Tsallis fits of charged hadron spectra

Classical phenomenological Tsallis distribution

$$\frac{1}{2\pi p_T}\frac{d^2N}{dp_T dy} = \frac{gV}{(2\pi)^3} m_T \cosh(y)\left(1+(q-1)\frac{m_T\cosh(y)-\mu}{T}\right)^{-\frac{q}{q-1}}$$

The conversion from rapidity to pseudo-rapidity phase-space is made using a Jacobian (J (y, η)) of the form,

$$\frac{dN}{dp_T d\eta} = \sqrt{1-\frac{m^2}{m_T^2\cosh^2(y)}}\frac{dN}{dp_T dy} \Rightarrow \text{ at } y\approx 0 \quad \frac{dN}{dp_T d\eta} = \frac{p_T}{m_T}\frac{dN}{dp_T dy}$$

The expression of the Tsallis distribution for all charged particle pseudo-rapidity distribution at mid-rapidity can be given as,

$$\frac{1}{2\pi p_T}\frac{d^2N_{\rm ch}}{dp_T d\eta} = \frac{2p_T V}{(2\pi)^3}\sum_{i=1}^{3} g_i\left[1+(q-1)\frac{m_{T,i}-\mu}{T}\right]^{-\frac{q}{q-1}}$$

where i = (π^+, K^+, p) and $g_{\pi+}$ = 1, g_{K+} = 1 and g_p = 2. The factor 2 in front of the right hand side of this equation takes into account the contribution of the antiparticles (π^-, K^-, $\bar{p}$).

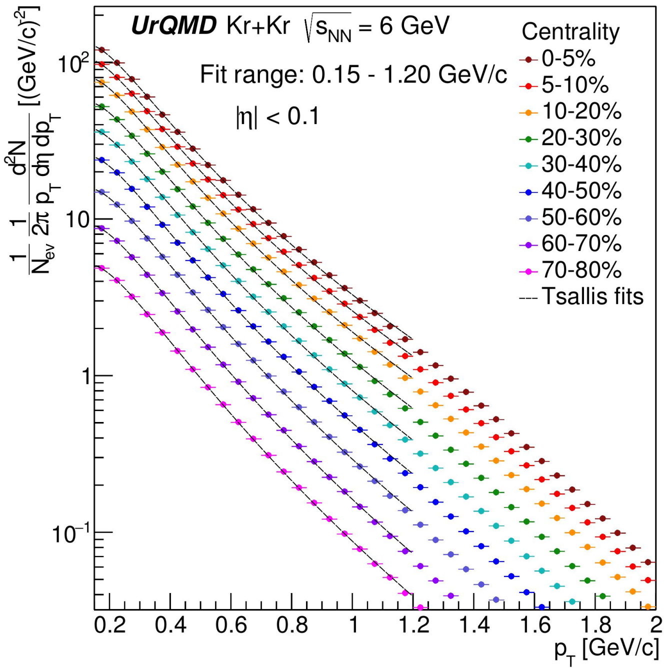
Classical phenomenological Tsallis distribution

The Tsallis distribution for all charged particle:

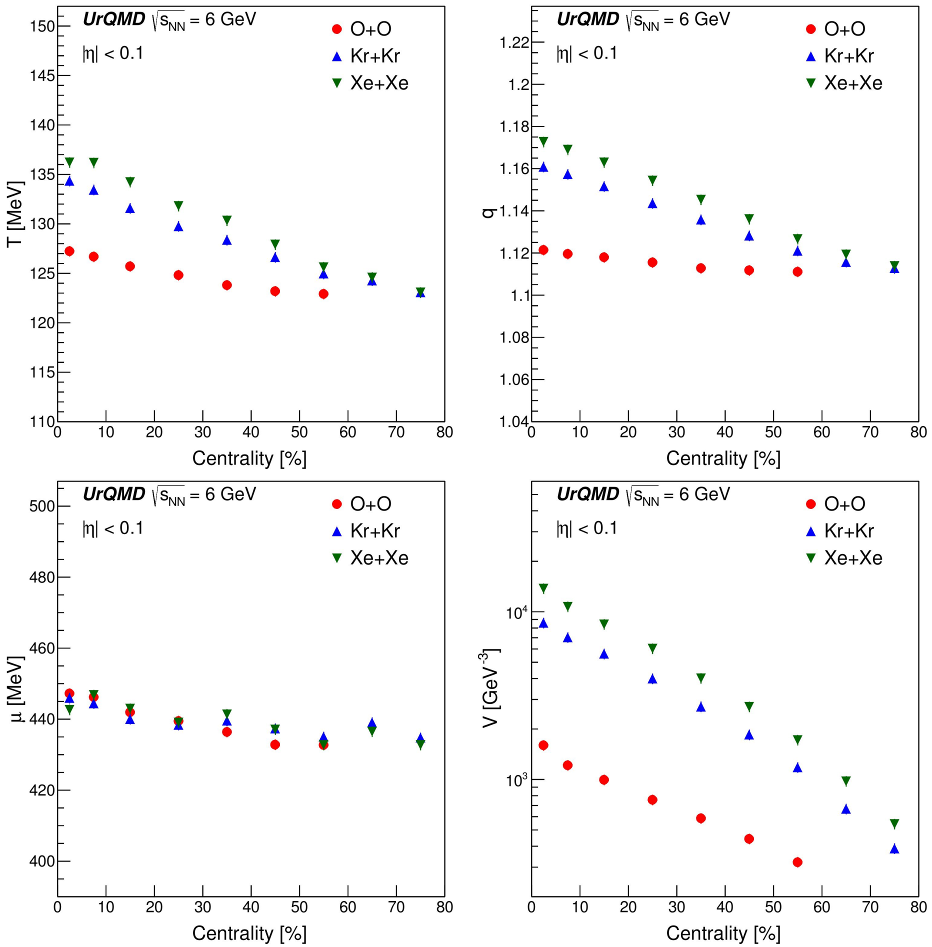
$$\frac{1}{2\pi p_T}\frac{d^2N_{\rm ch}}{dp_T d\eta} = \frac{2p_T V}{(2\pi)^3}\sum_{i=1}^{3} g_i\left[1+(q-1)\frac{m_{T,i}-\mu}{T}\right]^{-\frac{q}{q-1}}$$

- $i = (\pi^+, K^+, p)$
- $g_{\pi+} = 1$, $g_{K+} = 1$, $g_p = 2$

Fit parameters: $T,\ q,\ \mu,\ V$



Centrality dependence of extracted parameters



Tsallis fits of charged hadron spectra

Quantum phenomenological Tsallis distribution

$$\frac{d^2N}{dp_T dy} = \frac{gV}{(2\pi)^2} \frac{p_T m_T \cosh y}{\left[1-(1-q)\frac{m_T \cosh y - \mu}{T}\right]^{\frac{q}{q-1}} \pm 1}$$

$$\frac{dN}{dp_T d\eta} = \sqrt{1 - \frac{m^2}{m_T^2 \cosh^2(y)}} \frac{dN}{dp_T dy}$$ ⇨ at y ≈ 0 $$\frac{dN}{dp_T d\eta} = \frac{p_T}{m_T} \frac{dN}{dp_T dy}$$

$$\frac{1}{2\pi p_T} \frac{d^2 N_{\mathrm{ch}}}{dp_T d\eta} = \frac{2 p_T V}{(2\pi)^3} \sum_{i=1}^{3} g_i \left(\left[1 + (q-1)\frac{m_{T,i} - \mu_i}{T}\right]^{-\frac{1}{q-1}} \pm 1 \right)^{-q}$$

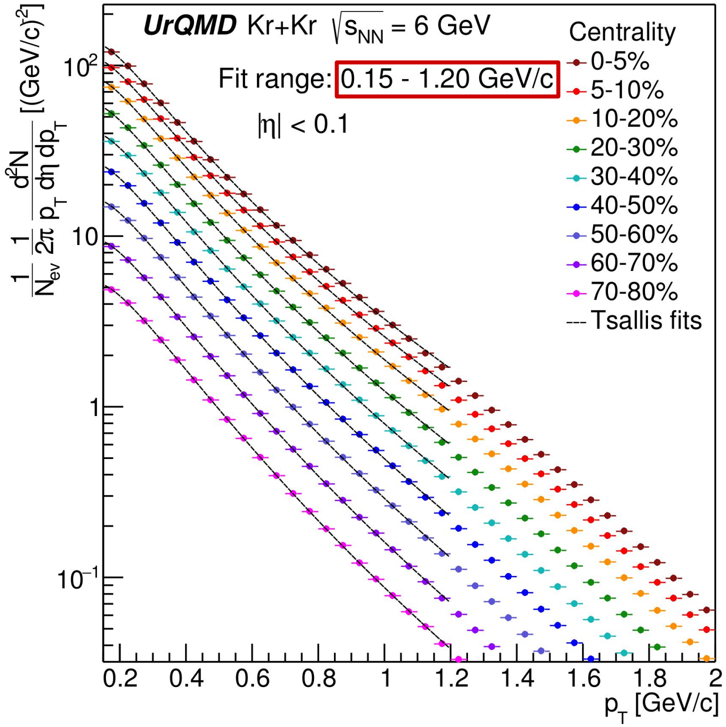
where i = (π^+, K^+, p) and $g_{\pi+}$ = 1, g_{K+} = 1 and g_p = 2. The factor 2 in front of the right hand side of this equation takes into account the contribution of the antiparticles (π^-, K^-, $\bar{p}$).

Quantum phenomenological Tsallis distribution

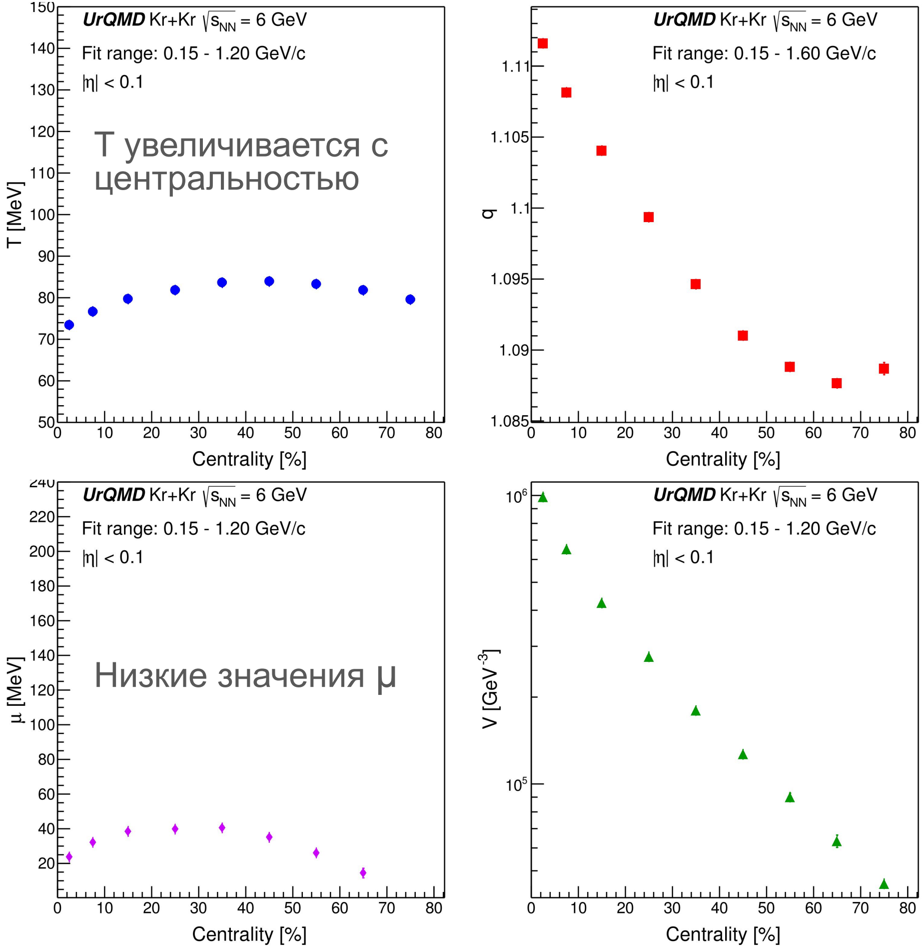
$$\frac{1}{2\pi p_T}\frac{d^2N_{\mathrm{ch}}}{dp_T d\eta} = \frac{2p_T V}{(2\pi)^3}\sum_{i=1}^{3} g_i \left(\left[1+(q-1)\frac{m_{T,i}-\mu_i}{T}\right]^{-\frac{1}{q-1}} \pm 1 \right)^{-q}$$

Fit parameters: $T,\ q,\ \mu,\ V$

μ общий для всех



Centrality dependence of extracted parameters

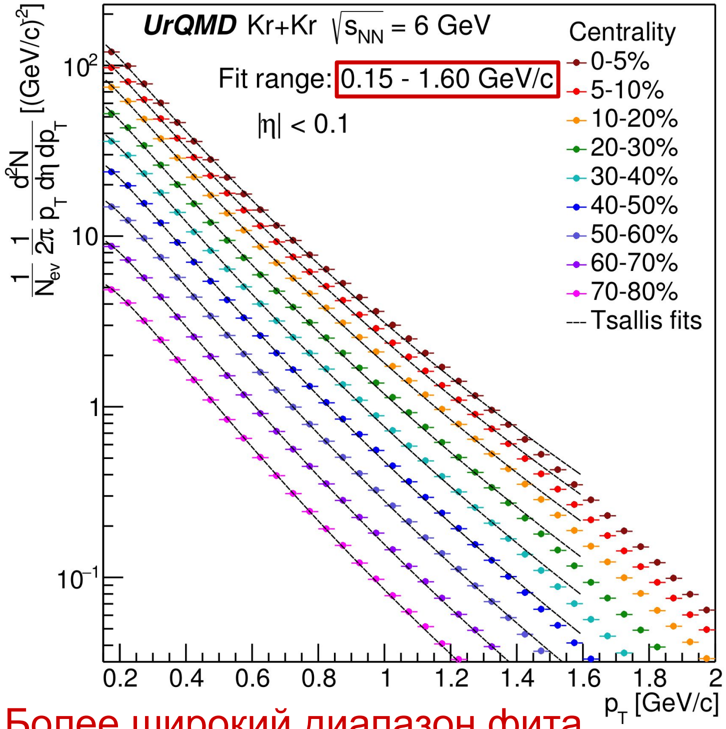


Quantum phenomenological Tsallis distribution

$$\frac{1}{2\pi p_T}\frac{d^2N_{\rm ch}}{dp_Td\eta} = \frac{2p_TV}{(2\pi)^3}\sum_{i=1}^{3} g_i\left(\left[1+(q-1)\frac{m_{T,i}-\mu_i}{T}\right]^{-\frac{1}{q-1}} \pm 1\right)^{-q}$$

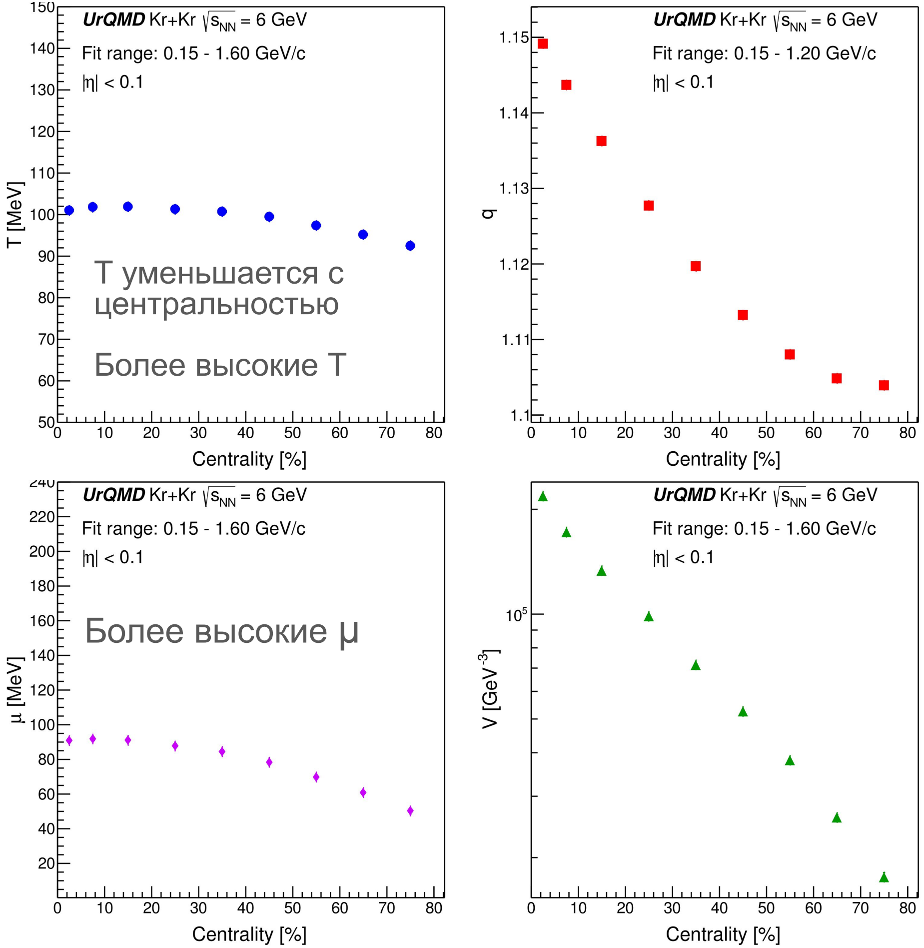
Fit parameters: $T,\ q,\ \mu,\ V$

μ общий для всех



Более широкий диапазон фита

Centrality dependence of extracted parameters



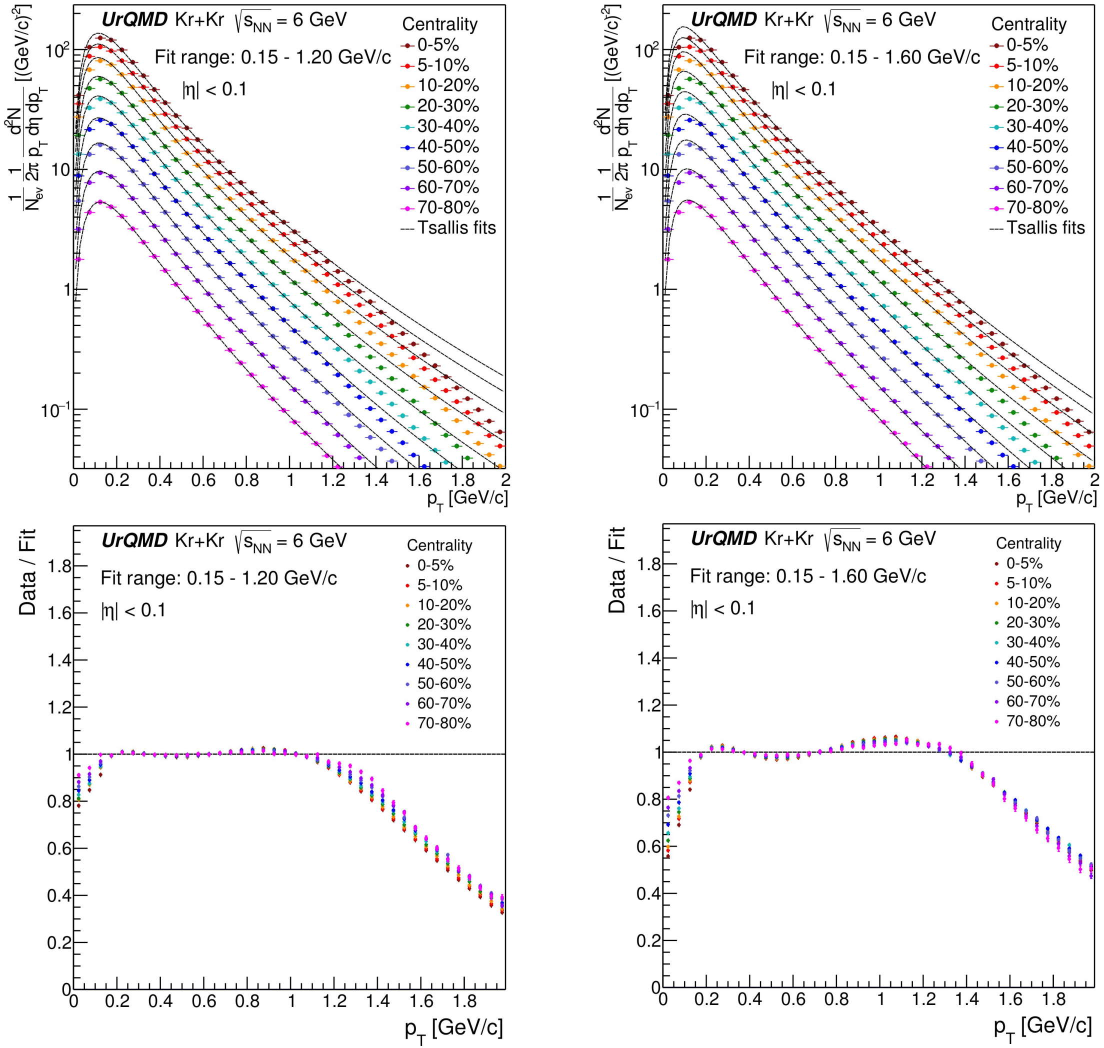
Quantum phenomenological Tsallis distribution

$$\frac{1}{2\pi p_T}\frac{d^2N_{\mathrm{ch}}}{dp_T\, d\eta} = \frac{2p_T V}{(2\pi)^3}\sum_{i=1}^{3} g_i\left(\left[1+(q-1)\frac{m_{T,i}-\mu_i}{T}\right]^{-\frac{1}{q-1}} \pm 1\right)^{-q}$$

Fit parameters: $T,\ q,\ \mu,\ V$

μ общий для всех

Отношение данных к функции фита для разных диапазонов фитирования

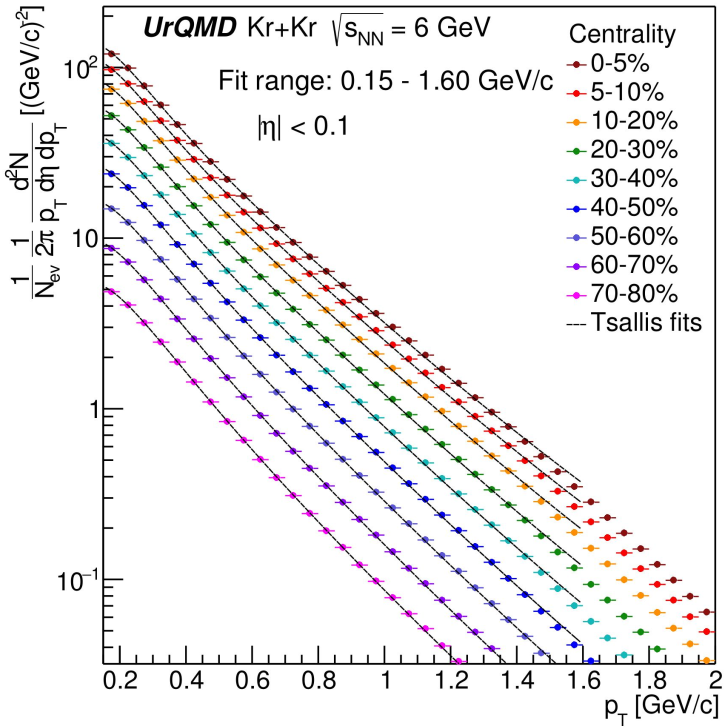


Quantum phenomenological Tsallis distribution

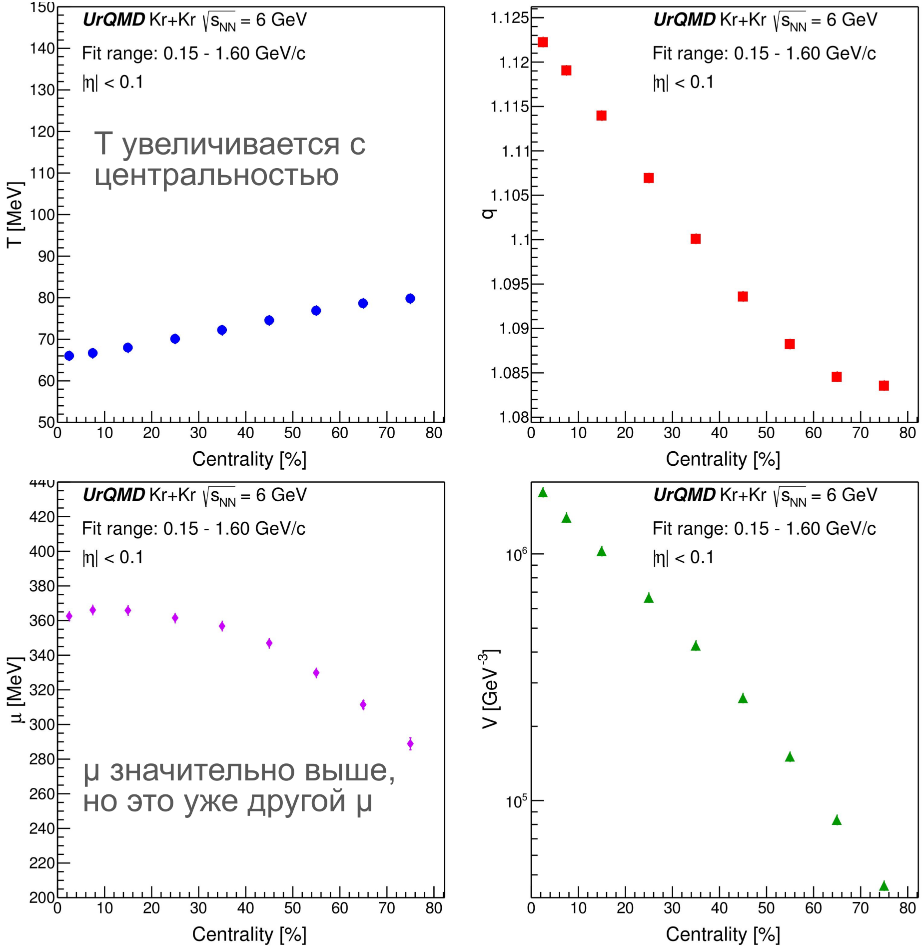
$$\frac{1}{2\pi p_T}\frac{d^2N_{\rm ch}}{dp_T d\eta} = \frac{2p_T V}{(2\pi)^3}\sum_{i=1}^{3} g_i\left(\left[1+(q-1)\frac{m_{T,i}-\mu_i}{T}\right]^{-\frac{1}{q-1}} \pm 1\right)^{-q}$$

Fit parameters: $T,\ q,\ \mu,\ V$

μ = 0 для пионов и каонов (в соотв. слагаемых, i = π^+, K^+)
μ – параметр фита в слагаемом для протонов (i = p)



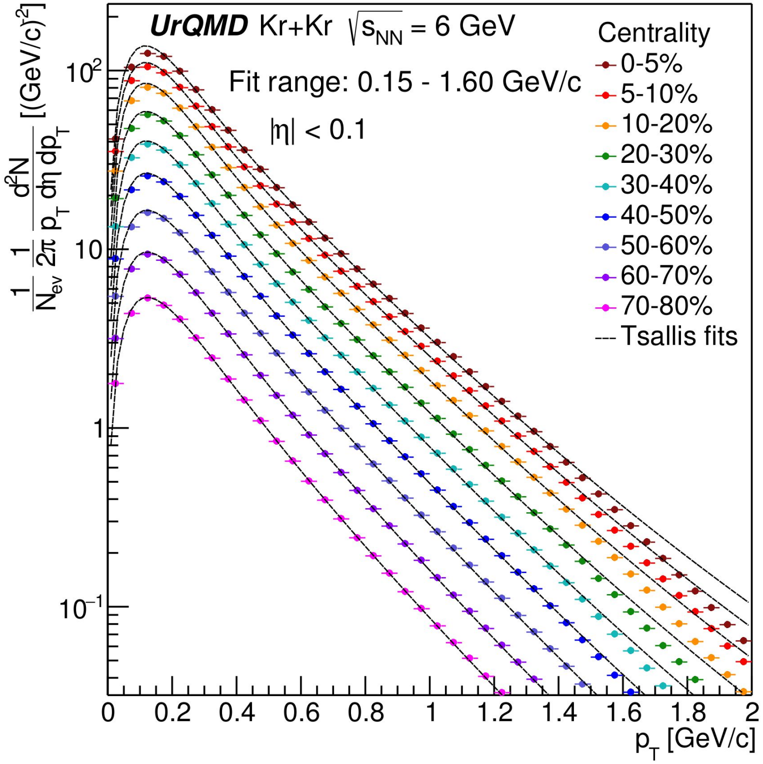
Centrality dependence of extracted parameters

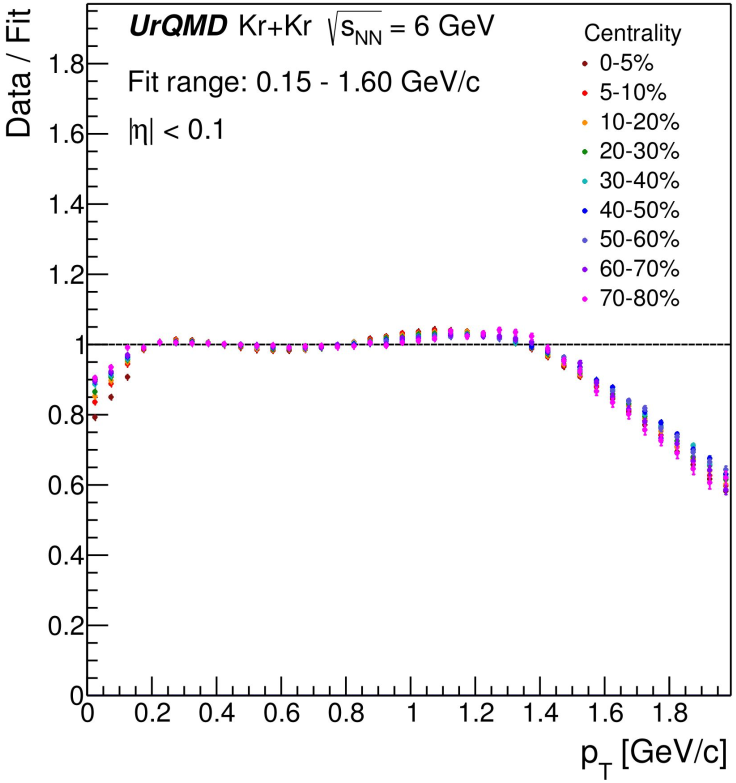


Quantum phenomenological Tsallis distribution

$$\frac{1}{2\pi p_T}\frac{d^2N_{\mathrm{ch}}}{dp_T d\eta} = \frac{2p_T V}{(2\pi)^3}\sum_{i=1}^{3} g_i\left(\left[1+(q-1)\frac{m_{T,i}-\mu_i}{T}\right]^{-\frac{1}{q-1}} \pm 1\right)^{-q}$$

μ = 0 для пионов и каонов (в соотв. слагаемых, i = π^+, K^+)
μ – параметр фита в слагаемом для протонов (i = p)





The exact Tsallis-3 classical distribution/Tsallis statistics with escort probabilities

Norm equations:

$$1 = \sum_{n=0}^{n_0} \frac{\omega^n}{n!\Gamma\left(\frac{1}{q-1}\right)} \int_0^\infty t^{\frac{2-q}{q-1}-n} e^{-t+\beta'(\Lambda+\mu n)} (K_2(\beta' m))^n dt,$$

$$\theta = \sum_{n=0}^{n_0} \frac{\omega^n}{n!\Gamma\left(\frac{q}{q-1}\right)} \int_0^\infty t^{\frac{1}{q-1}-n} e^{-t+\beta'(\Lambda+\mu n)} (K_2(\beta' m))^n dt$$

The transverse momentum distribution:

$$\frac{d^2N}{dp_T dy} = \frac{gV}{(2\pi)^2} p_T m_T \cosh y \frac{1}{\theta} \sum_{n=0}^{n_0} \frac{\omega^n}{n!\Gamma\left(\frac{q}{q-1}\right)} \int_0^\infty t^{\frac{1}{q-1}-n} \times e^{-t+\beta'(\Lambda - m_T \cosh y + \mu(n+1))} (K_2(\beta' m))^n dt.$$

Upper cut-off limit of summation: $n_0 = \left[\frac{\nu}{3}\left(\frac{2-q}{q-1}+\delta\right)\right]$

$\nu = 0.4$

$\delta = 0$

$$\beta' = \frac{-t(1-q)}{T\theta^2} \qquad \omega = \frac{gV}{2\pi^2}\frac{m^2 T\theta^2}{q-1}$$

The Maxwell-Boltzmann transverse momentum distribution of the Tsallis-3 statistics for q > 1 in the rapidity range $y_0 \le y \le y_1$:

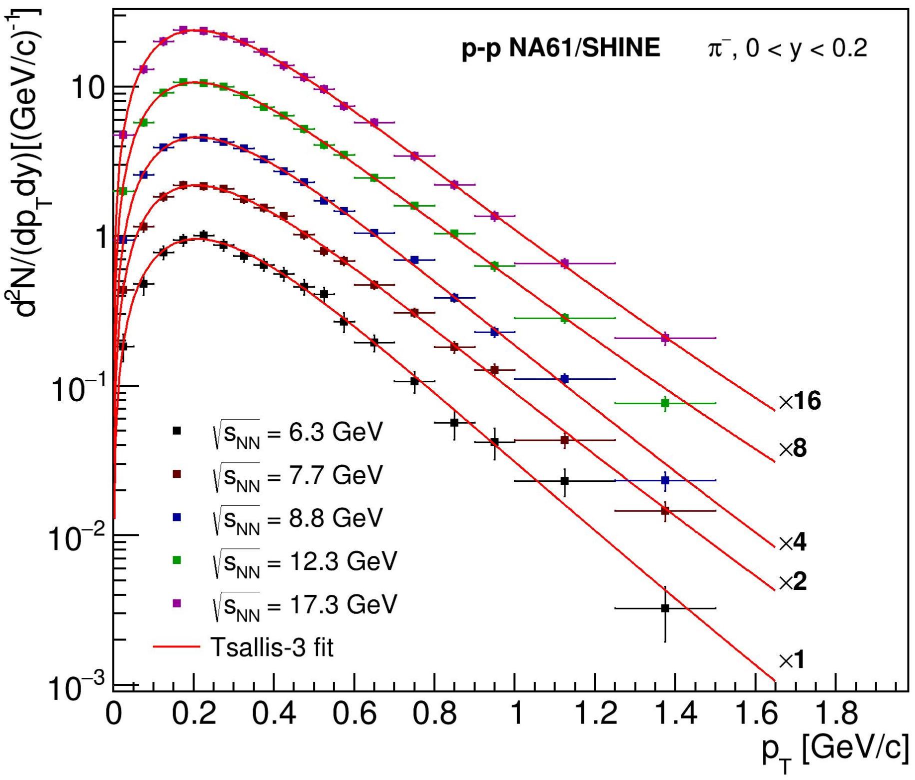
$$\left.\frac{d^2N}{dp_T dy}\right|_{y_0}^{y_1} = \frac{gV}{(2\pi)^2} p_T m_T \int_{y_0}^{y_1} dy \cosh y \frac{1}{\theta} \sum_{n=0}^{n_0} \frac{\omega^n}{n!\Gamma\left(\frac{q}{q-1}\right)} \int_0^\infty t^{\frac{1}{q-1}-n} \times e^{-t+\beta'(\Lambda - m_T \cosh y + \mu(n+1))} (K_2(\beta' m))^n dt.$$

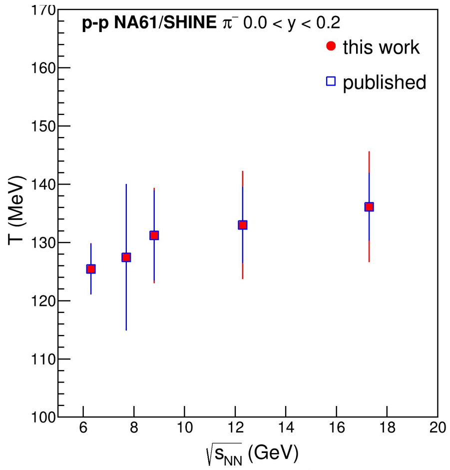
arXiv:2306.01003 [hep-ph]

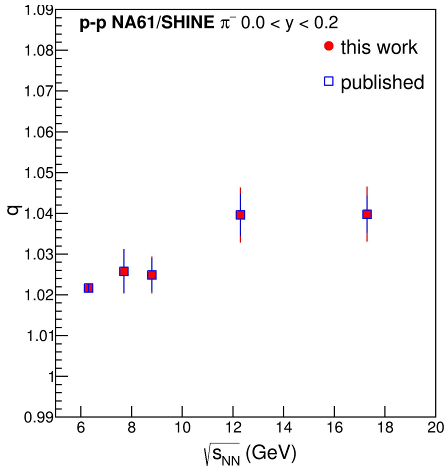
The exact Tsallis-3 classical distribution/Tsallis statistics with escort probabilities

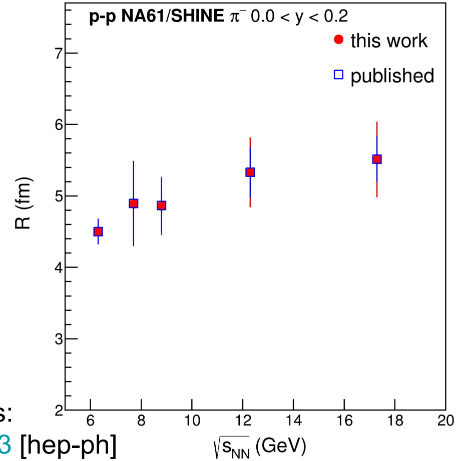
Результаты фита согласуются с опубликованными

Fit range: p_T = 0–1.4 GeV/c









Данные: Abgrall N. et al (NA61/SHINE Collaboration) 2014
Eur. Phys. J. C **74** 2794

Published results:
arXiv:2306.01003 [hep-ph]

The exact Tsallis-3 classical distribution/Tsallis statistics with escort probabilities

1 – my results

2 – published results (arXiv:2306.01003 [hep-ph])

$\sqrt{s_{NN}}$	6.3		7.7		8.8		12.3		17.3	
	1	2	1	2	1	2	1	2	1	2
T, MeV	125.45 ± 4.31	125.45 ± 4.23	127.45 ± 9.56	127.45 ± 12.49	131.20 ± 8.09	131.20 ± 7.66	133.02 ± 9.19	133.02 ± 6.47	136.13 ± 9.42	136.13 ± 5.74
q	1.0217 ± $1.9 \cdot 10^{-6}$	1.0217 ± $2 \cdot 10^{-6}$	1.0258 ± 0.0041	1.0258 ± 0.0053	1.0249 ± 0.0044	1.0249 ± 0.0039	1.0396 ± 0.0066	1.0396 ± 0.0050	1.0398 ± 0.0066	1.0398 ± 0.0044
R, fm	4.50 ± 0.17	4.502 ± 0.171	4.89 ± 0.44	4.893 ± 0.589	4.86 ± 0.40	4.864 ± 0.371	5.33 ± 0.48	5.330 ± 0.342	5.51 ± 0.52	5.515 ± 0.310
χ^2/ndf	2.70/15	2.70/15	1.15/15	1.15/15	0.82/15	0.82/15	0.77/15	0.77/15	0.44/15	0.44/15

Backup

Quantum phenomenological Tsallis distribution

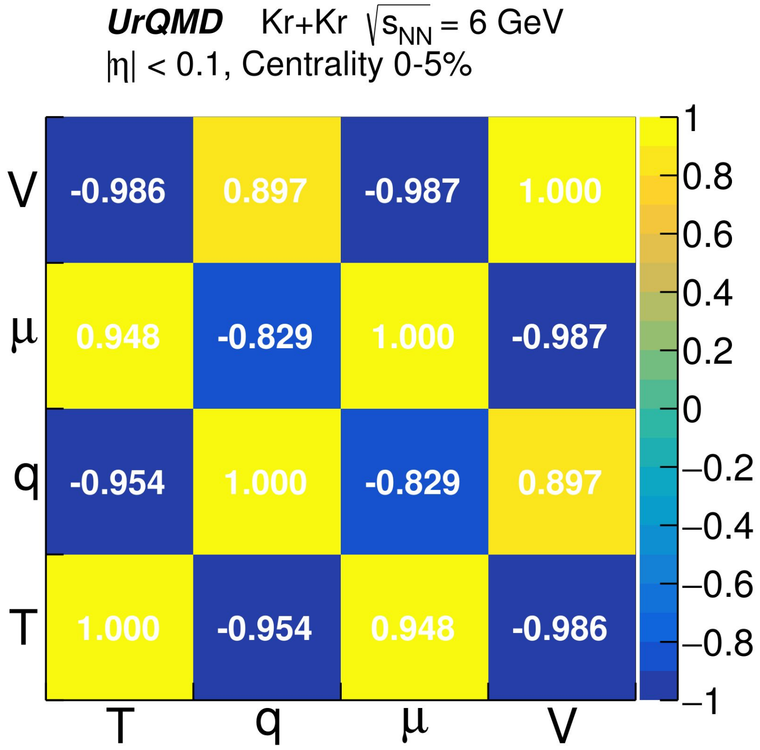
Корреляционные матрицы

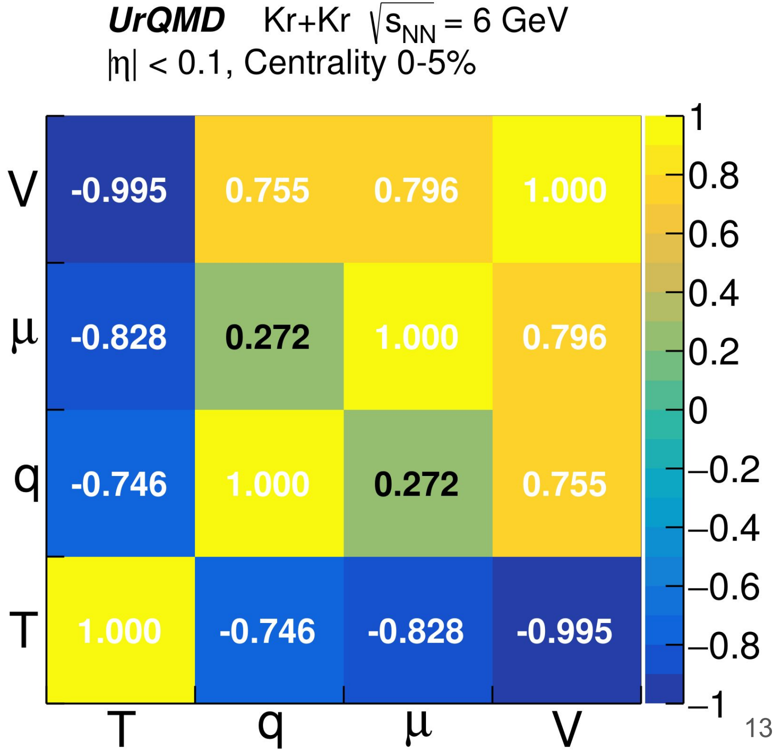
$$\frac{1}{2\pi p_T}\frac{d^2N_{\rm ch}}{dp_Td\eta} = \frac{2p_TV}{(2\pi)^3}\sum_{i=1}^{3} g_i\left(\left[1+(q-1)\frac{m_{T,i}-\mu_i}{T}\right]^{-\frac{1}{q-1}} \pm 1\right)^{-q}$$

Fit range: p_T = 0.15–1.6 GeV/c

μ – общий для всех параметр фита (есть во всех слагаемых)

μ = 0 для пионов и каонов (в соотв. слагаемых, i = π^+, K^+)
μ – параметр фита в слагаемом для протонов (i = p)

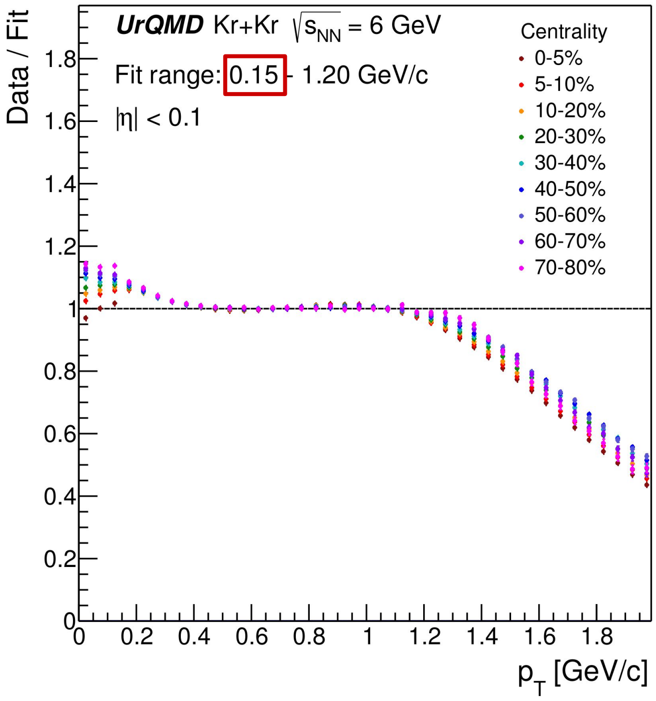


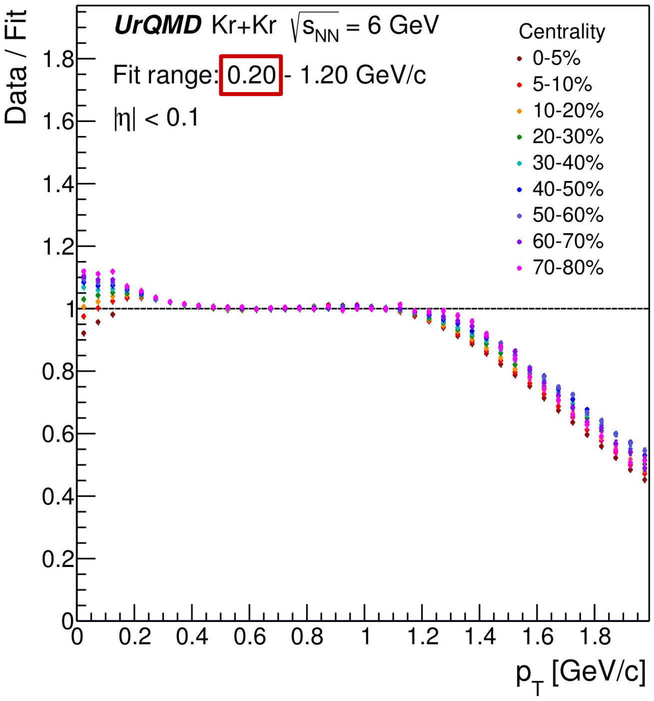


Quantum phenomenological Tsallis distribution

$$\frac{1}{2\pi p_T}\frac{d^2N_{\mathrm{ch}}}{dp_T d\eta} = \frac{2p_T V}{(2\pi)^3}\sum_{i=1}^{3} g_i\left(\left[1+(q-1)\frac{m_{T,i}-\mu_i}{T}\right]^{-\frac{1}{q-1}} \pm 1\right)^{-q}$$

μ = 0 для пионов и каонов (в соотв. слагаемых, i = π^+, K^+)
μ – параметр фита в слагаемом для протонов (i = p)





Quantum phenomenological Tsallis distribution

$$\frac{1}{2\pi p_T}\frac{d^2 N_{\mathrm{ch}}}{dp_T d\eta} = \frac{2p_T V}{(2\pi)^3}\sum_{i=1}^{3} g_i \left(\left[1 + (q-1)\frac{m_{T,i}-\mu_i}{T} \right]^{-\frac{1}{q-1}} \pm 1 \right)^{-q}$$

μ = 0 для пионов и каонов (в соотв. слагаемых, i = π^+, K^+)
μ – параметр фита в слагаемом для протонов (i = p)