Noise suppression for Baikal-GVD using spectral properties of event graphs

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Baikal-GVD Collaboration



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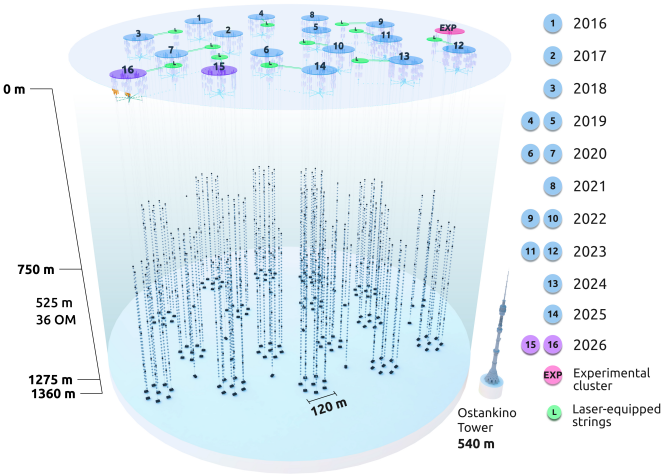


Figure 1: Baikal-GVD telescope, 2026 configuration.

PoS(ICRC2019) 875 · arXiv:1908.06509

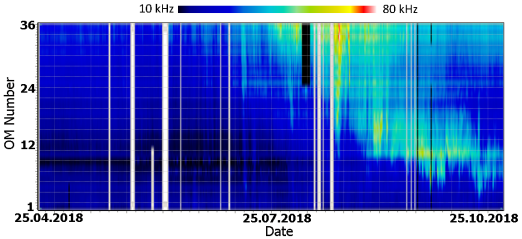


Figure 2: Noise rate by optical module, April–October 2018.

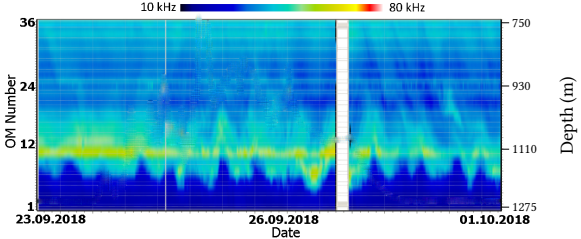


Figure 3: Short-term noise variations, 23 September–1 October 2018.

Noise Sources

- Dark current (8 kHz)
- Afterpulses
- Natural luminescence

PoS(ICRC2021) 1113 · arXiv:2107.14211

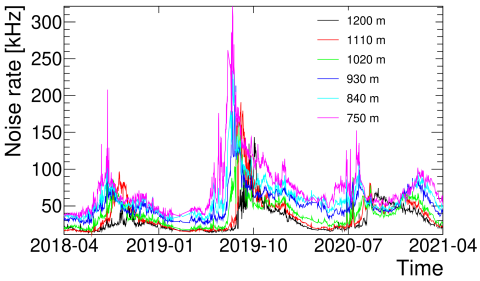


Figure 4: Noise rates at different depths, 2018–2021.

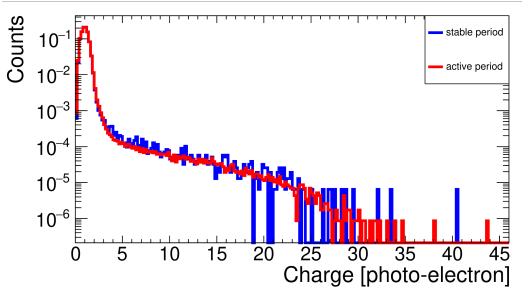


Figure 5: Noise-hit charge distributions during stable and active periods.

Channel noise levels may reach ~ 500 kHz

For a Fixed Muon Direction

$$\Delta t_{ij} \in \left[\frac{-KR_{ij} - \Delta z_{ij}}{c} - \tau,\ \frac{KR_{ij} - \Delta z_{ij}}{c} + \tau \right]$$

$$K = \frac{c}{v_g} \frac{1}{\sin \theta_c} - \frac{1}{\tan \theta_c}$$

$$R_{ij} < 3L_{\text{abs}}$$

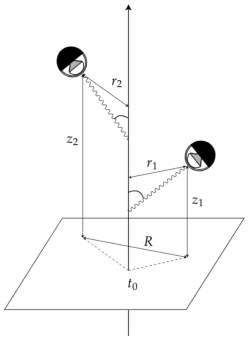
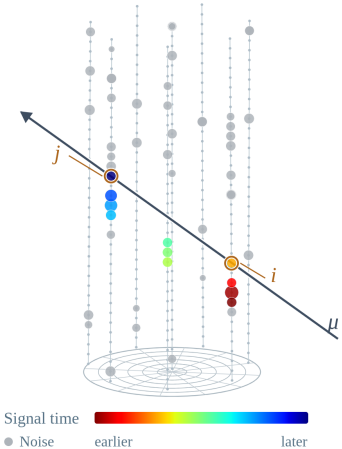


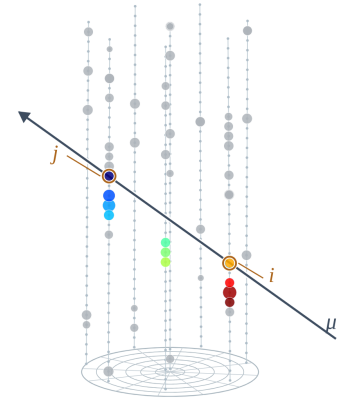
Figure 3.3: *Schematic view of a muon passing two optical modules.*

R. Bruijn, "The Antares Neutrino Telescope: Performance Studies and Analysis of First Data"

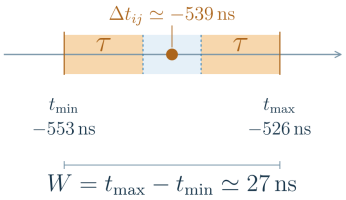
Constructing the Causality Graph



Constructing the Causality Graph



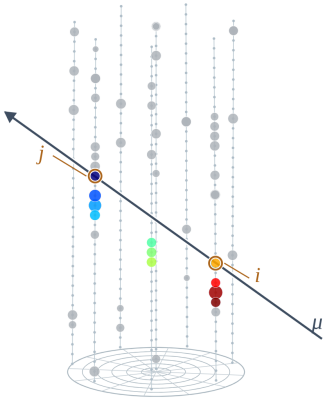
The allowed time window for this pair



The pair passes → draw an edge.



Constructing the Causality Graph



Signal time

Noise earlier later

The allowed time window for this pair

$\Delta t_{ij} \simeq -539\,\text{ns}$

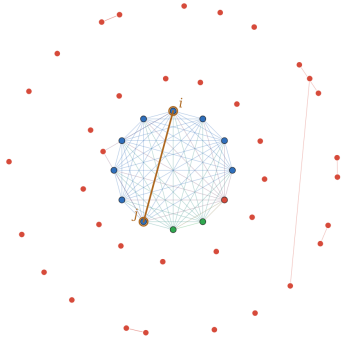
$\mathcal{T}$ $\mathcal{T}$

$t_{\min}$ $-553\,\text{ns}$ $t_{\max}$ $-526\,\text{ns}$

$W = t_{\max} - t_{\min} \simeq 27\,\text{ns}$

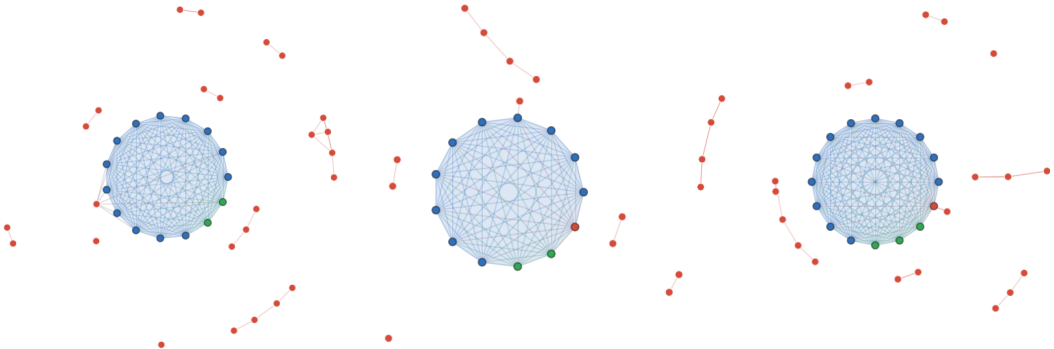
The pair passes → draw an edge.





Muon Cascade Noise

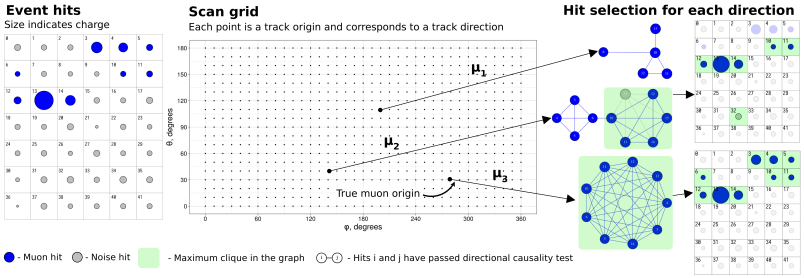
Colors show MC origin; edges use position and time.



Key observation

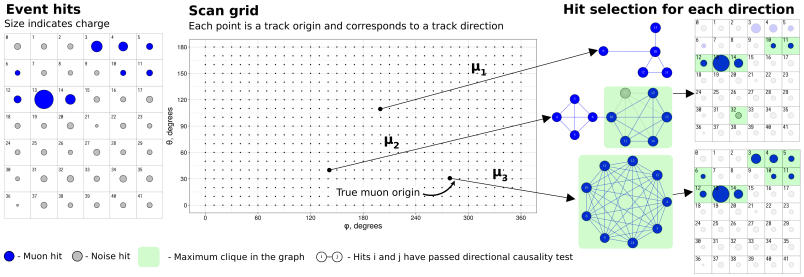
For the true muon direction, Cherenkov hits form a complete subgraph (a clique).

PoS(ICRC2021) 1063 · arXiv:2108.00208



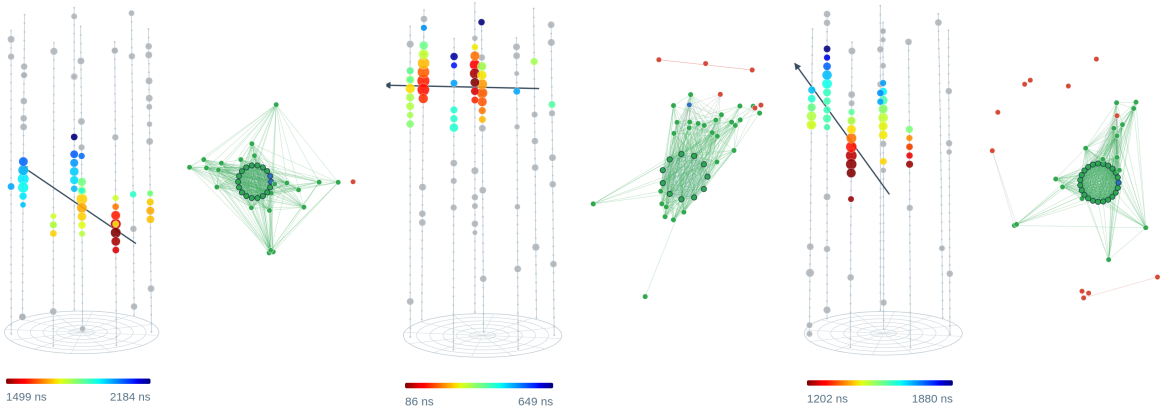
Sample	Average purity	Average efficiency
Atmospheric neutrino sample (hit selection from [6])	0.99	0.50
Atmospheric neutrino sample	0.95	0.95
High energy (E^{-2}) neutrino sample	0.98	0.74
High energy (E^{-2}) neutrino sample (at least 8 track hits)	0.98	0.89

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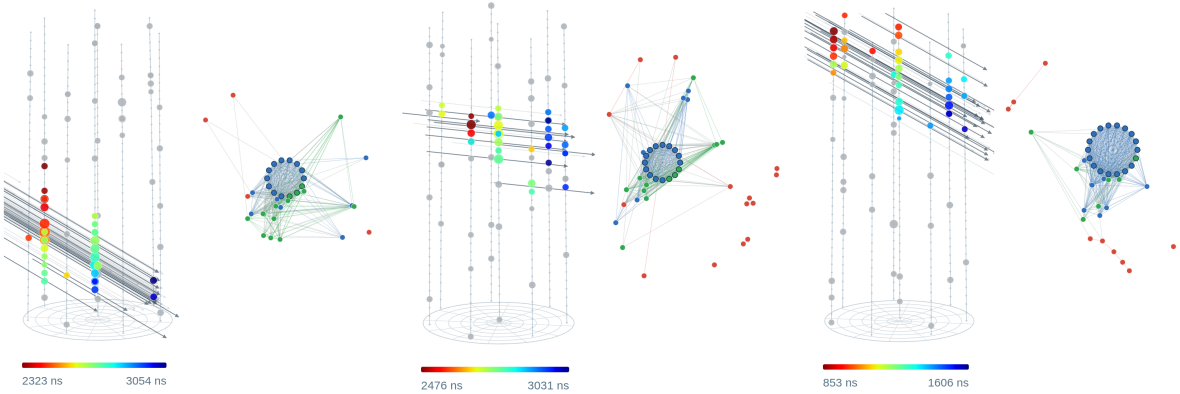


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Graph examples: astrophysical (E^{-2}) neutrino events

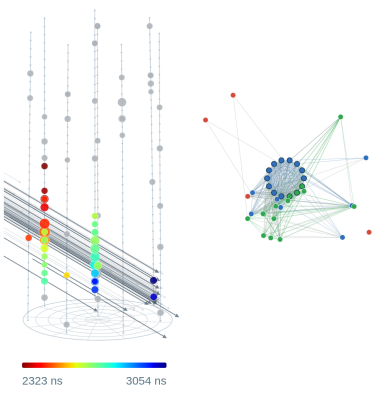


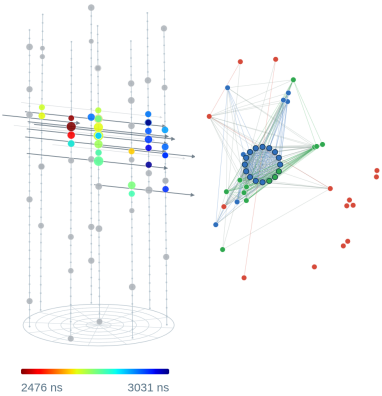
At higher energies delayed hits from stochastic losses contribution becomes significant, decreasing efficiency of clique-based selection.

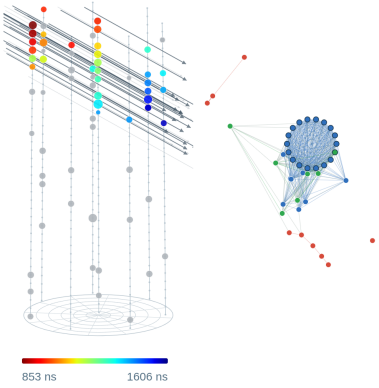


For atmospheric muon groups contributions from multiple muons can also lie outside of the clique.

Graph examples: atmospheric muon groups



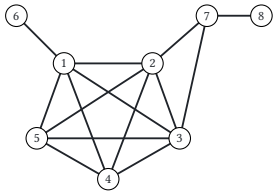




To increase hit selection efficiency for brighter events we should relax the clique requirement

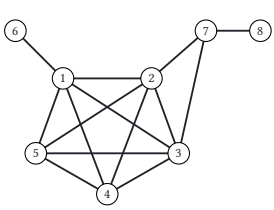
1. **Adjacency matrix**

$$A_{ij} = \begin{cases} 1, & \text{if } i \sim j, \\ 0, & \text{otherwise.} \end{cases}$$



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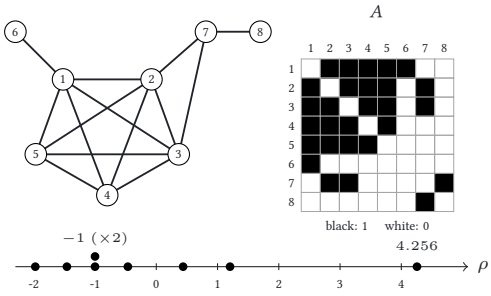


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2. **Graph spectrum**: eigenvalues of A

$$\sigma(A) = \{\rho_1, \ldots, \rho_N\}, \qquad Au_k = \rho_k u_k.$$



1. **Adjacency matrix**

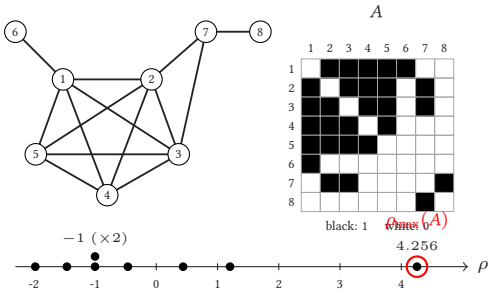
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3. **Perron value** (spectral radius)

$$\rho_{\max}(A) = \max_k |\rho_k| = \max_k \rho_k.$$



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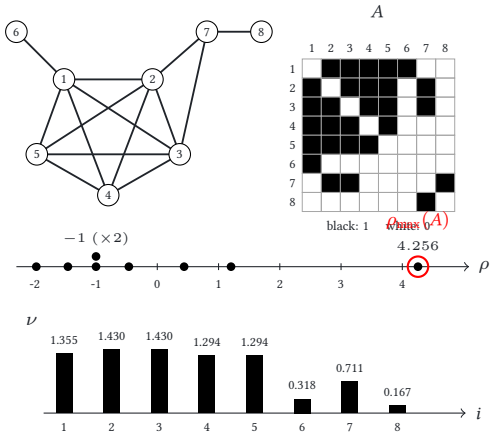
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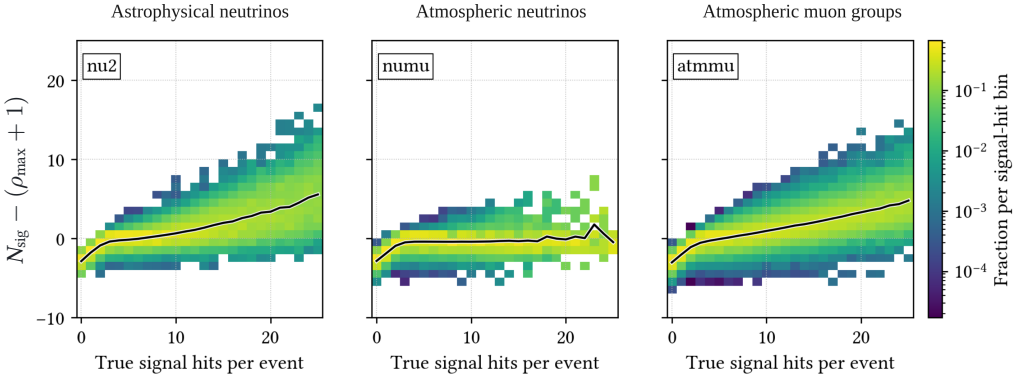
4. **Perron vector**

$$A\nu = \rho_{\max}(A)\nu, \qquad \frac{1}{N}\sum_i \nu_i = 1.$$



Perron value as hit multiplicity estimator

$$N_{\text{sig}} \simeq \rho_{\max} + 1 \qquad (N_{\text{sig}} \leq 12)$$



$\rho_{\max} \leq 12$: dim events

$\rho_{\max} > 12$: bright events

1. Identify a direction

Maximize the Perron value

$$\hat{\boldsymbol{\mu}} = \arg\max_{\boldsymbol{\mu}} \rho_{\max}(A(\boldsymbol{\mu}))$$

Use $A = A(\hat{\boldsymbol{\mu}})$

Spectral Filter: the algorithm

1. Identify a direction

Maximize the Perron value

$$\hat{\boldsymbol{\mu}} = \arg\max_{\boldsymbol{\mu}} \rho_{\max}(A(\boldsymbol{\mu}))$$

Use $A = A(\hat{\boldsymbol{\mu}})$

2. Pick a scorer

Low multiplicity: $\rho_{\max}(A) \leq 12$

$$\text{score}_i = \nu_i$$

$$(A - P)\nu = \rho_{\max}(A - P)\nu$$

$$\frac{1}{N}\sum_i \nu_i = 1$$

Signal if $\text{score}_i > T_L = 1.75$

P - Poisson expectation of A

$$P_{ij} = 1 - \exp[-W_{ij}(\lambda_i + \lambda_j)], \quad P_{ii} = 0$$

Spectral Filter: the algorithm

1. Identify a direction

Maximize the Perron value

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Use $A = A(\hat{\boldsymbol{\mu}})$

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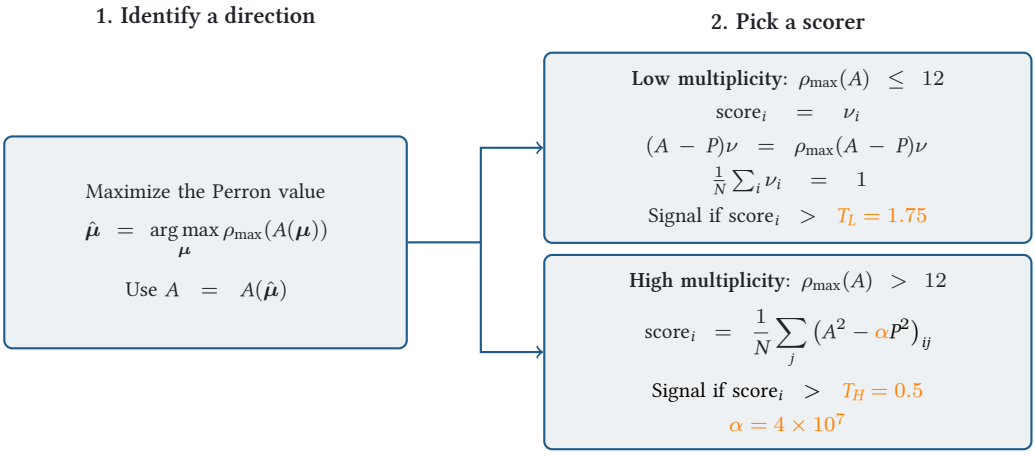
$$\text{score}_i = \frac{1}{N}\sum_j \left(A^2 - \alpha P^2\right)_{ij}$$

Signal if $\text{score}_i > T_H = 0.5$

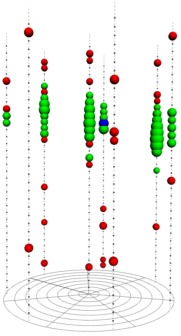
$\alpha = 4 \times 10^7$

P - Poisson expectation of A

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1. Identify a direction
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$\hat{\boldsymbol{\mu}} = \arg\max_{\boldsymbol{\mu}} \rho_{\max}(A(\boldsymbol{\mu}))$
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Low multiplicity: $\rho_{\max}(A) \leq 12$
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Signal if $\text{score}_i > T_L = 1.75$
High multiplicity: $\rho_{\max}(A) > 12$
$\text{score}_i = \frac{1}{N}\sum_j \left(A^2 - \alpha P^2\right)_{ij}$
Signal if $\text{score}_i > T_H = 0.5$
$\alpha = 4 \times 10^7$
Instead of scanning for direction with the largest clique
Instead of picking hits within the largest clique

Spectral Filter Algorithm: Example



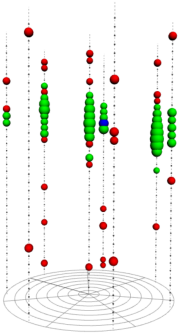
Hit positions and times

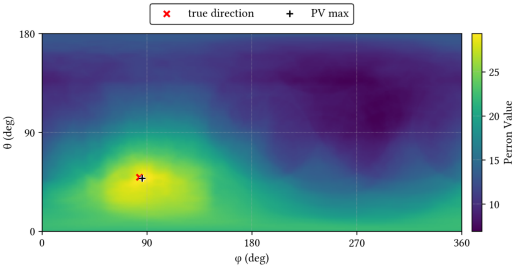
Channel noise rates λ_i

Parameters: α, T_L, T_H

● Noise ● Muon ● Cascades

Spectral Filter Algorithm: Example





Hit positions and times

Channel noise rates λ_i

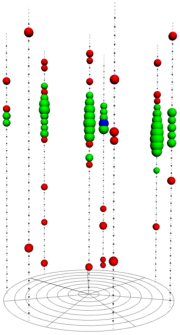
4° from the MC direction

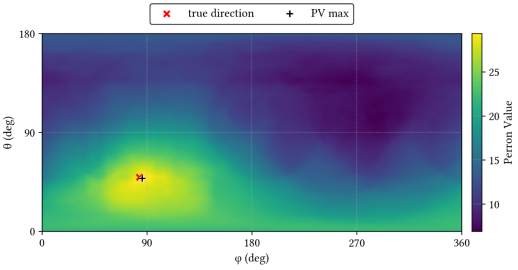
Parameters: $\alpha, \quad T_L, \quad T_H$

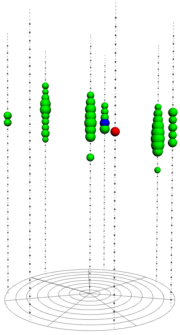
$$\boldsymbol{\mu}^* = \arg\max_{\boldsymbol{\mu}} \rho(A(\boldsymbol{\mu}))$$

● Noise ● Muon ● Cascades

Spectral Filter Algorithm: Example







Hit positions and times

Channel noise rates λ_i

Parameters: $\alpha, \quad T_L, \quad T_H$

4° from the MC direction

$$\boldsymbol{\mu}^* = \arg\max_{\boldsymbol{\mu}} \rho(A(\boldsymbol{\mu}))$$

$\rho \leq 12: \; s_i^L = \nu_i$

$\rho > 12: \; s_i^H$ from A^2

$s_i < T_{L,H} \quad \Rightarrow \quad$ noise

● Noise ● Muon ● Cascades

	Spectral filter		ScanFit	
Dataset	Purity	Efficiency	Purity	Efficiency
Atmospheric neutrino	92%	94%	95%	95%
High-energy neutrino (E^{-2})	93%	93%	98%	74%
Atmospheric muon groups	93%	91%	97%	79%

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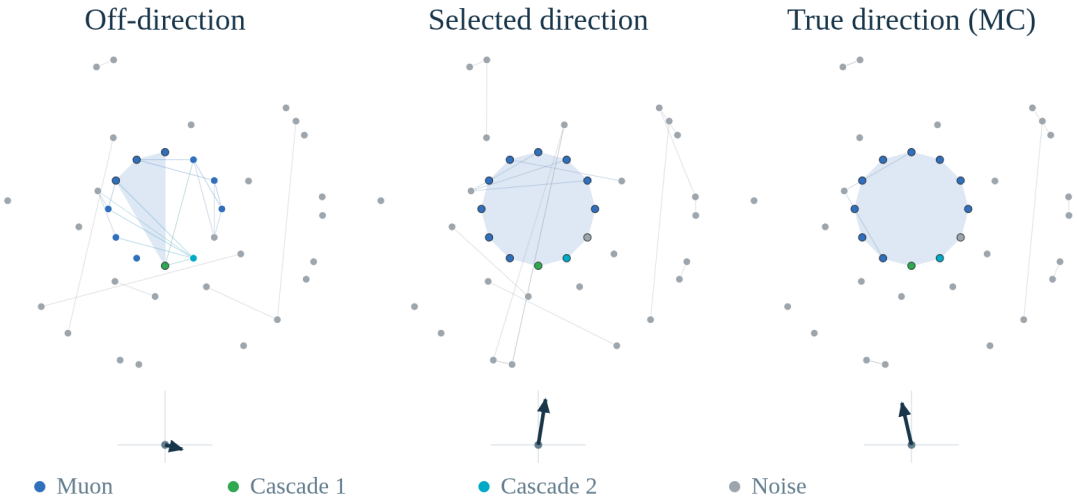
Results & Conclusions

	Spectral filter		ScanFit	
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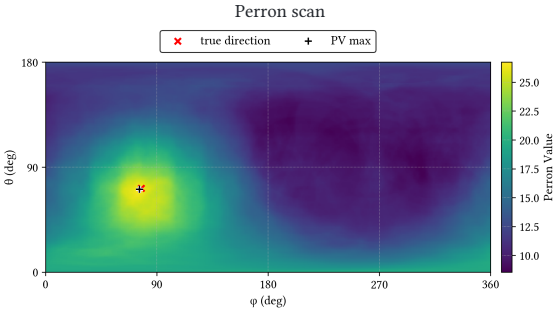
- Spectral graph theory helps relax the constraints of the clique-based noise suppression method.
- In Monte Carlo simulations, hit selection efficiency for astrophysical events and neutrinos increased by 20%.
- Processing speed no longer depends on clique enumeration; GPU processing is possible (500 μs per event with 300 hits). For ScanFit complexity of clique search could result in event processing time of over 1 minute.

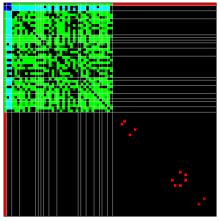
Backup

Causality Graphs for Different Directions

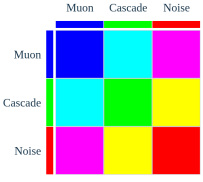


Example: Astrophysical Neutrino





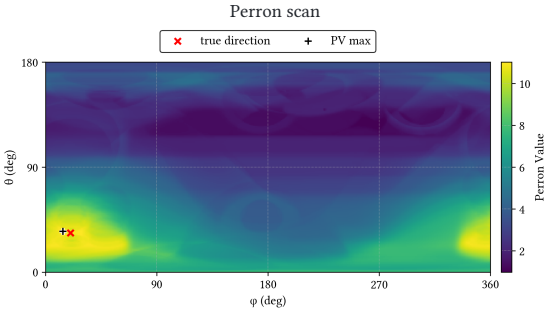
$A_{ij} = 1$

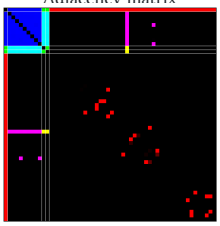


$A_{ij} = 0$

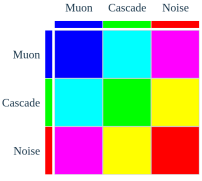
θ	ϕ	N_{hit}	N_{sig}	N_{μ}	MC P	MC E	fitted P	fitted E
71.77°	257.36°	79	40	2	100.0%	100.0%	100.0%	100.0%

Example: Atmospheric Neutrino



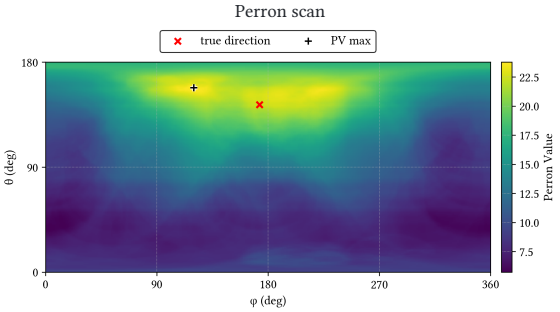


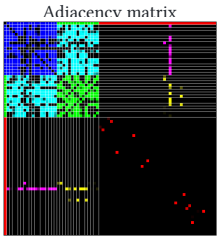
$A_{ij} = 1$



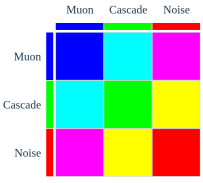
$A_{ij} = 0$

θ	ϕ	N_{hit}	N_{sig}	N_{μ}	MC P	MC E	fitted P	fitted E
33.64°	200.11°	55	11	9	91.7%	100.0%	91.7%	100.0%





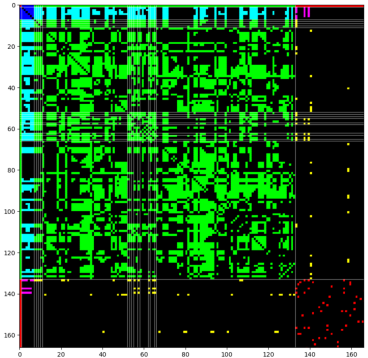
$A_{ij} = 1$



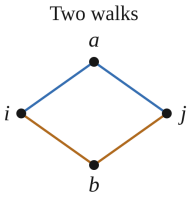
$A_{ij} = 0$

θ	ϕ	N_{hit}	N_{sig}	N_{μ}	MC P	MC E	fitted P	fitted E
143.50°	353.03°	75	33	18	94.3%	100.0%	86.8%	100.0%

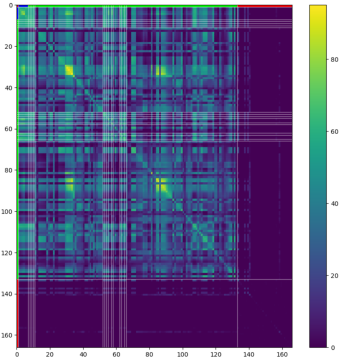
Bright Events: Hit Scoring with A^2



A



$(A^2)_{ij} = 2$

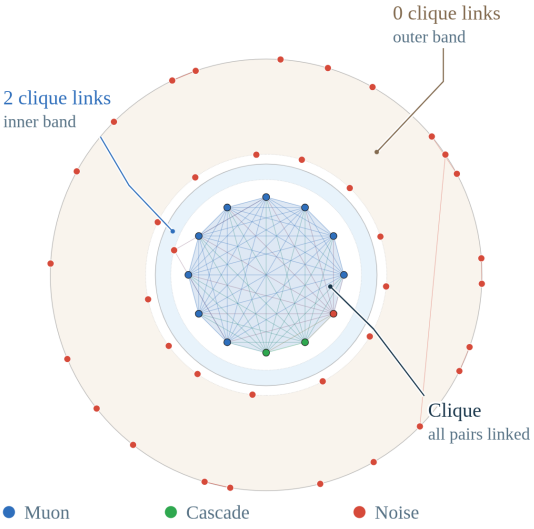


A^2

$(A^2)_{ij}$ counts walks of length 2 from i to j.

$$\text{score}_i = \frac{1}{N} \sum_j \left(A^2 - \alpha P^2\right)_{ij}$$

How to Read a Causality Graph



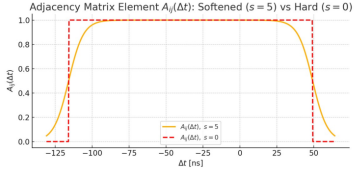
Smoothing

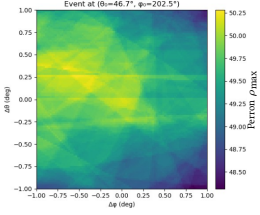
14/30

$$\Delta z_{i,j} - kR - w \leq c\Delta t_{i,j} \leq \Delta z_{i,j} + kR + w$$

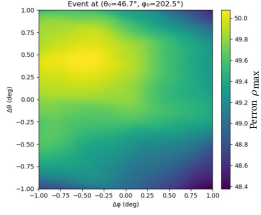
$$\Updownarrow$$

$$A_{ij}(\theta,\phi) = \begin{cases} \mathbb{I}\left[\Delta t_{ij} \in [\mathrm{low}_{ij},\, \mathrm{high}_{ij}],\ R_{ij}^{\perp} \leq 3r_{\mathrm{absorb}}\right] & \text{if smoothness } = 0,\ i \neq j \\ \sigma\left(\frac{\Delta t_{ij} - \mathrm{low}_{ij}}{s}\right)\sigma\left(\frac{\mathrm{high}_{ij} - \Delta t_{ij}}{s}\right)\sigma\left(\frac{3r_{\mathrm{absorb}} - R_{ij}^{\perp}}{s}\right) & \text{if smoothness } > 0,\ i \neq j \\ 0 & i = j \end{cases}$$



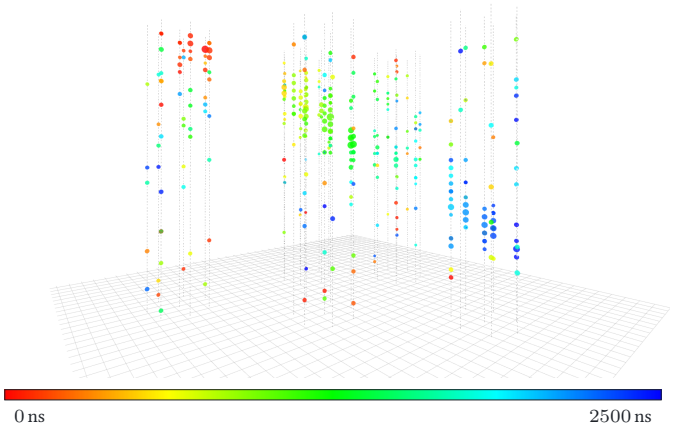


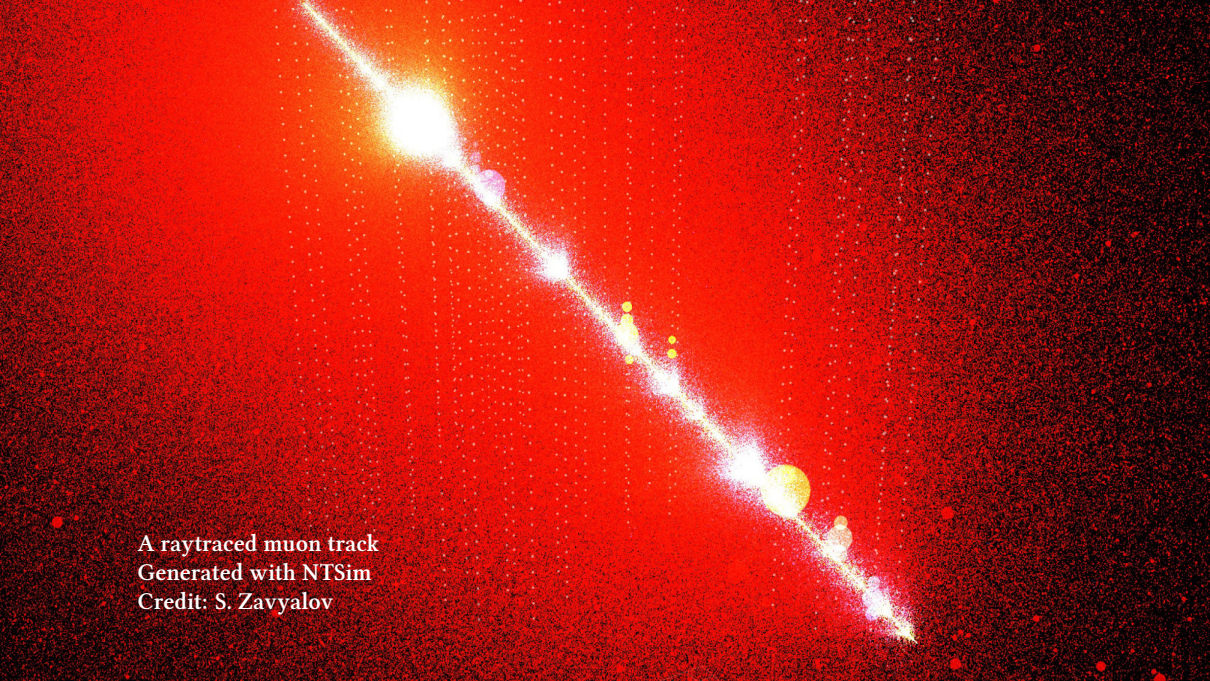
S = 0



S = 5

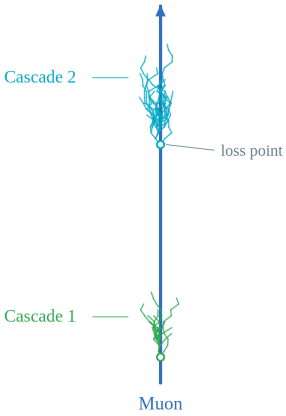
Event example



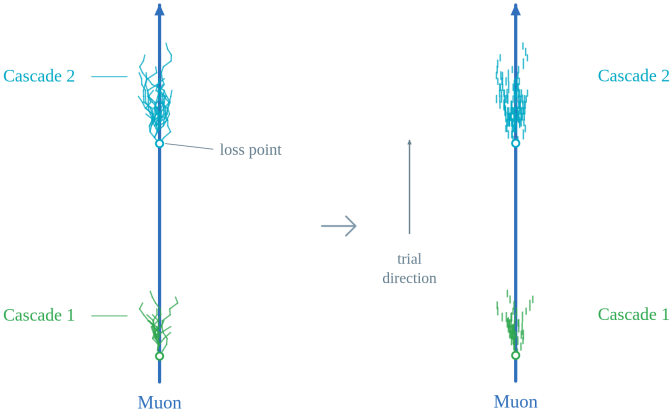


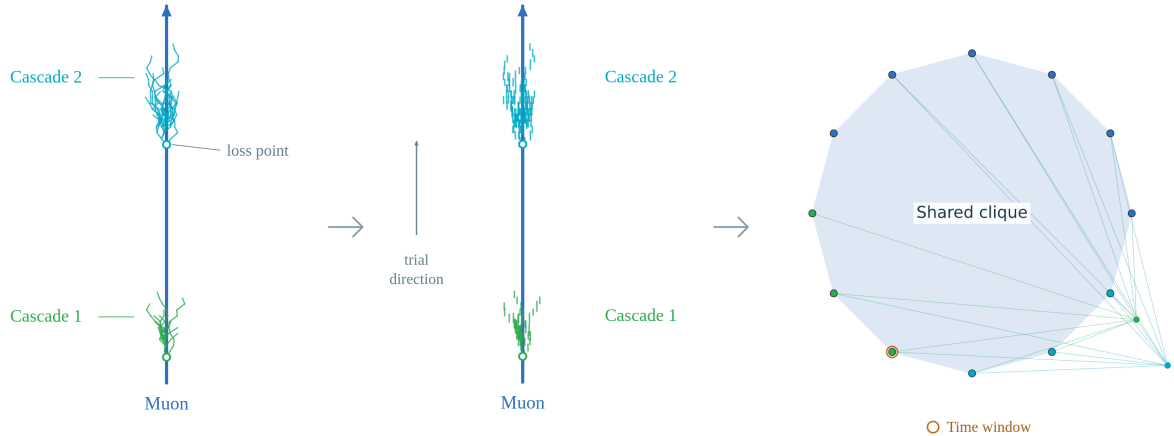
A raytraced muon track
Generated with NTSim
Credit: S. Zavyalov

Parallel Approximation for the Muon and Cascades



Parallel Approximation for the Muon and Cascades




Cascade 2
loss point
Cascade 1
Muon
trial
direction
Muon
Cascade 2
Cascade 1
Shared clique
Time window

For the Adjacency Matrix A

$$A\nu = \rho_{\max}(A)\nu, \qquad \nu_i > 0$$

If the graph with adjacency matrix A is connected, then

- The Perron eigenvalue $\rho_{\max}(A)$ is positive and has multiplicity 1.
- The Perron vector ν is positive and unique up to normalization.

2.2 Perron-Frobenius Theory

Let T be a real $n \times n$ matrix with nonnegative entries. T is called *primitive* if for some k we have $T^k > 0$; T is called *irreducible* if for all i, j there is a k such that $(T^k)_{ij} > 0$. Here, for a matrix (or vector) A, $A > 0$ (≥ 0) means that all its entries are positive (nonnegative).

The matrix $T = (t_{ij})$ is irreducible if and only if the directed graph Γ_T with vertices $\{1, \ldots, n\}$ and edges (i, j) whenever $t_{ij} > 0$ is strongly connected.

(A directed graph (X, E) is *strongly connected* if for any two vertices x, y there is a directed path from x to y, i.e., there are vertices $x_0 = x, x_1, \ldots, x_m = y$ such that $(x_{i-1}, x_i) \in E$ for $1 \leq i \leq m$.)

Note that if T is irreducible, then $I + T$ is primitive.

The *period* d of an irreducible matrix T is the greatest common divisor of the integers k for which $(T^k)_{ii} > 0$. It is independent of the i chosen.

Theorem 2.2.1 *Let $T \geq 0$ be irreducible. Then there is a (unique) positive real number θ_0 with the following properties:*

(i) There is a real vector $x_0 > 0$ with $Tx_0 = \theta_0 x_0$.
(ii) θ_0 has geometric and algebraic multiplicity one.
(iii) For each eigenvalue θ of T we have $|\theta| \leq \theta_0$. If T is primitive, then $|\theta| = \theta_0$ implies $\theta = \theta_0$. In general, if T has period d, then T has precisely d eigenvalues θ with $|\theta| = \theta_0$, namely $\theta = \theta_0 e^{2\pi i j/d}$ for $j = 0, 1, \ldots, d-1$. In fact the entire spectrum of T is invariant under rotation of the complex plane over an angle $2\pi/d$ about the origin.
(iv) Any nonnegative left or right eigenvector of T has eigenvalue θ_0. More generally, if $x \geq 0$, $x \neq 0$ and $Tx \leq \theta x$, then $x > 0$ and $\theta \geq \theta_0$; moreover, $\theta = \theta_0$ if and only if $Tx = \theta x$.
(v) If $0 \leq S \leq T$ or if S is a principal minor of T, and S has eigenvalue σ, then $|\sigma| \leq \theta_0$; if $|\sigma| = \theta_0$, then $S = T$.
(vi) Given a complex matrix S, let $|S|$ denote the matrix with elements $|S|_{ij} = |S_{ij}|$. If $|S| \leq T$ and S has eigenvalue σ, then $|\sigma| \leq \theta_0$. If equality holds, then $|S| = T$, and there are a diagonal matrix E with diagonal entries of absolute value 1 and a constant c of absolute value 1, such that $S = cETE^{-1}$.

Figure: A. Brouwer, W. Haemers, *Spectra of Graphs*