Exact Event Time Reconstruction for Time-of-Flight Particle Identification

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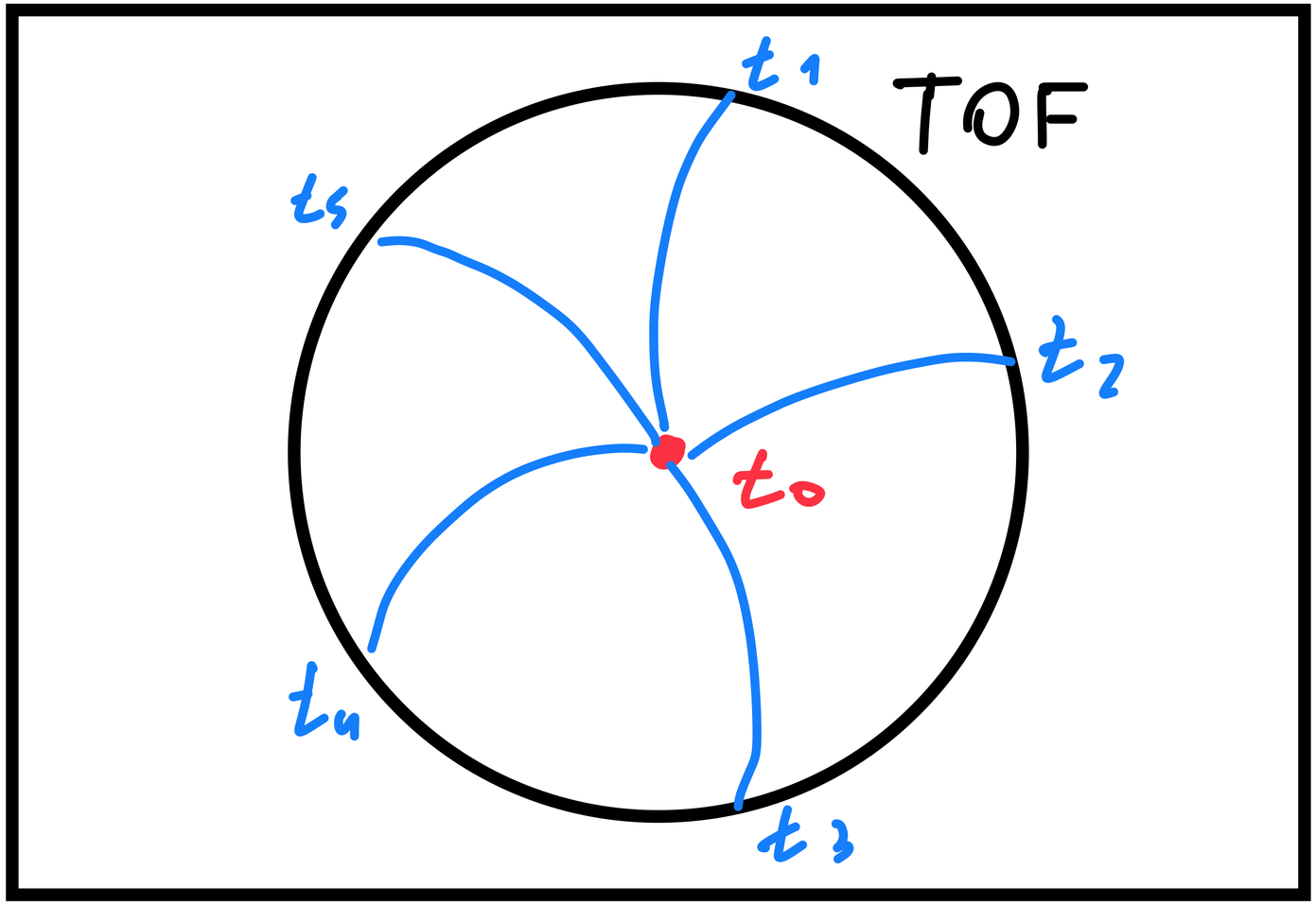
TOF particle identification



$$tof_i = t_i - t_0$$

$$\downarrow$$

$$m_i^2 = p_i^2 \left(\frac{c^2 \cdot tof_i^2}{L_i^2} - 1 \right)$$

But what if we don't know t_0?

t_0 reconstruction methods

1. Signal from the collider

2. Dedicated t_0 detector

3. Algorithmic reconstruction using TOF signals

Brute force (enumeration)

1. Choose allowed particle types: $\left[\pi^{\pm}, K^{\pm}, p^{\pm}\right]$ and for each track calculate:

$$tof_{ik} = \frac{L_i}{c}\sqrt{1 + \frac{m_k^2}{p_i^2}},$$

2. For each event check all possible mass hypotheses combinations - 3^N

3. For every combination calculate:

$$\chi^2 = \sum^{N} \frac{(t_0 + tof_{ik} - t_i)^2}{\sigma_{ik}^2}, \quad t_0 = \frac{1}{w}\sum^{N} \frac{t_i - tof_{ik}}{\sigma_{ik}^2}, \quad w = \sum^{N} \frac{1}{\sigma_{ik}^2}$$

4. Time complexity $O\left(N \cdot 3^N\right)$ - **very slow!**

Exact solution in polynomial time - 1

Crucial observation:

Fixing the mass hypothesis determines the optimal t_0



Fixing t_0 determines the optimal mass hypothesis.

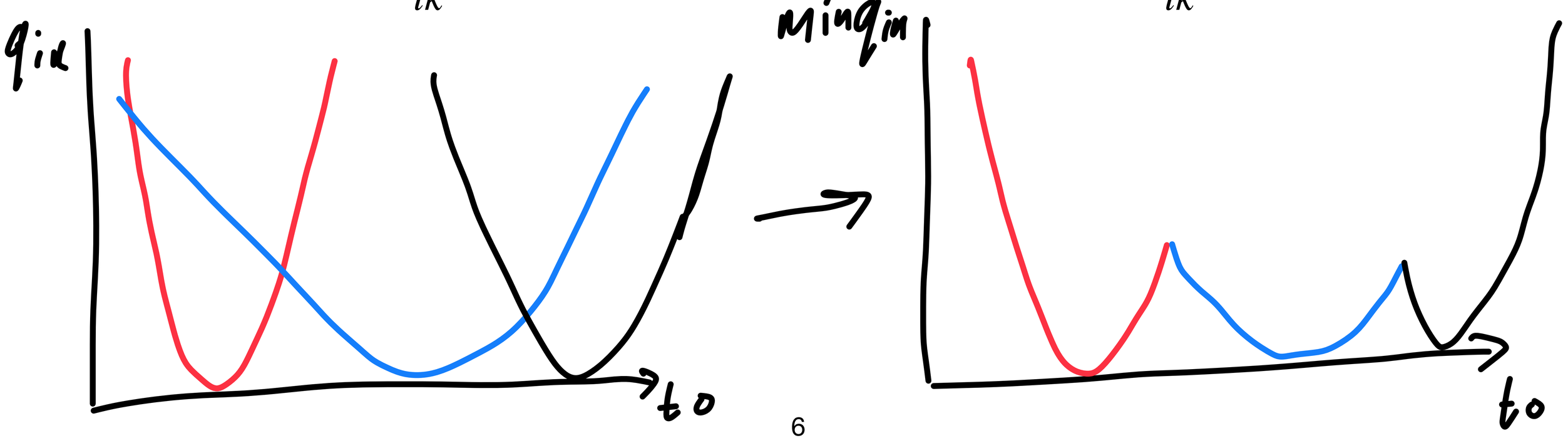


So our task is now a 1D optimization problem in t_0

A closer look at one track

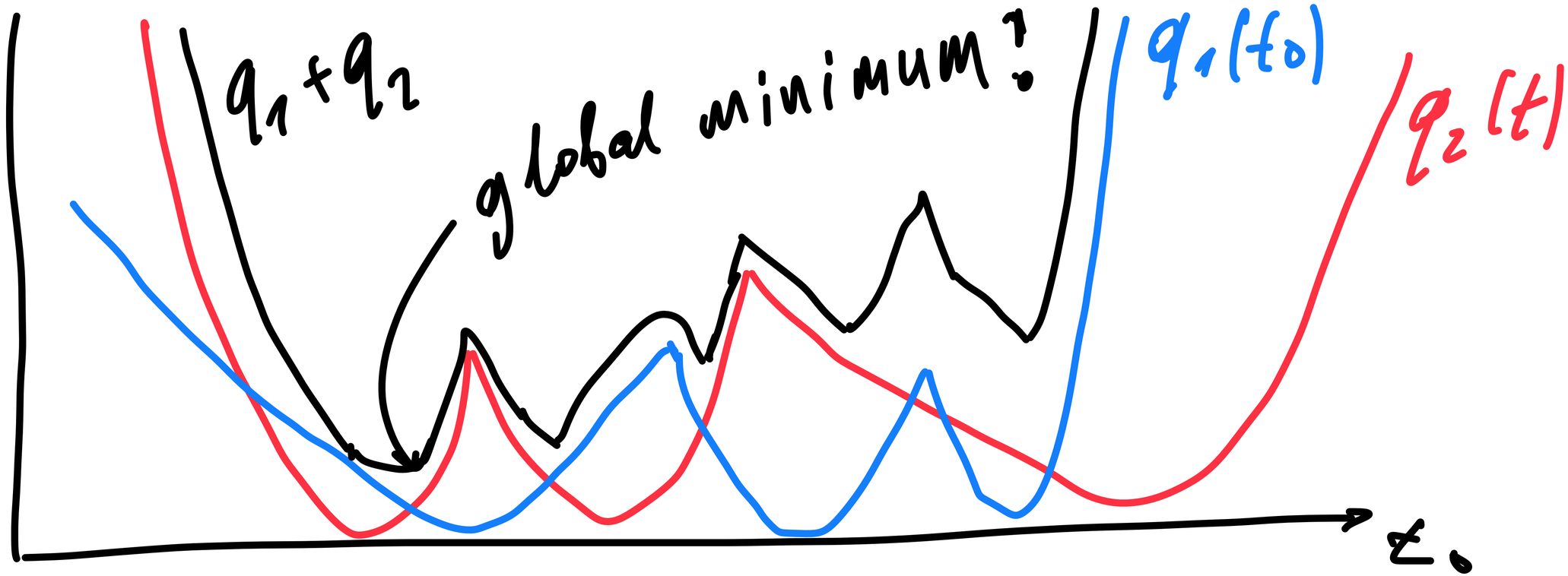
Contribution to χ^2 from single track:

$$q_{ik}(t_0) = \frac{(t_0 + tof_{ik} - t_i)^2}{\sigma_{ik}^2} = C_{ik}(t_0 - a_{ik})^2, \;\; C_{ik} = \frac{1}{\sigma_{ik}^2}, \;\; a_{ik} = t_i - tof_{ik}$$



Exact solution in polynomial time - 2

$$\min_{t_0,k_1,\ldots,k_N} \sum^{N} q_{i,k_i}(t_0) = \min_{t_0} \sum^{N} \min_{k} q_{ik}(t_0)$$



Exact solution in polynomial time - 3

1. Find all hypotheses change points x_i
2. Sort them such that $-\infty < x_{(1)} < x_{(2)} < \ldots < x_{(K)} < \infty$
3. Inside each interval k,

$$\chi^2_k = A_k t_0^2 - 2B_k t_0 + C_k$$

$$A_k = \sum^N w_{ik}, \;\; B_k = \sum^N w_{ik} a_{ik}, \;\; C_k = \sum^N w_{ik} a_{ik}^2$$

4. Update A, B, C in $O(1)$ at each hypothesis switch
5. Find minimum of χ^2 inside each interval and choose the smallest one
6. Time complexity $O(N \log N)$

ALICE method

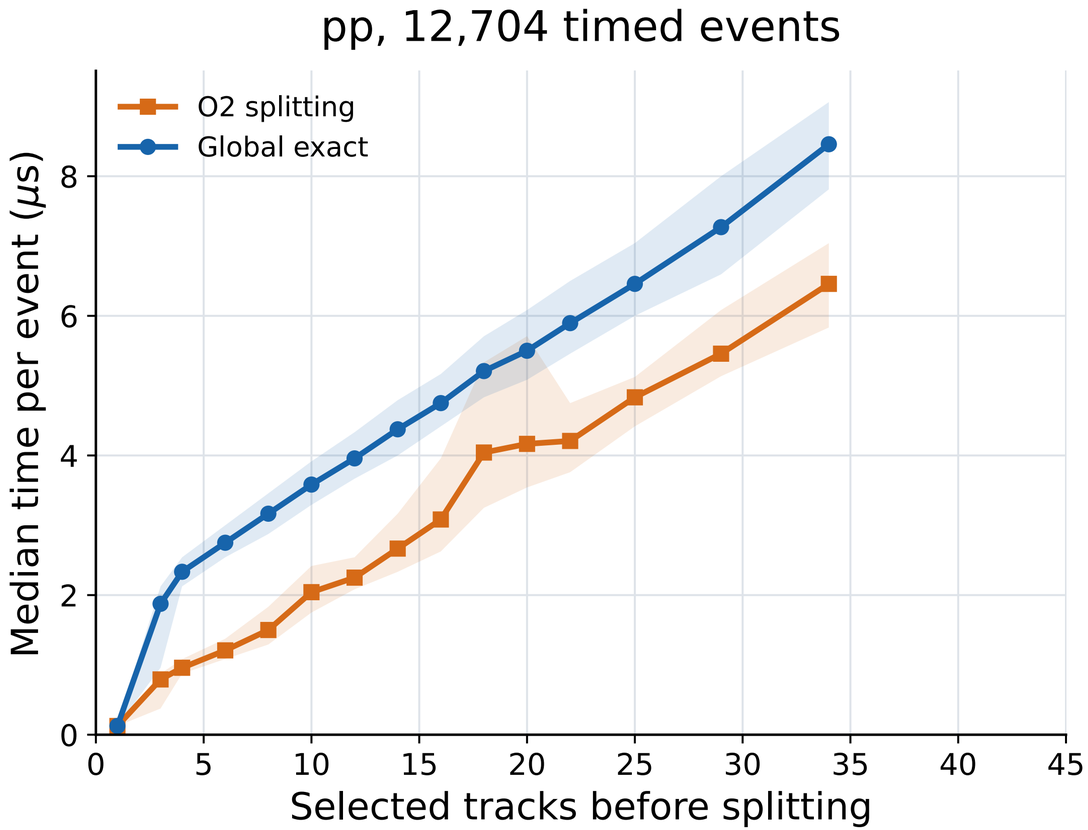
1. Split the event into sets of at most M tracks.

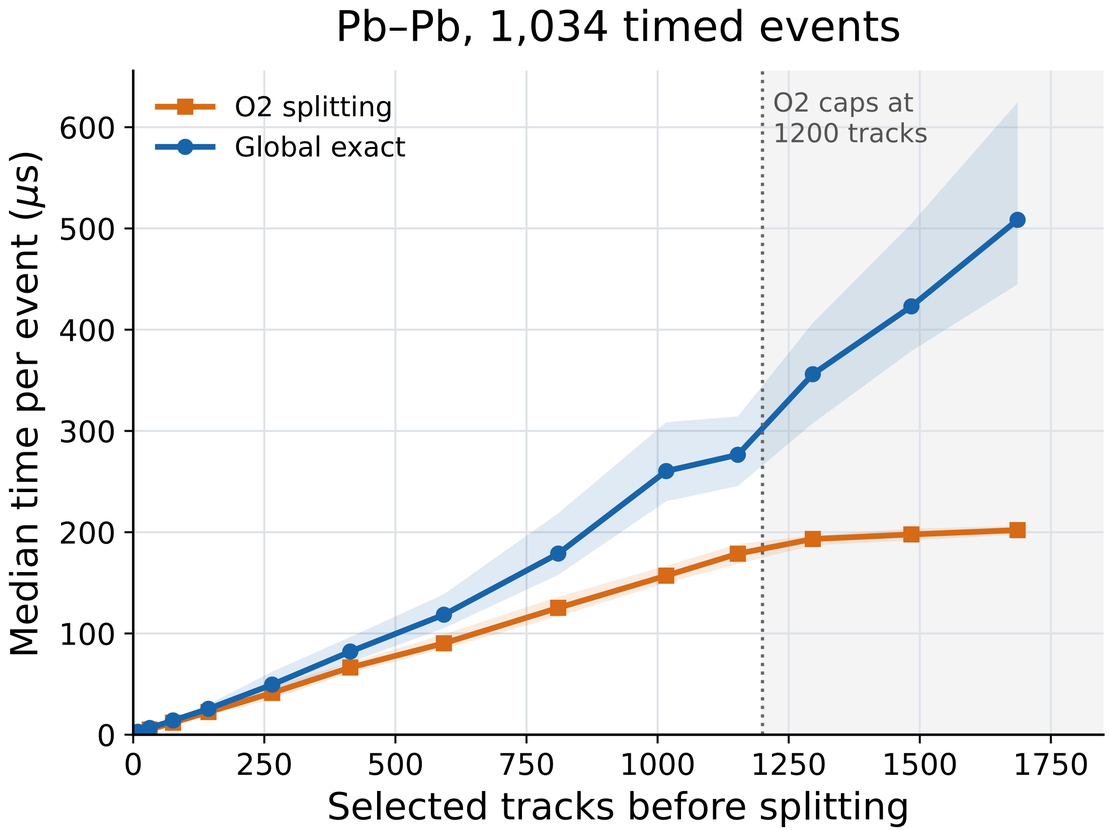
2. In each set find t_{0m} using enumeration.

3. Take weighted average between sets as the answer.

Global optimality is not guaranteed, there is a counterexample.

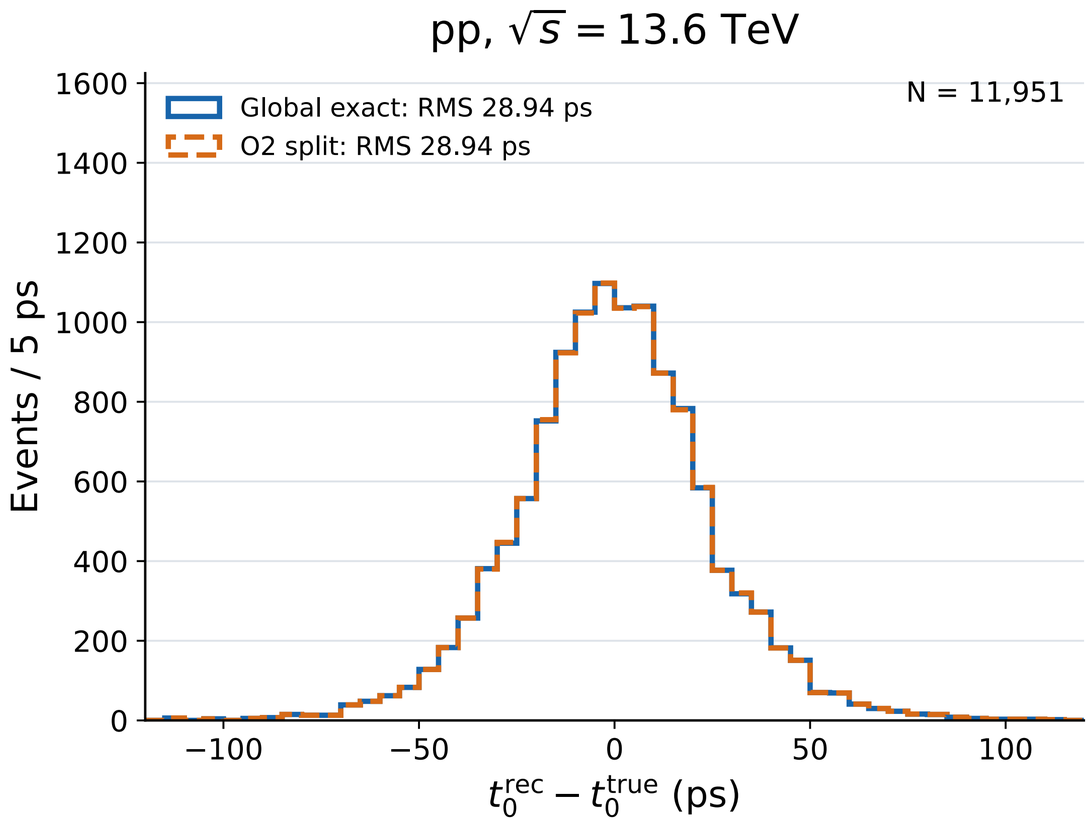
Results - 1

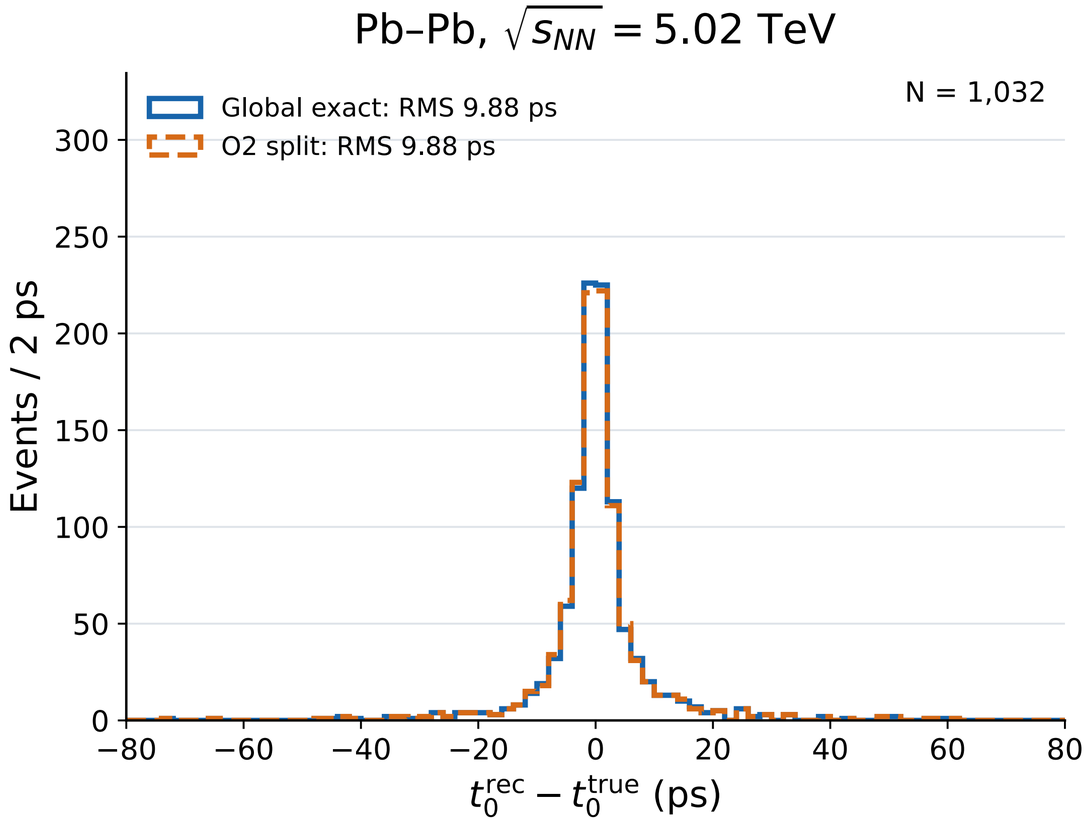




PYTHIA 8 + simple detector toy model

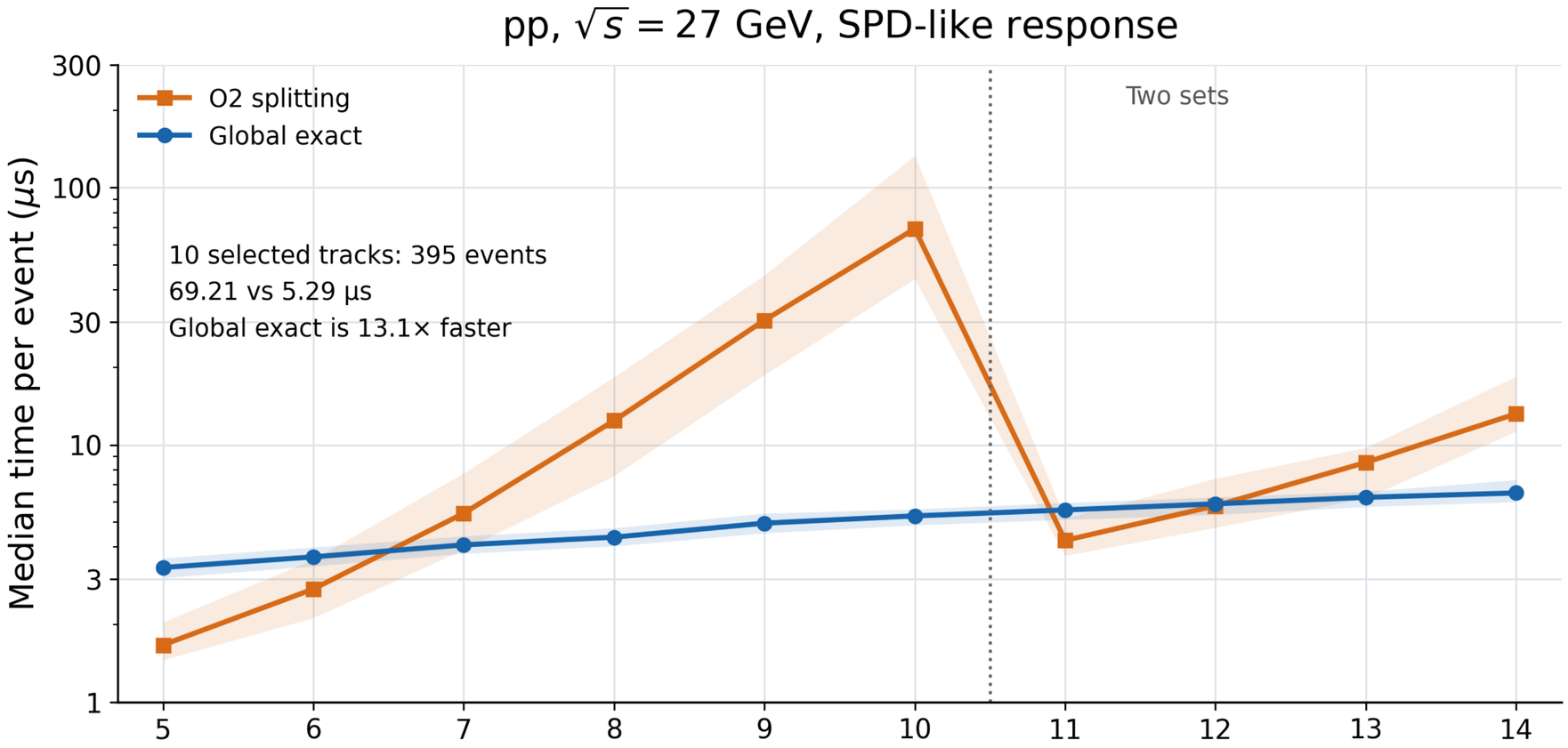
Results - 2





PYTHIA 8 + simple detector toy model

Results - 3



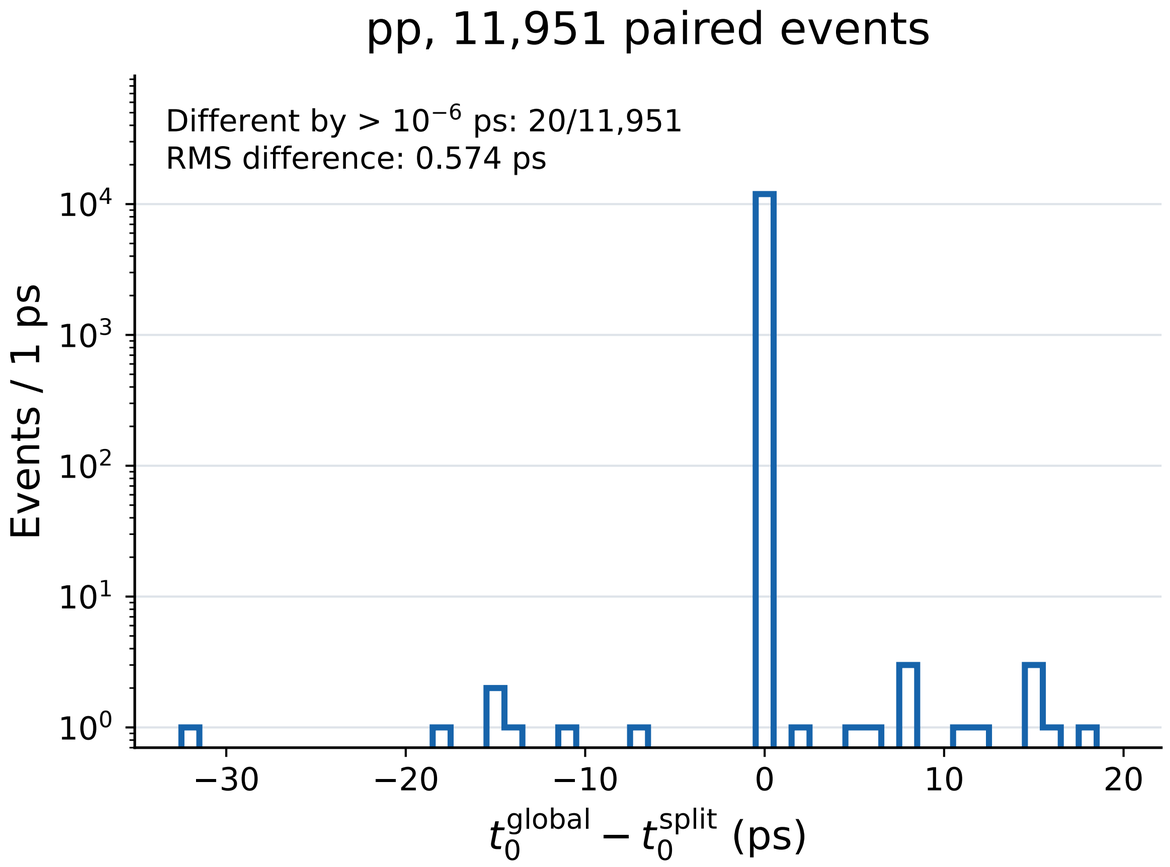
PYTHIA 8 + simple detector toy model

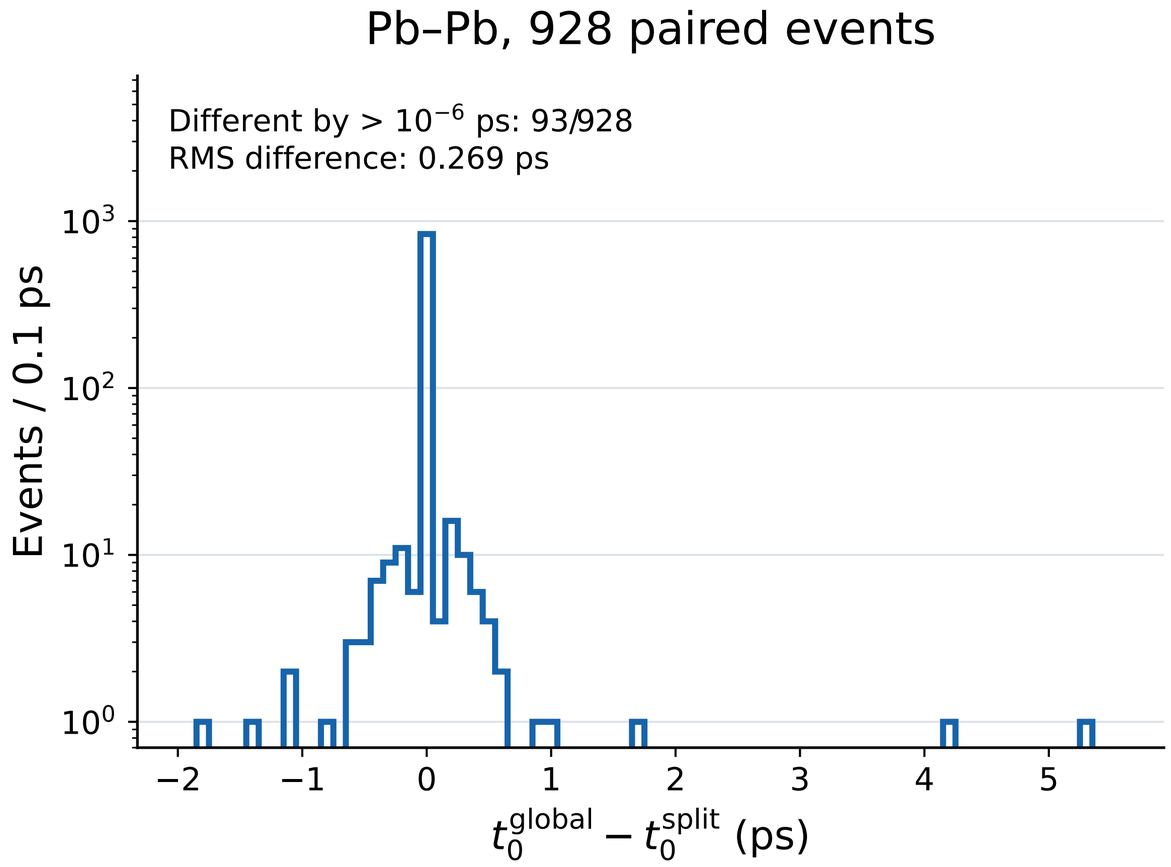
Conclusions

1. An exact $O(N \log N)$ algorithm solves the joint event-time and mass-assignment problem.
2. The implementation has practical runtimes on the tested simulated event samples.
3. The exact solution provides a reference for validating heuristic reconstruction methods.

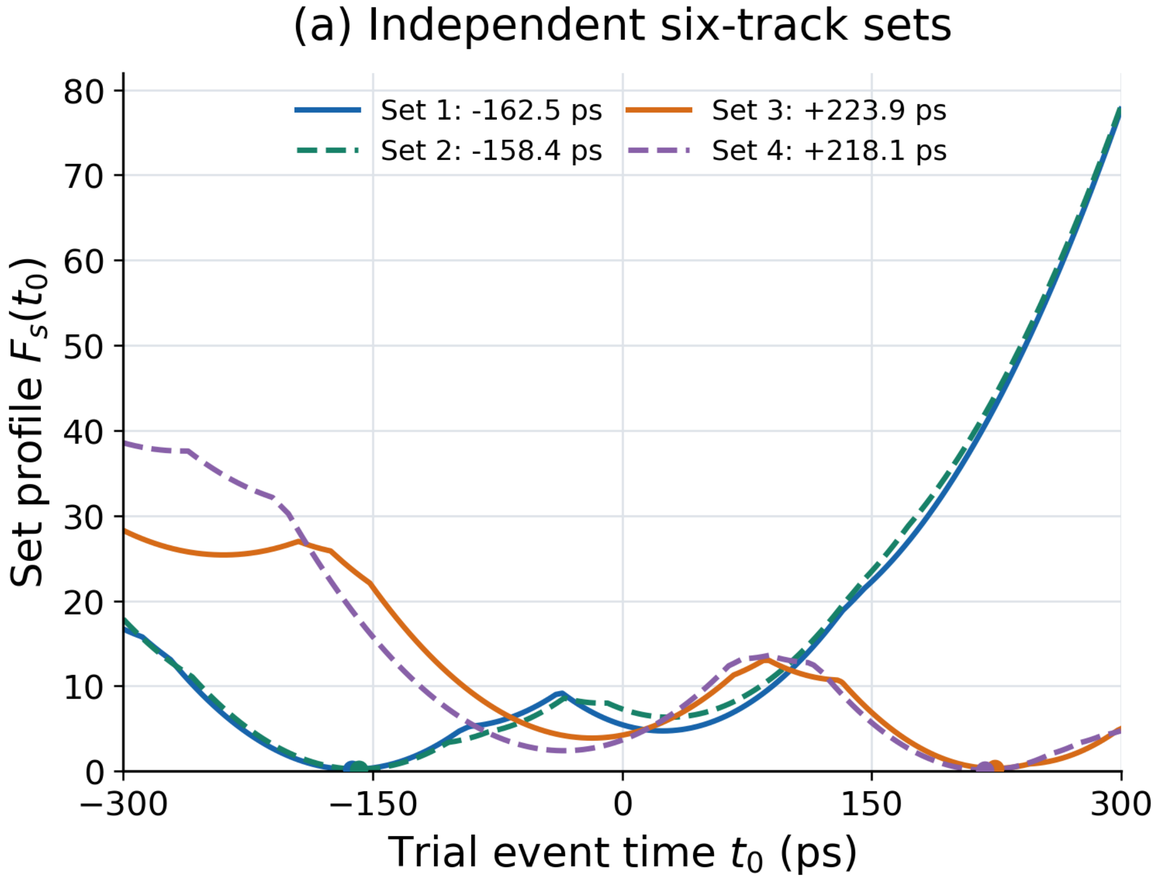
Thank you for your attention!

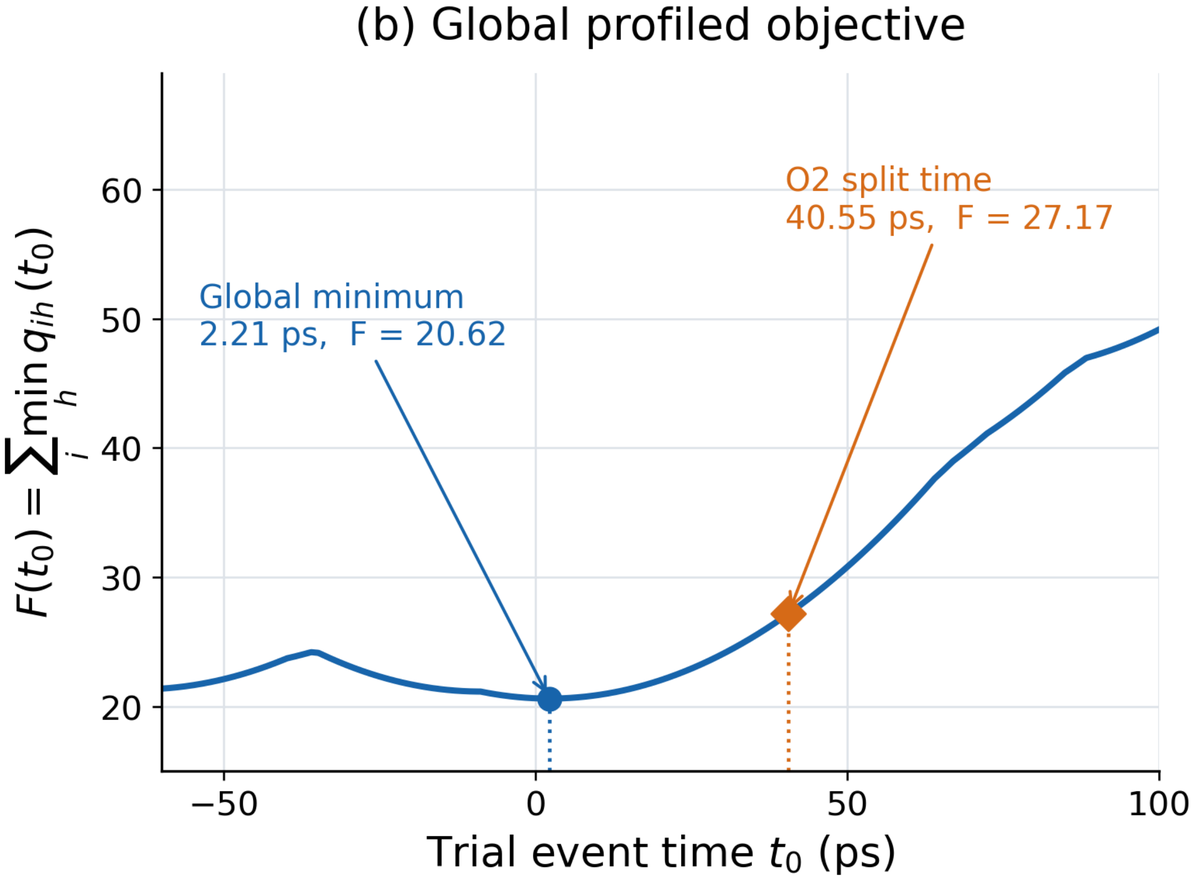
Backup - 1


pp, 11,951 paired events
Different by > 10^{-6} ps: 20/11,951
RMS difference: 0.574 ps
Events / 1 ps
10^4
10^3
10^2
10^1
10^0
−30
−20
−10
0
10
20
$t_0^{\text{global}} - t_0^{\text{split}}$ (ps)


Pb–Pb, 928 paired events
Different by > 10^{-6} ps: 93/928
RMS difference: 0.269 ps
Events / 0.1 ps
10^3
10^2
10^1
10^0
−2
−1
0
1
2
3
4
5
$t_0^{\text{global}} - t_0^{\text{split}}$ (ps)

Backup - 2





Constructed SPD-like counterexample