

Problems of vacuum energy and their possible solution

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Eur.Phys.J.C 85 (2025) 8, 912; arXiv: 2608.25860 [gr-qc]

Supported by state funding for neutrino physics FSUS-2025-0019

The 8th International Conference
on Particle Physics and Astrophysics (ICPPA-2026)

27 – 30 October 2026, MEPhI, Moscow, Russia

Cosmological Constant & Vacuum Energy Problem

Einstein-Hilbert action with the Λ -term (Einstein, 1917):

$$S = -\frac{M_{Pl}^2}{16\pi} \int d^4x \sqrt{-g} (R + \Lambda) \equiv -\frac{M_{Pl}^2}{16\pi} \int d^4x \sqrt{-g} R - \rho_{vac} \int d^4x \sqrt{-g}$$

Today Λ -term is identified with the **vacuum energy density**, acting as **dark energy** (DE) with the equation of state $P = -\rho$.

The Core Contradiction between **theoretical estimates** of the value of ρ_{vac} and **observational constraints** on its possible observable value:

- **Observations:** DE makes $\sim 70\%$ of the total cosmological energy density:

$$\rho_{DE} \sim 1 \text{ keV}/\text{cm}^3 \approx 10^{-47} \text{ GeV}^4$$

- **Theoretical estimates:** Give either **infinitely large values** or, in the case of cancellations of vacuum energies of bosonic and fermionic fluctuations (SUSY), a **value exceeding observations by 55 orders of magnitude**:

$$\rho_{SUSY}^{vac} \sim m_{SUSY}^4 \sim 10^{55} \rho_{DE}, \quad m_{SUSY} \sim 100 \text{ GeV}$$

Structure of the QCD-vacuum

Proton mass $m_p \sim 1 \text{ GeV}$:

- $p = uud$, $m_q \sim 5 \text{ MeV} \implies m \approx (15\text{MeV} - E_{bind}) \gtrsim 0.01m_p - ???$
 E_{bind} is the binding energy of quarks in a proton.

The missing contribution:

- nontrivial properties of the QCD vacuum $\implies$ condensates of quark and gluon fields:

$$\langle \bar{q}q \rangle \neq 0, \quad \langle G_{\mu\nu} G^{\mu\nu} \rangle \neq 0, \quad \varrho_{vac}^{(cond)} \approx 1\text{GeV}^4$$

- Quarks inside a proton destroy the condensates and the proton mass:

$$m_p = 2m_u + m_d - \varrho_{vac}^{(cond)} l_p^3 \sim 1\text{GeV}, \quad l_p \sim 1/\text{GeV} \text{ is the proton size}$$

The energy density of the condensate must be negative and by 47 orders of magnitude larger than the observed value $\varrho_{DE} \approx 10^{-47} \text{ GeV}^4$.

Something else, except for quarks and gluons, "lives" in vacuum and this "something" (new field – ??) compensates ϱ_{vac} by 47 orders of magnitude.

The first model of dynamical reduction of vacuum energy

- Dolgov, 1982 (Nuffield Workshop on the Very Early Universe, proceedings)

$$S_\phi = \int d^4x \sqrt{-g} \mathcal{L} = \int d^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - U(\phi, R) \right]$$

Equation of motion for the homogeneous field ϕ in FLRW-metric:

$$\ddot{\phi} + 3H\dot{\phi} + \partial U / \partial \phi = 0, \quad U = \frac{1}{2} (\beta R + m^2) \phi^2$$

When $\beta R < 0$ and $|\beta R| > |m^2|$ the EoM in de Sitter space-time has unstable solutions, exponentially rising with time, since $(m_\phi^{(eff)})^2 < 0$ at $R = const$.

With rising ϕ , its backreaction on the cosmological expansion transforms the initial exponential law into a power-law one:

$$a(t) \sim \exp(H_{vac} t) \implies a(t) \sim t^\kappa \quad (\kappa = const)$$

The Friedmann expansion is restored despite the nonzero vacuum energy.

Shortcomings & Other Attempts

Shortcomings of the simple 1982 model:

- The energy-momentum tensor of field ϕ is not proportional to $g_{\mu\nu}$:
 $T_{\mu\nu} \neq \Lambda g_{\mu\nu} \implies$ the vacuum energy does not vanish even asymptotically.
- The change in the expansion regime is achieved purely due to the weakening of the gravitational interaction (G_N decreases with time as $G_N \sim 1/t^2$).

Other attempts:

- Dolgov, Kawasaki (2003): Singular interaction $L_{int} \propto (\partial_\mu \phi \partial^\mu \phi)/R^2$.
- Dolgov, Urban (2008): Various interaction potentials $U(\phi, R)$.

The main problem: In all these cases, the transition to the canonical cosmology dominated by usual matter was not realized.

Generalisation of the model

$$\mathcal{L}_F = \frac{1}{2}\phi^2 [\beta R F(R, \phi) + m^2]$$

Arbuzova, Dolgov (Eur.Phys.J.C 85 (2025) 8, 912, arXiv: 2502.05581 [gr-qc]):

$$\mathcal{L}_f = \frac{1}{2}\phi^2 [\beta R f(\phi) + m^2] \equiv \frac{1}{2} [\phi^2 m^2 + \beta R Q(\phi)], \quad Q(\phi) = \phi^2 f(\phi)$$

Equation of motion for homogeneous field ϕ :

$$g^{\mu\nu} D_\mu D_\nu \phi + m^2 \phi + \frac{1}{2} \beta R \partial_\phi Q = \ddot{\phi} + 3H\dot{\phi} + m^2 \phi + \frac{1}{2} \beta R \partial_\phi Q = 0$$

- D_μ is the covariant derivative in FLRW-metric and $\partial_\phi Q = \partial Q / \partial \phi$.
- $H(t) = \dot{a}/a$ is the Hubble parameter, $a(t)$ is the scale factor.

The energy–momentum tensor of ϕ is defined as:

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}}$$

Energy-momentum tensor

Correspondingly

$$T_{\mu\nu} = (\partial_\mu\phi)(\partial_\nu\phi) - (1/2)g_{\mu\nu} \left[g^{\alpha\beta}(\partial_\alpha\phi)(\partial_\beta\phi) - m^2\phi^2 \right] \\ -\beta Q(\phi) (R_{\mu\nu} - g_{\mu\nu}R/2) + \beta (D_\mu D_\nu - g_{\mu\nu}D^2) Q(\phi)$$

The covariant derivatives of $Q(\phi)$:

$$D_\mu Q = (\partial_\phi Q)\partial_\mu\phi, \quad D^2 Q = \partial_\phi^2 Q \partial_\mu\phi \partial^\mu\phi + \partial_\phi Q D^2\phi$$

The trace of the energy-momentum tensor:

$$T^\nu_\nu = -(\partial\phi)^2(3\beta\partial_\phi^2 Q + 1) + 2m^2\phi^2 + \beta QR + 3\beta [(\partial_\phi Q)m^2\phi + \beta R(\partial_\phi Q)^2/2]$$

For a special case $Q = \phi^2$ the trace is:

$$T^\nu_\nu = -(6\beta + 1)(\partial_\mu\phi)(\partial^\mu\phi) + \beta(6\beta + 1)R\phi^2 + 2(1 + 3\beta)m^2\phi^2$$

- This is the well known result. Note, that $T^\nu_\nu = 0$ for $\beta = -1/6$ and $m = 0$.

More Complicated Model

To cure the problems of a simpler model described above we consider:

$$\bar{Q}(\phi, M_0, k) = \phi^2(1 + \sigma\phi^2/M_0^2)^k,$$

- M_0, k are some constant parameters to be fixed below. We take $\sigma = \pm 1$.

The derivatives of $\bar{Q}$ over ϕ are given by:

$$\begin{aligned}\partial_\phi \bar{Q} &= \frac{2k\sigma\phi^3(1 + \sigma\phi^2/M_0^2)^{k-1}}{M_0^2} + 2\phi(1 + \sigma\phi^2/M_0^2)^k, \\ \partial_\phi^2 \bar{Q} &= \frac{4(k-1)k\sigma^2\phi^4(1 + \sigma\phi^2/M_0^2)^{k-2}}{M_0^4} + \\ &\quad \frac{10k\sigma\phi^2(1 + \sigma\phi^2/M_0^2)^{k-1}}{M_0^2} + 2(1 + \sigma\phi^2/M_0^2)^k\end{aligned}$$

Curvature Scalar

Taking trace of the Einstein equations

$$\frac{M_{Pl}^2}{8\pi} \left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) = \varrho_{vac} g_{\mu\nu} + T_{\mu\nu}^{(matt)}$$

we find the algebraic relation:

$$R \left(\beta \bar{Q} + \frac{3\beta^2 (\partial_\phi \bar{Q})^2}{2} + \frac{M_{Pl}^2}{8\pi} \right) = (\partial\phi)^2 (3\beta \partial_\phi^2 \bar{Q} + 1) - 2m^2 \phi^2 - 3\beta m^2 \phi (\partial_\phi \bar{Q}) - 4\varrho_{vac} - \tilde{T}_\nu^\nu$$

$\tilde{T}_\nu^\nu$ is the trace of the energy-momentum tensor of the other kinds of matter.

This equation allows to express R through ϕ and its first derivative.

Dimensionless Variables

Two differential equations that govern cosmological evolution:

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi + \frac{1}{2}\beta R \partial_{\phi}\bar{Q} = 0,$$
$$\dot{H} + 2H^2 = -R/6$$

- Here R is a known function of ϕ and its first derivative.

Dimensionless variables:

$$\tau = tH_0, \quad \varphi = \phi/H_0, \quad h = H/H_0, \quad \frac{d}{dt} = H_0 \frac{d}{d\tau},$$
$$R = rH_0^2, \quad \varrho_{vac} = H_0^4 \lambda, \quad \bar{Q} = H_0^2 q, \quad M_0 = H_0 \mu.$$

- H_0 is an arbitrary normalisation constant.
- We fix $H_0^2 = 8\pi \varrho_{vac}/(3M_{Pl}^2)$, where ϱ_{vac} is the original large vacuum energy.

In what follows we denote derivative over τ by prime: $df/d\tau \equiv f'$.

Dimensionless Equations

Finally we come to following two differential equations

$$h' + 2h^2 = -\frac{r}{6}; \quad \varphi'' + 3h\varphi' + \beta r q_1/2 = 0$$

Dimensionless curvature scalar

$$r = \frac{(3\beta q_2 + 1)(\varphi')^2 - 2(m/H_0)^2 \varphi^2 - 3\beta(m/H_0)^2 \varphi q_1 - 4\lambda - \tilde{T}/H_0^4}{3\beta^2 q_1^2/2 + M_{Pl}^2/(8\pi H_0^2) + \beta q}$$

- Here $q \equiv \bar{Q}/H_0^2$, $q_1 \equiv \partial_\phi \bar{Q}/H_0$, $q_2 \equiv \partial_\phi^2 \bar{Q}$ are dimensionless functions

We assume:

Field ϕ is massless: $m = 0$, the matter is relativistic: $\tilde{T}_\nu^\nu = 0$.

The curvature turns into:

$$r_0 = \frac{(3\beta q_2 + 1)(\varphi')^2 - 4\lambda}{3\beta^2 q_1^2/2 + M_{Pl}^2/(8\pi H_0^2) + \beta q}$$

Compensation of positive vacuum energy (de Sitter)

First, let us consider the case of **positive vacuum energy**, $\rho_{vac} > 0$, which corresponds to a de Sitter space-time with initial curvature $R < 0$.

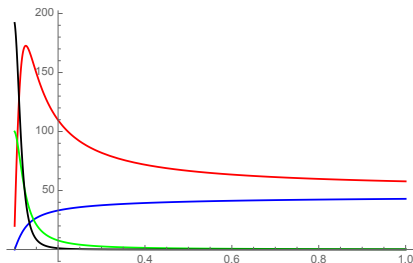
- The detailed study of this regime was published in: EA, A.D. Dolgov, "Dynamical mechanism of vacuum energy compensation", Eur. Phys. J. C 85 (2025) 912, e-Print: 2502.05581 [gr-qc]
- To compensate for the positive vacuum energy, the mechanism requires a **positive coupling constant**:

$$\beta = 1$$

- Let us look at the numerical solutions for a large initial vacuum energy $\lambda^{(in)} = 10^4$ and with initial $h_{in} = h(0) = 2$.

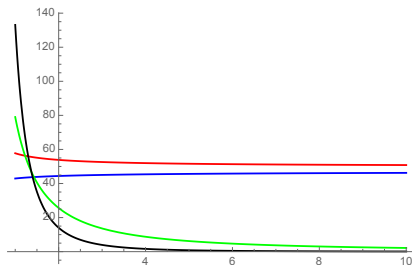
Numerical solutions for dS: $\lambda^{(in)} = 10^4$, $\beta = 1$

Small time τ



Evolution of $10^2 \tau h$, $10^3 \varphi$, $10^2 \varphi'$, $(-r_0)$

Large time τ



Evolution of $10^2 \tau h$, $10^3 \varphi$, $3 \cdot 10^4 \varphi'$, $(-10^5 r_0)$

- The calculations are performed with $\mu = 1$, $\sigma = 1$, $k = 3$.

Numerical calculations demonstrate that the curvature r_0 quickly tends to zero, meaning the **positive vacuum energy is completely compensated**.

From de Sitter to anti-de Sitter

The presented model successfully eliminates **positive vacuum energy**, turning the de Sitter expansion into a power-law expansion governed by relativistic matter.

What happens if the **initial vacuum energy is negative** ($\rho_{vac} < 0$)?

Key dynamical differences:

- **de Sitter (dS):** $\rho_{vac} > 0$. Acts as a cosmic repulsion, leading to an eternal **exponential expansion**.
- **anti-de Sitter (AdS):** $\rho_{vac} < 0$. Acts as an extra attraction, inevitably leading to an expansion halt, contraction, and a **Big Crunch**.

New questions arise:

- How does the compensating scalar field behave in the AdS space-time?
- Can the same dynamical mechanism prevent the inevitable gravitational collapse naturally associated with AdS geometry?

Properties of pure anti-de Sitter expansion

In contrast to de Sitter space, AdS space-time is characterized by **negative vacuum energy**:

$$\varrho_{vac} < 0 \implies R = -\frac{32\pi}{M_{Pl}^2} \varrho_{vac} > 0$$

For an **empty space** filled only with negative vacuum energy, **the dimensionless equation for the Hubble parameter** takes the form:

$$h' + 2h^2 = -\frac{r_0}{6}$$

- Here $r_0 > 0$ represents the initial dimensionless curvature

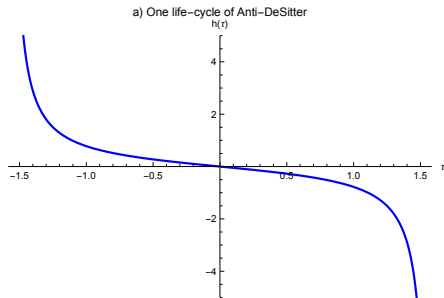
Analytical solution for the Hubble parameter:

$$h(\tau) = -\sqrt{\frac{r_0}{12}} \tan \left[\sqrt{\frac{r_0}{3}} (\tau - \tau_0) \right]$$

A universe with a negative vacuum energy first expands, then stops, starts to contract, and finally collapses to a **Big Crunch singularity**.

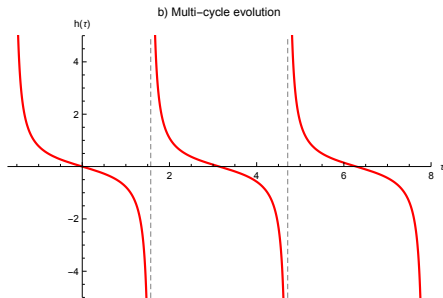
Evolution of pure anti-de Sitter universe

Single life-cycle



Small values of $(\tau - \tau_0)$. It covers the entire lifetime of the anti-de Sitter universe: expansion ($h > 0$), stop ($h = 0$), and contraction ($h < 0$) back to a singularity.

Hypothetical multi-cycle



Hypothetical evolution of several life-cycles of the anti-de Sitter universe, assuming it could somehow pass through the singularity. (Phoenix Universe)

Evolution of the dimensionless Hubble parameter h as a function of time τ .

Compensation mechanism for AdS space

To adapt the dynamical compensation mechanism for the **negative vacuum energy** of anti-de Sitter space, the coupling parameters must be adjusted:

- The sign of the coupling constant is reversed: $\beta = -1$.
- The function parameters are set to $\sigma = 1$ and the scalar field is assumed to be massless ($m = 0$).

The system of dimensionless equations governing the cosmological evolution:

$$h' + 2h^2 = -\frac{r}{6}; \quad \varphi'' + 3h\varphi' + \beta r q_1/2 = 0$$

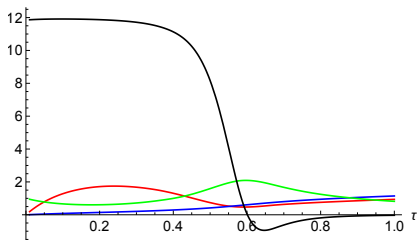
The dimensionless curvature r accounting for the negative vacuum energy ($\varrho_{\text{vac}} \rightarrow -\lambda$) is given by:

$$r = \frac{(3\beta q_2 + 1)(\varphi')^2 + 4\lambda}{\frac{3}{2}\beta^2 q_1^2 + \beta q + \frac{\lambda}{3}}$$

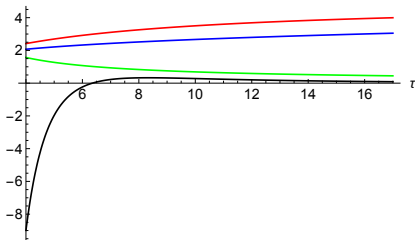
- $\lambda_{\text{in}} = |\varrho_{\text{vac}}|/H_0^4$ is the initial magnitude of the dimensionless vacuum energy.

Numerical solutions for AdS: $\lambda^{(in)} = 10^2$, $\beta = -1$

Small time τ



Large time τ



Evolution of $10\tau h$, φ , φ' , r

Evolution of $10\tau h$, φ , $10\varphi'$, $10^6 r$

- $h(\tau)$ becomes positive and decreases roughly as $\sim 1/\tau$.
- The curvature r tends down to zero.

The catastrophic collapse (Big Crunch) is prevented. The anti-de Sitter evolution successfully turns into a power-law expansion.

The Big Crunch Threat

- For a very large initial vacuum energy, e.g. $\lambda^{(in)} \sim 10^4$, the compensating field φ is not strong enough under the standard coupling ($\beta = -1$).
- The curvature r and the Hubble parameter h remain negative.
- $h \rightarrow -\infty$, which corresponds to the universe contraction.

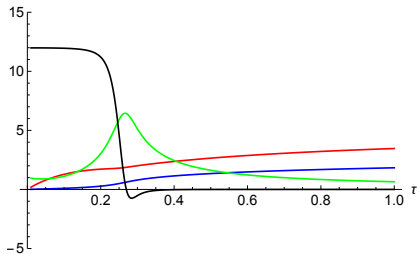
The universe collapses into a **Big Crunch singularity** faster than the compensation mechanism can complete its work and stabilize the curvature.

- This dynamics predicts the existence of a **critical vacuum energy threshold**, $|\lambda_{crit}|$.
- If the initial negative vacuum energy exceeds this value, the dynamic compensation fails to prevent the ultimate collapse.

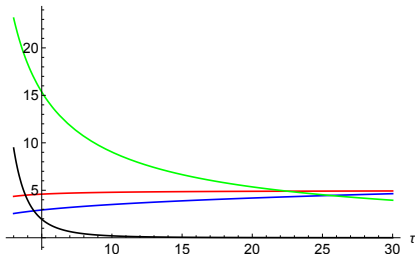
However, this threshold problem can be resolved by increasing the magnitude of the coupling constant β .

Numerical solutions for AdS: $\lambda^{(in)} = 10^4$, $\beta = -10$

Small time τ



Large time τ



Evolution of $10\tau h$, φ , φ' , r

Evolution of $10\tau h$, φ , $10^2 \varphi'$, $10^7 r$

- To overcome the failure, the coupling parameter is enhanced: $\beta = -10$.
- Even huge absolute initial values of r quickly tend down to zero.

The enhanced mechanism works efficiently. The huge initial negative vacuum energy is completely compensated, preventing the singularity.

Conclusion: Universal Compensation & Asymptotics

The presented model efficiently compensates the effect of **any original vacuum energy** down to zero, resulting in a realistic cosmology.

- **de Sitter** ($\varrho_{vac} > 0$): Initial exponential expansion is dynamically transformed into a power-law expansion.
- **anti-de Sitter** ($\varrho_{vac} < 0$): The forthcoming catastrophic collapse (Big Crunch) is prevented, driven to the same power-law regime.

Asymptotical Solutions ($R \rightarrow 0$):

- The dimensionless equations for $r = 0$ are solved analytically:
 $h(\tau) \rightarrow [2(\tau + \tau_0)]^{-1}$ and $\varphi \rightarrow const.$
- $H(t) \sim 1/(2t)$ and scale factor $a(t) \sim t^{1/2} \implies$ these are the canonical expressions for a cosmology dominated by **relativistic matter**.

Important Clarification: Where does the vacuum energy go?

It is crucial to emphasize that the physical vacuum energy itself does not disappear anywhere.

- Its enormous gravitational impact is dynamically neutralized by the interaction of the curvature scalar R with the scalar field ϕ .
- The compensation mechanism drives the curvature to zero ($R \rightarrow 0$), rather than destroying the vacuum energy itself.

As a result of this compensation, the universe safely enters a standard power-law expansion regime, evolving exactly as if the vacuum energy were completely absent.

Next steps

- We need to include non-relativistic matter into the model and check if the transition to a matter-dominated cosmology could be successfully achieved.
- Another related problem is a possibility of describing cosmological dark energy in the proposed framework.
- Presumably, it can be realized by introducing a more complicated dependence on the curvature scalar, R , analogously to the known description of dark energy by modified gravity through $F(R)$ generalization of GR.

This is supposed to be the matter of future studies.

THANK YOU
for your attention!