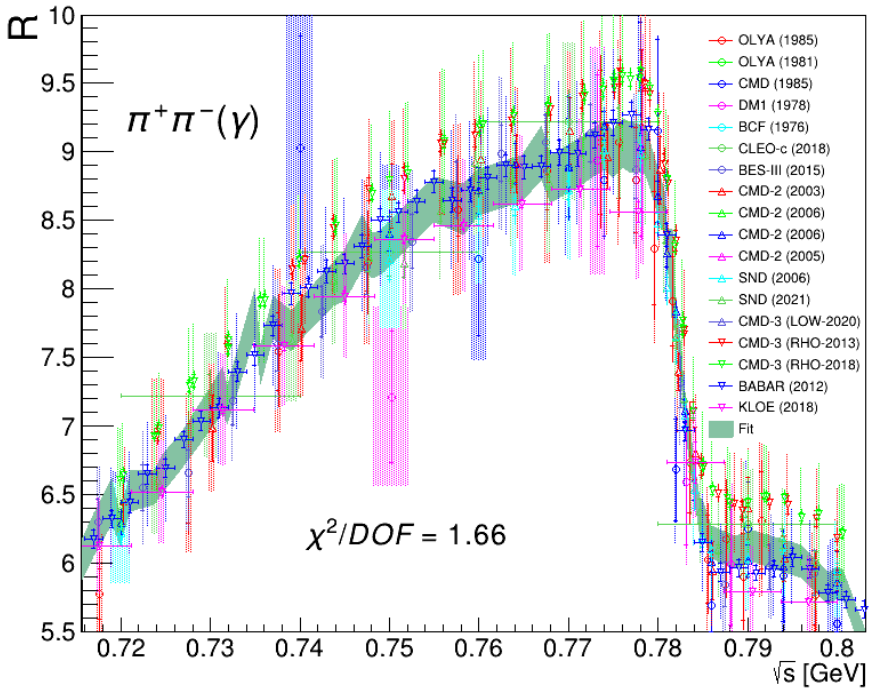
- Some heuristics: take experiments with $\Delta\chi^2_{sys} > 10$, compute their integral pulls relative to the fit,

$$\text{Pull} = \int (R_{exp}(s) - R_{fit}(s))\, ds \Big/ \int R_{fit}(s) ds\,,$$

and estimate the missing systematic uncertainty as $\epsilon = \sqrt{\langle \text{Pull}^2 \rangle}$. This extra uncertainty has to be assigned to *all* experiments (neglect possible correlations):

$$C_{mod} = \text{diag}(\sigma_{stat\,1}^2, \ldots, \sigma_{stat\,n}^2) + v_{sys} v_{sys}^T + \epsilon^2 R_{fit} R_{fit}^T$$

- Now repeat the fit with the modified error matrices $\Rightarrow$ $\Delta\chi^2_{sys}$ "peaks" are reduced as expected:



Exp.	Year	n_p	$\frac{\Delta\chi^2}{n_p}$	$\Delta\chi^2_{sys}$	$\frac{\Delta\chi^2_{res}}{n_p}$	Pull
CMD-3	2020	13	4.08	44.06	0.95	0.048
	2018	114	2.00	93.80	1.12	0.071
	2013	82	1.79	54.15	1.00	0.072
BaBar	2012	323	1.32	13.45	1.23	0.020
KLOE	2018	85	2.97	1.94	3.00	-0.009
BES-III	2015	60	0.86	0.29	0.85	0.002
CLEO-c	2018	35	0.67	1.36	0.63	0.016
CMD-2	2003	43	0.90	8.37	0.81	0.018
	2005	36	0.75	0.46	0.66	0.071
	2006	10	1.71	8.10	1.35	-0.052
	2006(ρ)	29	1.14	1.83	1.05	0.011
SND	2006	45	1.54	0.01	1.55	-0.001
	2021	36	2.15	2.73	2.05	0.013
CMD	1985	24	1.56	0.14	1.58	-0.007
OLYA	1985	79	0.97	0.00	0.97	0.004
DM1	1978	16	0.82	1.49	0.75	-0.042

$\epsilon = \sqrt{\langle \text{Pull}^2 \rangle} = 0.057$

Exp.	Year	n_p	$\frac{\Delta\chi^2}{n_p}$	$\Delta\chi^2_{sys}$	$\frac{\Delta\chi^2_{res}}{n_p}$	Pull
CMD-3	2020	13	1.11	1.00	0.96	0.052
	2018	114	1.11	5.36	1.01	0.058
	2013	82	1.14	3.20	0.98	0.059
BaBar	2012	323	1.19	0.05	1.18	0.007
KLOE	2018	85	2.14	1.39	2.27	-0.022
BES-III	2015	60	0.80	0.02	0.81	-0.011
CLEO-c	2018	35	0.66	0.02	0.65	0.003
CMD-2	2003	43	0.87	0.06	0.85	0.007
	2005	36	0.74	0.12	0.68	0.037
	2006	10	1.21	1.10	1.29	-0.056
	2006(ρ)	29	1.01	0.00	1.01	-0.001
SND	2006	45	1.37	0.08	1.42	-0.012
	2021	36	2.08	0.00	2.07	0.002
CMD	1985	24	1.56	0.00	1.56	-0.017
OLYA	1985	79	0.87	0.07	0.90	-0.009
DM1	1978	16	0.75	0.65	0.77	-0.053

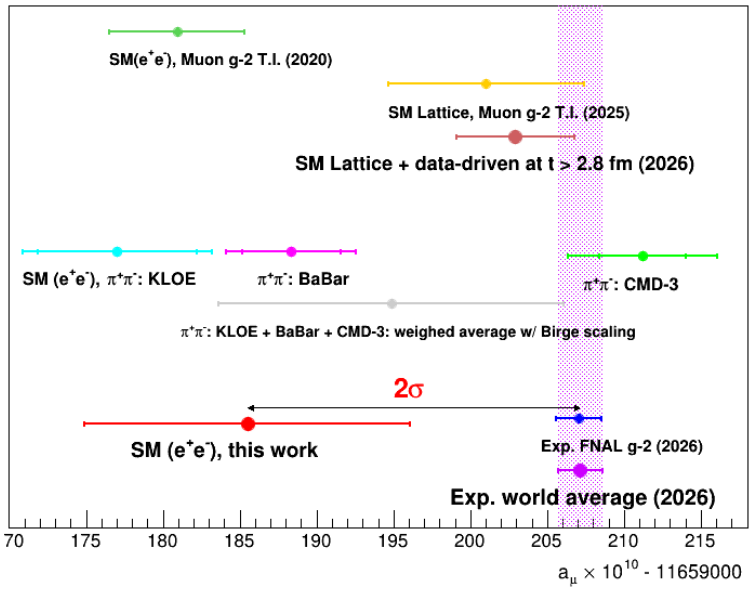
↑ Stop upon this iteration as all $\Delta\chi^2_{sys} < 10$

Missing $\sigma_{\rm tot}(e^+e^- \to \textit{hadrons})$ systematics and $a_\mu^{\rm LO-HVP}$

- The technique outlined on previous slides is used to estimate missing systematics in all channels affected by incompatible measurements of the $\sigma_{\rm tot}$: $\pi^+\pi^-(\gamma)$, $\pi^+\pi^-\pi^0$, K^+K^-, $\eta\gamma$, $\omega\pi$. The extra systematic uncertainty is then propagated to $a_\mu^{\rm LO-HVP}$. Residual tensions in the fits are accounted for by scaling the uncertainty of $R_{fit}^{\rm had}(s)$ by Birge factor $\sqrt{\chi^2/n_{\rm dof}}$, in case $P(\chi^2, n_{\rm dof}) < 0.05$. [*Open questions, todo's ...*]
- In the literature, uncertainties related to incompatible $\sigma_{\rm tot}(s)$ measurements are usually described in terms of the $a_\mu^{\rm LO-HVP}$ dispersive integral in the given s range using $\sigma_{\rm tot}(s)$ from individual (mutually incompatible) experiments: e.g., let's integrate over CMD-3 $\pi^+\pi^-(\gamma)$ data at $\sqrt{s} = 0.6$–0.9 GeV, then compare this to the integral over CMD-2, SND, BaBar, KLOE, *etc.*, estimate an extra uncertainty of $a_\mu^{\rm LO-HVP}$ from the discrepancies between the integrals.
- Our technique allows for a more detailed estimate of the extra uncertainty at a level of $\sigma_{\rm tot}(s)$ fits rather than dispersion integrals over individual $\sigma_{\rm tot}(s)$ measurements. The uncertainty of the extra uncertainty (`xsys`) is estimated by varying the threshold indicator of missing systematics in $6 < \Delta\chi^2_{thr} < 25$ range (the nominal indicator is $\Delta\chi^2_{sys} > 10$).

In fact, our technique involves simultaneous extremization of both χ^2 and its "entropy": to make χ^2 distribution over d.o.f. uniform, the fit is "warmed up" by extra systematics accounting for incompatibility between experiments.

Result: $a_\mu^{\mathrm{LO-HVP}} = (697.7 \pm 9.8_{e^+e^-} \pm 1.1_{xsys} \pm 2.3_{par} \pm 2.5_{rad}) \times 10^{-10}$



- Assigning extra systematic uncertainties to $\sigma_{tot}(e^+e^- \to hadrons)$ measurements we account for incompatibilities between the experiments and obtain $a_\mu^{\mathrm{LO-HVP}}$ with a realistic uncertainty.
- $a_\mu^{SM} - a_\mu^{exp}$ tension is reduced to 2σ. The dispersive $a_\mu^{\mathrm{LO-HVP}}$ is compatible with lattice QCD.
- $g_\mu - 2$ experimental accuracy is ahead of theory. Is the long standing $g_\mu - 2$ "anomaly" resolved? Refined $\sigma_{tot}(e^+e^- \to \pi^+\pi^-, \dots)$ measurements needed *(CMD-3, SND, KEDR, ... ?)*

Final state	$a_\mu(\mathrm{had,LO})\times10^{10}$ (exp.)	(χ^2_{thr})	(par.)	(rad.)	$\sqrt{s}$ [GeV]	ϵ, %	$\frac{\chi^2}{\mathrm{dof}}$	d.o.f.
$\pi^+\pi^-(\gamma)$	509.006 (9.631)	(0.372)	(2.280)	(2.294)	0.3 ÷ 1.937	$5.7^{+0.8}_{-0.5}$	1.66	754
$\pi^+\pi^-\pi^0$	48.238 (1.785)	(0.572)	(0.201)	(0.063)	0.56024 ÷ 1.937	$9.7^{+0.0}_{-3.5}$	1.40	531
$\pi^+\pi^-2\pi^0$	19.294 (0.436)	(0.000)	(0.063)	(0.064)	0.85 ÷ 1.937		1.17	76
$2\pi^+2\pi^-$	14.411 (0.174)	(0.000)	(0.183)	(0.012)	0.6125 ÷ 1.93		1.41	124
K^+K^-	23.198 (0.203)	(0.447)	(0.089)	(0.008)	0.985 ÷ 1.937	$0.0^{+6.6}_{-0.0}$	2.35	188
K_SK_L	13.106 (0.106)	(0.000)	(0.000)	(0.000)	1.00028 ÷ 1.937		0.95	157
$\pi^0\gamma$	4.359 (0.093)	(0.000)	(0.049)	(0.000)	0.59986 ÷ 1.38		1.70	75
$K_SK^+\pi^- + K_SK^-\pi^+$	1.814 (0.100)	(0.000)	(0.000)	(0.000)	1.24 ÷ 1.937		0.99	20
$2\pi^+2\pi^-\pi^0$	1.219 (0.076)	(0.000)	(0.017)	(0.001)	1.0125 ÷ 1.937		0.58	29
$2\pi^+2\pi^02\pi^-$	1.381 (0.141)	(0.000)	(0.011)	(0.000)	1.3125 ÷ 1.937		1.49	2
$2\pi^+2\pi^-3\pi^0$	0.099 (0.013)	(0.000)	(0.002)	(0.001)	1.575 ÷ 1.937		0.57	1
$3\pi^+3\pi^-$	0.254 (0.014)	(0.000)	(0.002)	(0.012)	1.3125 ÷ 1.937		1.52	54
$3\pi^+3\pi^-\pi^0$	0.020 (0.004)	(0.000)	(0.001)	(0.000)	1.6 ÷ 1.937		0.65	1
$\eta\gamma$	0.640 (0.062)	(0.002)	(0.047)	(0.000)	0.59986 ÷ 1.354	$5.9^{+0.0}_{-5.9}$	2.05	122
$\eta\pi^+\pi^-$	0.575 (0.019)	(0.000)	(0.000)	(0.000)	1.15 ÷ 1.937		1.18	72
$K^+K^-\pi^0$	0.202 (0.050)	(0.000)	(0.000)	(0.001)	1.44 ÷ 1.937		0.54	16
$K^+K^-\pi^0\pi^0$	0.100 (0.010)	(0.000)	(0.000)	(0.000)	1.5 ÷ 1.937		1.32	8
$K^+K^-\pi^+\pi^-$	0.810 (0.038)	(0.000)	(0.009)	(0.000)	1.4125 ÷ 1.937		1.91	38
$K^+K^-\pi^+\pi^-\pi^0$	0.129 (0.019)	(0.000)	(0.000)	(0.000)	1.6125 ÷ 1.937		1.63	11
$K_SK_L\eta$	0.238 (0.052)	(0.000)	(0.000)	(0.000)	1.575 ÷ 1.937		1.31	6
$K_SK_L\pi^0$	0.839 (0.093)	(0.000)	(0.000)	(0.000)	1.425 ÷ 1.937		1.50	6
$K_SK_L\pi^0\pi^0$	0.137 (0.043)	(0.000)	(0.000)	(0.000)	1.35 ÷ 1.937		0.00	0
$K_SK_L\pi^+\pi^-$	0.166 (0.028)	(0.000)	(0.000)	(0.000)	1.425 ÷ 1.937		0.00	0
$K_SK^+\pi^-\pi^0 + K_SK^-\pi^+\pi^0$	0.640 (0.044)	(0.000)	(0.000)	(0.000)	1.51 ÷ 1.937		1.08	15
$K_SK_S\pi^+\pi^-$	0.066 (0.006)	(0.000)	(0.000)	(0.000)	1.63 ÷ 1.937		1.37	4
$\omega(783)\eta$	0.035 (0.002)	(0.000)	(0.000)	(0.000)	1.34 ÷ 1.937		0.85	42
$\omega(783) < \pi^0\gamma > \pi^0$	0.906 (0.022)	(0.020)	(0.102)	(0.000)	0.75 ÷ 1.937	$0.0^{+9.2}_{-0.0}$	1.56	135
$\omega(783) < \pi^+\pi^-\pi^0 > \pi^+\pi^-$	0.092 (0.005)	(0.000)	(0.000)	(0.000)	1.15 ÷ 1.937		0.90	67
$\omega\eta\pi^0$	0.189 (0.045)	(0.000)	(0.091)	(0.000)	1.5 ÷ 1.937		0.30	8
$\pi^+\pi^-2\pi^0\eta$	0.117 (0.019)	(0.000)	(0.000)	(0.000)	1.625 ÷ 1.937		0.85	5
$\pi^+\pi^-3\pi^0$	1.067 (0.112)	(0.000)	(0.000)	(0.000)	1.125 ÷ 1.937		0.68	9
$\pi^+\pi^-\pi^0\eta$	0.663 (0.075)	(0.000)	(0.000)	(0.000)	1.394 ÷ 1.937		0.82	25
$\phi(1020) < X - K\bar{K} > \eta$	0.068 (0.003)	(0.000)	(0.001)	(0.000)	1.56 ÷ 1.937		0.97	74
$p\bar{p}$	0.033 (0.003)	(0.000)	(0.001)	(0.000)	1.889 ÷ 1.937		0.70	19
$n\bar{n}$	0.026 (0.004)	(0.000)	(0.000)	(0.000)	1.89 ÷ 1.937		1.48	8
2hadron (hadrons)	42.704 (0.552)	(0.000)	(0.235)	(0.000)	1.937 ÷ 11.199		1.39	314
pQCD	2.065		(0.002)		> 11.1990			
ChPT $\pi\pi, \pi^0\gamma$	0.538		(0.013)		0.2792 ÷ 0.3000			
$\Psi(1S)$	6.495		(0.124)		3.0969			
$\Psi(2S)$	1.631		(0.057)		3.6861			
$\Upsilon(1S)$	0.054		(0.002)		9.4604			
$\Upsilon(2S)$	0.021		(0.003)		10.0234			
$\Upsilon(3S)$	0.014		(0.002)		10.3551			
$\Upsilon(4S)$	0.010		(0.001)		10.5794			
Total	**697.671** (9.829)	(1.103)	(2.274)	(2.455)				

Summary

- Using an up-to-date compilation of $\sigma_{\rm tot}(e^+e^- \to \textit{hadrons})$ data we estimated the LO hadronic contribution to the muon $g-2$.
- Incompatibilities between independent $\sigma_{\rm tot}(e^+e^- \to \textit{hadrons})$ measurements are mitigated by extra systematic uncertainties estimated using the requirement of uniformity of χ^2 distribution over degrees of freedom.
- Tensions in the e^+e^- input data translate into an expanded uncertainty of the $a_\mu^{\rm LO-HVP}$:

$$a_\mu^{\rm LO-HVP} = (697.7 \pm 9.8_{e^+e^-} \pm 1.1_{xsys} \pm 2.3_{par} \pm 2.5_{rad}) \times 10^{-10}$$

Given this, we obtain the SM prediction for the muon $g-2$

$$a_\mu^{\rm SM} = 11659185.5(10.6) \times 10^{-10}$$

lower than the experimental world average $a_\mu^{\rm exp}$ at 2σ level.

Accuracy of both dispersive and lattice $a_\mu^{\rm LO-HVP}$ *is behind the experiment.*

New $e^+e^- \to \pi^+\pi^-(\gamma), 3\pi, 4\pi, \ldots 2hadron(hadrons)$ *data?*

Backup

Fitting the $R^{\rm had}$ data

A standard χ^2 minimization:

$$\chi^2 = \sum_{i,j} \left[\frac{1}{\Delta\sqrt{s_i}} \int_{\Delta\sqrt{s_i}} R^{\rm had}_{fit}(s) d\sqrt{s} - R^{\rm had}_i \right] \times {\rm COV}^{-1}_{ij} \times \left[\frac{1}{\Delta\sqrt{s_j}} \int_{\Delta\sqrt{s_j}} R^{\rm had}_{fit}(s) d\sqrt{s} - R^{\rm had}_j \right],$$

where $R^{\rm had}_{fit}(s)$ is the fitted parameterisation, $R^{\rm had}_i$ are the measurements in $\Delta\sqrt{s_i}$ bins, and ${\rm COV}_{ij}$ is the full covariance matrix between measurements:

$$\begin{aligned} {\rm COV}_{ij} = \delta_{ij}\sigma^2_{{\rm stat},i} \quad + \quad & \frac{1}{\Delta\sqrt{s_i}} \int_{\Delta\sqrt{s_i}} R^{\rm had}_{fit}(s) d\sqrt{s} \times \frac{1}{\Delta\sqrt{s_j}} \int_{\Delta\sqrt{s_j}} R^{\rm had}_{fit}(s) d\sqrt{s} \times \\ & \times \left\{ \begin{array}{l} \Delta_{{\rm sys},i}\Delta_{{\rm sys},j}, \text{ if } i,j \text{ are from the same experiment} \\ \Delta_{{\rm sys},i}\Delta_{{\rm sys},j} \times ({\rm cross-experiment\ covariation}), \\ \qquad\qquad \text{if } i,j \text{ are from different experiments} \end{array} \right\}, \end{aligned}$$

where $\Delta_{{\rm sys},i}$ are relative systematic uncertainties as quoted in the original papers.

Why $R^{\rm had}_{fit}(s)$ in the systematic term of ${\rm COV}_{ij}$? Naively taking individual measurements $R^{\rm had}_{i,j}$ for the systematic uncertainty leads to a biased ${\rm COV}_{ij}$ and to a biased fit, as far as $R^{\rm had}_{i,j}$ *are already biased themselves* – a manifestation of the well known *Peele's Pertinent Puzzle (PPP)*: "...a phenomenon exhibiting unexpected mean values for experimental data affected by statistical and systematic errors" [*R. Frühwirth et al, EPJ Web of Conf., Vol. 27 (2012), 00008*]

The problem: $\delta\chi^2/\delta R^{\rm had}_{fit}(s)$ is non-linear w.r.t. $R^{\rm had}_{fit}(s) \Rightarrow$ run the fit iteratively $\longrightarrow$ ...

Fitting the $R^{\rm had}$ data *(continued)*

. . . $\longrightarrow$ run the fit iteratively:

1. Make the fit ignoring the systematic uncertainties to get zeroth approximation for $R^{\rm had}_{fit}(s)$. Though $\chi^2/{\rm dof}$ is awful, there's no PPP bias in the fit using a diagonal covariance matrix.
2. Rebuild the full covariance matrix using the obtained $R^{\rm had}_{fit}(s)$.
3. Repeat the fit with the full covariance matrix.
4. In practice, the procedure converges after these 2 iterations.

Caveats:

- A simplified parameterization of error matrices $C_{ij} = \delta_{ij}\sigma^2_{{\rm stat},i} + R_i R_j \cdot \Delta_{{\rm sys},i}\Delta_{{\rm sys},j}$, where $\Delta_{{\rm sys},i}$ is the quadratic sum of relative systematic uncertainties from all sources in the i-th point, is not always adequate. Some experiments publish full covariance matrices which, generally speaking, cannot be parameterized as above. Our parameterization is actually an artifact of the simplified output format from the PPDS CrossSection database.
- Covariance between different experiments: common MC for acceptance calculation, common background models, the same GEANT4 for detector simulation, common modeling of large angle Bhabha scattering for luminosity determination, etc. Need to be an expert to estimate a degree of covariance between different experiments. We neglect cross-dataset covariations, except for CMD-2 $\pi^+\pi^-\pi^0$ measurements in three scans at different $\sqrt{s}$, and CMD-2 (2003) / SND (2006) $\pi^+\pi^-(\gamma)$ measurements (based on expert estimate by S. Eidelman).
- It is not always clear how to scale statistical errors. If $R^{\rm had}(s)$ measurement gets a normalization bias from luminosity, acceptance, efficiency, then its statistical uncertainty should be scaled in the fit just as systematics (cf. the previous slide). Additive bias from background subtraction is more problematic. So far, we conservatively keep original $\sigma_{{\rm stat},i}$.

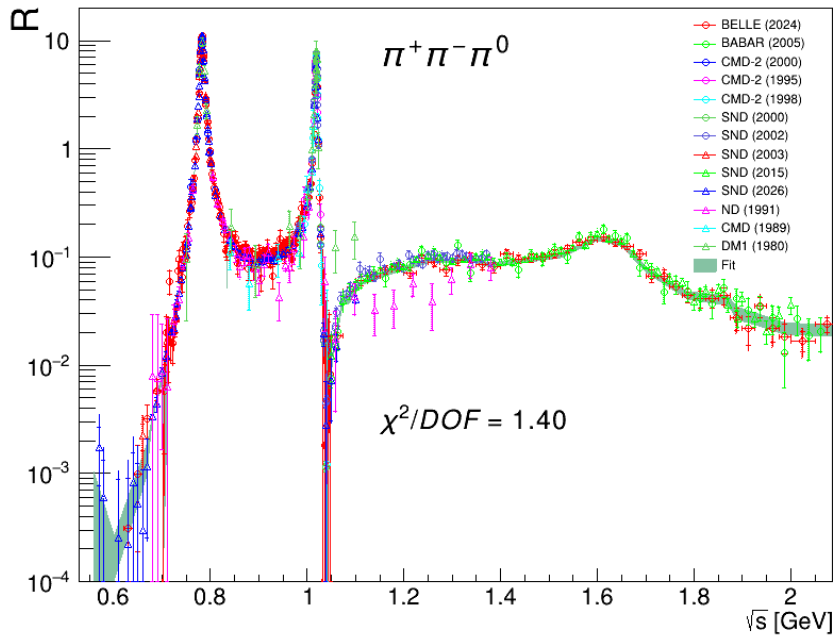
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Missing systematic uncertainties: $\pi^+\pi^-\pi^0$

- Initial fit with unmodified error matrices

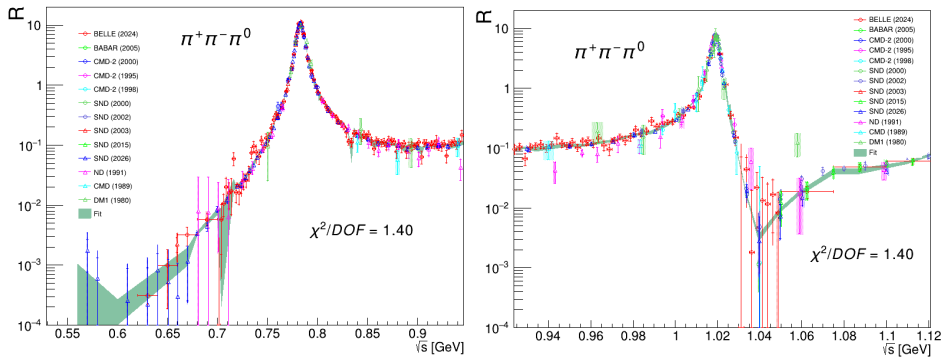
(global $\chi^2/dof = 1.78$):

Exp.	Year	n_p	$\frac{\Delta\chi^2}{n_p}$	$\Delta\chi^2_{sys}$	$\frac{\Delta\chi^2_{res}}{n_p}$	Pull
Belle	2024	212	1.45	69.92	1.23	0.048
SND	2003	67	2.37	150.81	1.48	0.073
SND	2026	102	0.47	5.56	0.47	-0.002
	2000	32	2.09	3.06	1.64	0.071
	2003	49	1.39	0.01	1.38	0.004
	2015	40	1.13	1.48	0.99	0.071
BaBar	2005	75	1.04	4.39	0.80	0.120
CMD-2	2000	13	2.37	5.52	2.32	-0.028
	1998	16	2.08			
	1995	14	1.13			
ND	1991	39	1.37	1.23	1.63	-0.125
CMD	1989	7	1.28	2.78	2.10	0.154
DM1	1989	26	1.07	0.08	1.04	0.019



- Fit with extra 9.7% systematics: →

Exp.	Year	n_p	$\frac{\Delta\chi^2}{n_p}$	$\Delta\chi^2_{sys}$	$\frac{\Delta\chi^2_{res}}{n_p}$	Pull
Belle	2024	212	1.09	2.95	0.98	0.025
SND	2003	67	1.32	15.21	1.17	0.058
SND	2026	102	0.41	2.80	0.45	-0.035
	2000	32	1.74	0.43	1.58	0.056
	2003	49	1.10	0.16	1.17	-0.021
	2015	40	0.90	0.48	0.79	0.067
BaBar	2005	75	0.92	1.76	0.73	0.105
CMD-2	2000	13	2.76	8.53	2.48	-0.050
	1998	16	1.33	2.82	0.94	0.050
	1995	14	0.93	1.24	0.81	0.026
ND	1991	39	1.34	1.42	1.68	-0.146
CMD	1989	7	1.19	1.78	1.61	0.138
DM1	1989	26	1.12	0.00	1.13	-0.001



← For the SND (2003) dataset the condition $\Delta\chi^2_{\rm sys} < 10$ was not achieved in four iterations, with the estimated extra systematic uncertainty $\epsilon = 9.7\%$. The process was aborted at this point as further expansion of ϵ resulted in growth of $\Delta\chi^2_{\rm sys}$. This indicates that the adopted approximation for the extra uncertainty is insufficient to account for tensions present in the channel.

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Open questions, todo's ...

- Missing systematics:
  - Parameterization of the extra uncertainty accounting for incompatible measurements is not unique.
  - We stick to the simplest estimate: assign single additional normalization uncertainty to all experiments contributing to the fit in the given channel.
  - Expert knowledge required. Is extra systematics (partially) correlated between experiments? Is it multi-component (e.g., some components are correlated due to common MC, some other components are not)?
  - $\sqrt{s}$ dependence. We can restrict fit in the given channel to some $\sqrt{s}$ range and find missing systematics in $\sqrt{s}$ ranges.
  - Minimum χ^2 & maximum χ^2 entropy criterion:
    - ★ Formulate it using a strict definition of the entropy of the fit.
    - ★ Make missing systematics true free parameter(s) of the fit.
    - ★ This would allow to drop our heuristics and implement a well justified numerical procedure.
  - Ultimately, an estimate of the missing uncertainties should give hints to instrumental origins of incompatibilities between experiments *(currently, it does not)*.
- Covariance matrices:
  - The parameterization $C \propto \mathrm{diag}(\sigma^2_{stat}) + \sigma_{sys} \otimes \sigma^T_{sys}$ is insufficient: σ_{stat} correlations are neglected, the systematic part is better approximated as $\sum_i \sigma_{sys,i} \otimes \sigma^T_{sys,i}$, etc.
  - Recent experiments publish full covariance matrices.
  - Covariations between experiments: expert knowledge needed.
- Missing channels?
- **In progress:** an online $a_\mu^{\mathrm{LO-HVP}}$ calculator /D.V. Golubkov/